Landau-Ginzburg models for complete intersections

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(joint work with Thomas Reichelt)

We report on ongoing work concerning the B-model for nef complete intersections in a smooth toric variety X_Σ. Such an intersection is given as the zero locus of a generic section of a sum $\mathcal{E} = \mathcal{L}_1 \oplus \ldots \oplus \mathcal{L}_s$ of s say, ample, line bundles $\mathcal{L}_1, \ldots, \mathcal{L}_s$. We suppose also that $-K_{X_\Sigma} - \mathcal{L}_1 - \ldots - \mathcal{L}_s$ is numerically effective. One can consider the so-called twisted Gromov-Witten invariants for these data, that is, correlators

$$\langle \alpha_1, \ldots, \widetilde{\alpha_k}, \ldots, \alpha_n \rangle_{0,n,\beta} := \int_{[\overline{\mathcal{M}}_{0,n,\beta}(X)]^{virt}} \bigcup_{i=1}^{n} \mathrm{ev}_i(\alpha_i) \cup e(\widetilde{\mathcal{E}}_k),$$

where $\widetilde{\mathcal{E}}_k$ is the bundle on $\overline{\mathcal{M}}_{0,n,\beta}(X)$ with fibre at $[C, (x_1, \ldots, x_n), f]$ being the subspace of $H^0(C, f^*\mathcal{E})$ of sections which vanishes at x_k. This gives rise to the twisted Gromov-Witten quantum product. Notice that the bilinear pairing used in the definition of this twisted quantum product defined by $(\alpha, \beta) := \int_X \alpha \cup \beta \cup e(\mathcal{E})$, hence, it is in general degenerate. One may consider the quotient $H^*(X_\Sigma, \mathbb{C})/(ker(e(\mathcal{E}) \cdot -))$ on which this pairing is non-degenerate. Then the twisted quantum product descends to the so-called reduced one. It is known (see [CG07]) that the reduced one is the quantum product on the *ambient cohomology* of the subvariety defined by a generic section of $\mathcal{E}$.

Classical constructions of Dubrovin and Givental associate a family of holomorphic vector bundles on $\mathbb{P}^1$ to the quantum cohomology of a smooth projective variety, this is the so-called quantum $\mathcal{D}$-module. A similar construction exists for the twisted and reduced invariants, and a concrete description of these $\mathcal{D}$-modules in the toric case has recently been given in [MM11]. The aim of our work is to reconstruct these differential systems from an appropriate Landau-Ginzburg model. For this, we rely on our earlier paper [RS10] in which we describe the Gauß-Manin system of a generic family of Laurent polynomials by hypergeometric differential equations. For a complete intersection, the corresponding Laurent polynomials are constructed from an extended fan Σ', which is the fan of the total bundle of $\oplus_{j=1}^s \mathcal{L}_j^{-1}$. For the family of Laurent polynomials $\varphi_{\Sigma'} : (\mathbb{C}^*)^{n+s} \times \mathbb{C}^{m+s} \to \mathbb{C} \times \mathbb{C}^{m+s}$ obtained we consider its so-called twisted de Rham cohomology, namely, the $n+s$-th cohomology $H^{n+s}(\varphi)$ of the complex $(\Omega^\bullet_{pr}[z], zd - d\varphi\wedge)$, where $\Omega^\bullet_{pr}$ is the relative de Rham complex $\Omega^\bullet_{(\mathbb{C}^*)^{n+s} \times \mathbb{C}^{m+s}/\mathbb{C}^{m+s}}$. We can relate this $\mathcal{D}$-module to the twisted quantum-$\mathcal{D}$-module from [MM11], and the reduced quantum-$\mathcal{D}$-module from loc.cit. can also be described as a certain intersection complex. We can conclude, using [Sab08], that it underlies a variation of non-commutative pure polarized Hodge structures.

References

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