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Welfare Effects of Commodity Taxation

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ABSTRACT

In reality firms most often face negatively sloped demand curves. Then, for a given level of consumers' surplus, levies on prices yield higher fiscal revenues than specific duties. Therefore, according to the prevailing view, the switch from unit to ad valorem taxation is supposed to generate more welfare; some even speak of an associated Pareto-improvement. However, this is not true because taxing prices merely transfers profits to the Treasury, while total rent remains unaffected. Since excise duties diminish the welfare gain in comparison with untaxed trade, an appropriately designed income tax allows all parties to benefit. Sales should be taxed only exceptionally.

KEYWORDS

Commodity taxation, Unit taxation, Ad valorem taxation

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1. Alternative taxes on expenditures

Duties on trade in goods are among the oldest forms of taxation whereas levies on personal earnings only spread after the Industrial Revolution.[1] Temporarily, income tax was even regarded as 'queen of taxes' because of a potentially high revenue and, if progressively designed, the suitability for redistribution. In a neoclassical framework, the 'Atkinson-Stiglitz theorem' supports this view: '… where the utility function is separable between labor and all commodities, no indirect taxes need be employed' (Atkinson and Stiglitz 1976, p. 74).

The subsequent discussion, however, has confirmed once again that different (more or less convincing) assumptions produce different results. After all, a contribution by Saez reversed the Atkinson-Stiglitz-theorem: under non-linear income taxation and heterogeneous tastes, '… commodity taxation is a tool that allows the government to expand its taxation power and is therefore desirable' (Saez 2002, p. 228).

One advantage of indirect taxes is surely that they can be collected quite easily at the manufacturer or at the point of sale without considering the buyer's ability to pay. Yet, taxes on consumption generate regressive repercussions as low earners normally spend less in absolute terms but a relatively high share of their income, i.e. they save little if not nothing at all. This finding has been tried and tested empirically long since: '... the poorer a family is', wrote the Saxon statistician Ernst Engel more than 150 years ago, 'the greater must be the part of total expenditures devoted to the procurement of food ...'[2]

Nevertheless, a significant part of public revenue stems from the taxation of commercial trade.[3] At least, the concomitant regressive effects are taken into account by different rates for specified goods, such as a lower value added tax rate on expenses deemed essential.[4] Obviously, this approach creates a variety of definition problems as some bizarre examples illustrate.[5] In the following, we abstract from such matters of detail.

The analysis rests on the concept of *economic* exchange acts characterized by voluntariness and no personal preferences on both sides. Thus, such bargains certainly increase the business partners' utility (cf. Helmedag 2018, pp. 41-51). In order to avoid an inappropriate reduction of possible individual gains by taxing transactions it is necessary to know the exact welfare consequences for all parties involved.

In principle, excise duties occur in two alternative forms. They appear either as fixed amounts of money on physical units (e.g. cigarettes and sparkling wine) or as percentage markups on net prices ('ad valorem').[6]

Textbooks usually assert that the effects of both methods only coincide in the ideal case of perfect competition. Under realistic market conditions, however, a price tax is said to be superior to a quantity tax since the former entails benefits for all agents affected, i.e. firms, consumers and the government. Knut Wicksell enunciated this allegation quite early for a single seller facing a falling linear demand curve.[7] Suits and Musgrave (1953, p. 598-9) express the claimed advantageousness of ad valorem taxation by an often-adopted enumeration:

'1. The yield from any given unit tax is always smaller than the yield from the ad valorem tax which would result in the same final output and price.

2. The maximum yield which may be obtained from of a unit tax is smaller than the maximum yield possible from an ad valorem tax.

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1 Cf. Kiser and Karceski (2017) for an outline of taxation in premodern and modern times.

2 Cf. Engel (1857, p. 169) [own translation]. Schwabe (1870) makes an analogous statement about rented accommodations.

3 By now, about 45 % of the German fiscal revenue are indirect taxes (cf. Bundesministerium für Finanzen 2021). Stewart (2022, p. 17) provides the tax mix of selected OECD countries.

4 This reasoning corresponds to the separation into basic and luxury goods, which are financed out of different sources of disposable income, i.e. wages and profits. Cf. Helmedag (2019).

5 For instance, according to a judgment of the German Federal Finance Court, the pig ears dried in a fodder plant are also edible offal to which the reduced value added tax rate has to be applied. Cf. Bundesfinanzhof (2015).

6 Yet, the cumulative use of both types of taxation also happens as in the case of the former 'mineral oil tax' in Germany, which was renamed 'energy tax' in 2006.

7 Cf. Wicksell (1896, p. 10 ff.) With regard to the relevance of the analysis the author states in the preface: '… monopoly rent is not at all only an occasional phenomenon but occurs more or less in every trade …' (Wicksell 1896, p. IV, footnote 2) [own translation].

3. If the same yield is obtained from a unit and an ad valorem tax, the final price will be higher (the output smaller) under the unit tax.'

A more recent article summarizes: '... for any unit tax imposed on a monopoly, there exists an ad valorem tax that Pareto dominates it – that produces larger profit, tax revenue, and consumer surplus' (Skeath and Trandel 1994, p. 53). Meanwhile, it is considered a secure fact that in the case of a monopoly '... an ad valorem tax is welfare superior to a unit tax ...' (Anderson et al. 2001, p. 232). Of course, in reality a monopoly in its purest form is rare whereas often 'monopolistic competition' prevails. Such circumstances find expression in a falling demand function allowing autonomous price policy, at least in a certain interval (cf. Helmedag 1982). As a result, quantity taxation should be eliminated because this practice never engenders better results than value taxation.[8]

Actually, the assessment of such a policy requires thorough knowledge of the alternative quantitative outcomes for every group affected. Alas, the usual and quite generally formulated comparisons only provide incomplete information about the order of magnitudes. In contrast, the model below precisely indicates the welfare changes for all involved participants. Consequently, it is possible to derive conclusions for an expedient taxation of goods.

2. The reference market before taxation

Following Wicksell (and many others), we consider a representative supplier facing a linear relationship between the physically measured absolute output in a given period (q^{abs}) and a unit price expressed in money (p^{abs}). Furthermore, q_S^{abs} denotes the saturation quantity and *m* the slope of the falling demand curve:

$$q^{abs} = q_S^{abs} - mp^{abs} \tag{1}$$

The inverse demand function is interpreted as marginal willingness to pay:

$$p^{abs} = \frac{1}{m}\left(q_S^{abs} - q^{abs}\right) \tag{2}$$

The prohibitive or reservation price p_R^{abs} corresponds to a vanishing quantity:

$$p_R^{abs} = p^{abs}(0) = \frac{q_S^{abs}}{m} \tag{3}$$

Dividing equation (2) by expression (3) provides a link between price *p* as part of the maximum price and output *q* as a share of the saturation quantity[9]:

$$p = \frac{p^{abs}}{p_R^{abs}} = 1 - \frac{q^{abs}}{q_S^{abs}} = 1 - q \tag{4}$$

In order to simplify matters, we abstract from fixed costs and assume that marginal costs (c) are a constant percentage of the normalized reservation price ($0 < c < 1$). Profits (P) emerge as difference between revenue (R) and variable costs (C):[10]

$$P = R - C = pq - cq = (1 - q)q - cq = (1 - q - c)q \tag{5}$$

[8] A variety of studies examine the effects of ad valorem versus quantity taxation considering specifically differences in quality or costs, incomplete competition, uncertainty, etc. It may suffice here to mention Hoffmann and Runkel (2016) plus Li and Liu (2021). Besides, these articles provide further references to the relevant literature.

[9] Helmedag (2012) argues that in the absence of preferences monopoly pricing, i.e. marginal revenue equals marginal costs, usually arises irrespective of the number of suppliers.

[10] For positive fixed costs, *P* denotes the gross margin.

Profit maximization requires:

$$\frac{dP}{dq} = (1 - q - c) - q = 1 - 2q - c \stackrel{!}{=} 0 \tag{6}$$

The corresponding optimal quantity (q^*) amounts to[11]:

$$q^* = \frac{1 - c}{2} \tag{7}$$

The associated price (p^*) according to the normalized demand curve (4) is:

$$p^* = \frac{1 + c}{2} \tag{8}$$

In the absence of fixed costs, the maximum profit (P^*) becomes:

$$P^* = (p^* - c)q^* = \left(\frac{1 + c}{2} - c\right)\frac{1 - c}{2} = \frac{(1 - c)^2}{4} = \frac{8(1 - c)^2}{32} \tag{9}$$

The last fraction in equation (9) facilitates later comparisons. The consumers' monetary surplus, sometimes called 'rent', is obtained by the difference between the potential and actual expenditure. In case of a linear demand function, the formula for the area of a triangle can be employed to calculate the buyers' advantage (CR) at point q^*:

$$CR^* = \frac{1}{2}q^*(1 - p^*) = \frac{1}{2}q^*\big(1 - (1 - q^*)\big) = \frac{1}{2}(q^*)^2 = \frac{4(1 - c)^2}{32} \tag{10}$$

The supplier's profit is twice as high as the consumers' surplus. Total welfare (W^*) in the reference scenario adds up to:

$$W^* = P^* + CR^* = \frac{8(1 - c)^2}{32} + \frac{4(1 - c)^2}{32} = \frac{12(1 - c)^2}{32} \tag{11}$$

Now the Treasury comes into play. In the following, it remains an open question whether public expenditures will benefit the citizens more than they lose consequent upon taxation. Thus, issues concerning the size of fiscal revenue and its allocation are disregarded. The considerations below only deal with the consequences of taxing goods, which can be a markup imposed either on units sold or on revenues received. Hence, we speak of quantity or price taxation. The standardized reference market provides a yardstick for an assessment of the alternative levies.

3. Unit taxation

The surcharge of an *absolute* amount of money t_q to the net or producer price (p_q^n) gives the gross price (p) of a unit:

$$p = p_q^n + t_q \tag{12}$$

Inserting the final price in demand function (4) yields the related output(q_q):

$$p = 1 - q_q \tag{13}$$

[11] The sufficient condition applies. A linear marginal revenue curve intersects the output axis at half the saturation quantity. This is not always taken into account when value and unit taxes are graphically compared (see, for instance, Stiglitz (2000), p. 517).

The untaxed net price results from equations (12) and (13):

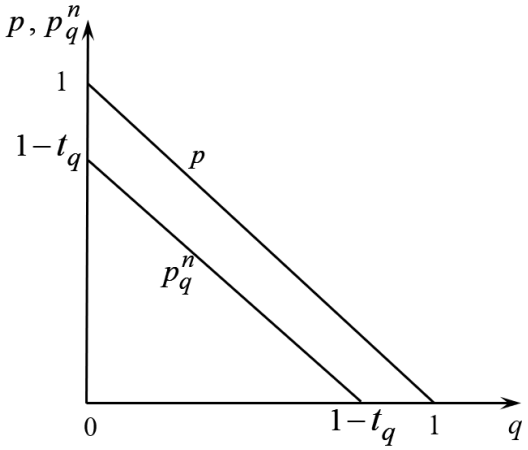
$$p_q^n = 1 - q_q - t_q \tag{14}$$

Rearranging the above expression informs about the sales volume:

$$q_q = 1 - p_q^n - t_q \tag{15}$$

Figure 1 shows that a unit tax shifts the demand curve parallel towards the origin.

Figure 1. The effect of a quantity tax.



Profits (P_q) amount to:

$$P_q = \left(p_q^n - c\right)q_q = \left(1 - q_q - t_q - c\right)q_q \tag{16}$$

From the optimization condition, $\frac{\partial P_q}{\partial q_q} \stackrel{!}{=} 0$, we get:

$$1 - t_q - 2q_q = c \tag{17}$$

Thus, the profit maximizing output $(q_q^*(t_q))$ reads:

$$q_q^*(t_q) = \frac{1 - c - t_q}{2} \geq 0 \; for \; 1 - c \geq t_q \tag{18}$$

Evidently, $q_q^*(t_q)$ falls with a rising quantity tax t_q up to its maximum (1 – *c*).
The tax revenue sums up to:

$$T_q = t_q\left(q_q^*(t_q)\right) = \frac{t_q\left(1 - c - t_q\right)}{2} \tag{19}$$

If we assume that the Treasury knows marginal costs *c*, the necessary condition for the greatest tax income is:

$$\frac{\partial T_q}{\partial t_q} = \frac{1 - 2t_q - c}{2} \stackrel{!}{=} 0 \tag{20}$$

Since the sufficient condition holds, one immediately calculates for the revenue maximizing unit tax rate (t_q^*):

$$t_q^* = \frac{1 - c}{2} \tag{21}$$

Resubstituting levy (21) into equation (18) gives the matching quantity (q_q^*):

$$q_q^* = \frac{1-c}{4} \tag{22}$$

Equation (14) now provides the profit optimizing net price (p_q^{n*}):

$$p_q^{n*} = 1 - \frac{1-c}{4} - \frac{1-c}{2} = \frac{3c+1}{4} \tag{23}$$

The gross price (p_q^*) corresponding to the maximum yield of a quantity tax is:

$$p_q^* = p_q^{n*} + t_q^* = \frac{3c+1}{4} + \frac{1-c}{2} = \frac{3+c}{4} \tag{24}$$

Using equation (16) the monopolist receives profit (P_q^*) to the tune of:

$$P_q^* = \frac{2(1-c)^2}{32} \tag{25}$$

The finance authority collects the maximum unit tax yield T_q^*:

$$T_q^* = t_q^* q_q^* = \left(\frac{1-c}{2}\right)\left(\frac{1-c}{4}\right) = \frac{4(1-c)^2}{32} \tag{26}$$

Obviously, by means of a quantity tax, the Treasury can earn maximum revenue equal to twice the profit. Again, consumers' surplus CR_q^* accounts for:

$$CR_q^* = \frac{1}{2} q_q^* \left(1 - p_q^*\right) = \frac{1}{2} q_q^* \left(1 - \left(1 - q_q^*\right)\right) = \frac{1}{2}\left(q_q^*\right)^2 = \frac{(1-c)^2}{32} \tag{27}$$

As in the case without taxation, the welfare increase of customers comes to half of the producer's rent.
The total welfare W_q^* arising from revenue maximizing unit taxation adds up to:

$$W_q^* = P_q^* + T_q^* + CR_q^* = \frac{7(1-c)^2}{32} \tag{28}$$

A look at equation (11) indicates that the overall surplus has fallen to 7/12 = 58.33 % of the tax-free situation.

4. Ad valorem taxation

Now we consider a levy that increases the net price of an item (p_p^n) by a *percentage* markup (t_p) yielding gross price (p):

$$p = p_p^n \left(1 + t_p\right) \tag{29}$$

As before, the normalized demand curve (4) applies:

$$p = 1 - q_p \tag{30}$$

Equalizing expressions (29) and (30) gives the net price in case of ad valorem taxation (p_p^n):

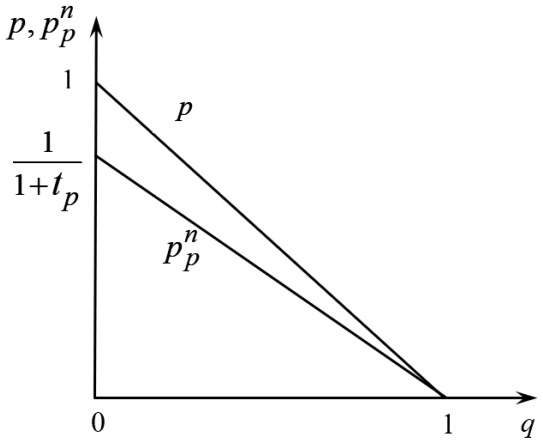
$$p_p^n = \frac{1-q_p}{1+t_p} \tag{31}$$

The corresponding quantity (q_p) emerges as:

$$q_p = 1 - p_p^n(1 + t_p) \tag{32}$$

Figure 2 illustrates that, under the present circumstances, the firm's price-sales function rotates inwards around the saturation quantity.

Figure 2. The effect of a price tax.



Profits (P_p) with price taxation run up to:

$$P_p = (p_p^n - c)q_p = \left(\frac{1 - q_p}{1 + t_p} - c\right)q_p \tag{33}$$

The necessary condition for an optimum reads:

$$\frac{\partial P_p}{\partial q_p} = \frac{1 - q_p}{1 + t_p} - c - \frac{q_p}{(1 + t_p)} \stackrel{!}{=} 0 \tag{34}$$

Hence, the trading volume is:

$$q_p^*(t_p) = \frac{1 - c(1 + t_p)}{2} \tag{35}$$

Inserting quantity (35) in equation (32) gives, after rearranging, the net price:

$$p_p^{n*}(t_p) = \frac{1 + c(1 + t_p)}{2(1 + t_p)} \tag{36}$$

From equations (35) and (36) the maximum profit $(P_p^*(t_p))$ ensues:

$$P_p^*(t_p) = \left(\frac{1 + c(1 + t_p)}{2(1 + t_p)} - c\right)\frac{1 - c(1 + t_p)}{2} = \frac{(c(1 + t_p) - 1)^2}{4(1 + t_p)} \tag{37}$$

If consumers are to pay the same final price (p_p^*) as with quantity taxation (p_q^*), the following requirement holds:

$$p_p^* = p_p^{n^*}(1 + t_p) = \frac{1 + c(1 + t_p)}{2} \stackrel{!}{=} \frac{3 + c}{4} = p_q^* \tag{38}$$

Solving entails the associated ad valorem tax rate (t_p^*):

$$t_p^* = \frac{1-c}{2c} \tag{39}$$

In consideration of the tax rate (39), equation (36) determines the appertaining net price (p_p^{n*}):

$$p_p^{n*} = \frac{1 + c\left(1 + \frac{1-c}{2c}\right)}{2\left(1 + \frac{1-c}{2c}\right)} = \frac{c(3+c)}{2(1+c)} \tag{40}$$

The corresponding quantity (q_p^*) reads:

$$q_p^* = \frac{1 - c\left(1 + \frac{1-c}{2c}\right)}{2} = \frac{1-c}{4} \tag{41}$$

Substituting t_p^* in equation (37) specifies profits (P_p^*), which only depend on marginal costs:

$$P_p^* = \frac{4c(1-c)^2}{32(1+c)} \tag{42}$$

The tax revenue (T_p^*) comes to:

$$T_p^* = t_p^* p_p^{n*} q_p^* = \left(\frac{1-c}{2c}\right)\left(\frac{c(3+c)}{2(1+c)}\right)\left(\frac{1-c}{4}\right) = \frac{(1-c)^2(2c+6)}{32(1+c)} \tag{43}$$

The consumers' surplus (CR_p^*) for a burden on units equals the buyers' rent under ad valorem taxation (27) since quantity and gross price coincide:

$$CR_p^* = CR_q^* = \frac{(1-c)^2}{32} \tag{44}$$

Then, total welfare adds up to:

$$W_p^* = CR_p^* + P_p^* + T_p^* = \frac{(1-c)^2}{32} + \frac{4c(1-c)^2}{32(1+c)} + \frac{(1-c)^2(2c+6)}{32(1+c)} = \frac{7(1-c)^2}{32} \tag{45}$$

Now the expressions are derived that allow a direct comparison of the two alternatives for taxing trade.

5. Putting commodity taxation to the test

Table 1 on the next page contains the relevant information for assessing the outcomes from a levy on quantities or prices, respectively. The third column reports the performance of untaxed trade. The figures in parentheses refer to the equation numbering.

The starting point of this study was the calculation of the maximum revenue from a unit tax and the associated price-quantity combination. An appropriately chosen ad valorem tax rate ensures the same output and an identical final price. Thus, consumers in both cases enjoy the same surplus and harbor therefore no preference for one or the other excise duty.

On this basis, the first proposition in the enumeration presented at the beginning can be examined. What about yields from alternative excise duties? The answer is given by the difference between equations (43) and (26), which indicate the fiscal revenue corresponding to price or unit taxation:

$$T_p^* - T_q^* = \frac{(1-c)^2(2c+6)}{32(1+c)} - \frac{4(1-c)^2}{32} = -\frac{(c-1)^3}{16(1+c)} > 0 \tag{46}$$

At the same gross price-quantity combination and for each (feasible) level of marginal costs *c*, ad valorem taxation leads to a higher public income than a levy on output. Since this was already the maximum yield of a quantity tax, the second statement on the just mentioned list is also proven. Therefore, value taxation allows to generate yields that exceed the greatest possible proceeds of a quantity tax. Additional receipts for the government up to this limit entail increasing final prices under both taxation regimes. If a value tax collects a certain amount of money, the unit tax gathers a smaller fiscal income at the corresponding price-quantity combination. In order to raise the same revenue as with price taxation the gross price for a burden on units must be higher and output will be lower. Consequently, the third assertion quoted at the outset is valid, too. As a result, ad valorem taxation reduces consumers' surplus less though identical yields accrue to the authorities. Seemingly, price taxation appears to be superior to quantity taxation.

Table 1. Market results in comparison.

	Unit taxation	Ad valorem taxation	No excise duty
Quantity	(22) $q_q^* = \frac{1-c}{4}$	(41) $q_p^* = \frac{1-c}{4}$	(7) $q^* = \frac{1-c}{2}$
Gross price	(24) $p_q^* = \frac{3+c}{4}$	(38) $p_p^* = \frac{3+c}{4}$	(8) $p^* = \frac{1+c}{2}$
Net price	(23) $p_q^{n*} = \frac{3c+1}{4}$	(40) $p_p^{n*} = \frac{c(3+c)}{2(1+c)}$	-
Consumers' surplus	(27) $CR_q^* = \frac{(1-c)^2}{32}$	(44) $CR_p^* = \frac{(1-c)^2}{32}$	(10) $CR^* = \frac{4(1-c)^2}{32}$
Tax revenue	(26) $T_q^* = \frac{4(1-c)^2}{32}$	(43) $T_p^* = \frac{(1-c)^2(2c+6)}{32(1+c)}$	-
Profit	(25) $P_q^* = \frac{2(1-c)^2}{32}$	(42) $P_p^* = \frac{4c(1-c)^2}{32(1+c)}$	(9) $P^* = \frac{8(1-c)^2}{32}$
Welfare	(28) $W_q^* = \frac{7(1-c)^2}{32}$	(45) $W_p^* = \frac{7(1-c)^2}{32}$	(11) $W^* = \frac{12(1-c)^2}{32}$

Hitherto, however, one aspect was left out of consideration. Namely, the third player – the firm – should also be taken into account. In fact, it is usually ignored how the seller fares under this or that excise duty.

Evidently, profits in case of ad valorem taxation (42) or unit taxation (25) are at variance. The difference amounts to:

$$P_p^* - P_q^* = \frac{4c(1-c)^2}{32(1+c)} - \frac{2(1-c)^2}{32} = \frac{(c-1)^3}{16(1+c)} < 0 \tag{47}$$

If the level of consumers' surplus is given and unit taxation prevails, profits exceed those associated with price taxation in the same amount as fiscal revenue is smaller. Hence, the last row of Table 1 exhibits the congruent total welfare for both taxation systems. The source of the larger public income is now revealed: a switch from unit to ad valorem taxation transfers the complete decrease in profits to the Treasury. Though this may be politically desired, it must not be extolled as a Pareto-improvement since the transition discriminates against firms.[12]

Obviously, the alleged hierarchy of alternatives to tax commodities cannot be maintained. But this insight should not surprise economists who habitually advance the commonplace: 'There is no such thing as a free lunch'.

[12] In contrast to a unit taxation aiming at maximum fiscal proceeds the Appendix elaborates on value taxation pursuing the same goal. This leads for all marginal costs *c* to greater consumers' surplus and higher public yield whereas profits are lower. Possibly, the Treasury later compensates firms for the loss, perhaps by granting a subsidy. Of course, tax policy then is no longer guided by revenue maximization but follows political deliberations.

In truth, it would be tantamount to a miracle if just the change in the method of commodity taxation causes advantages for every involved party without burdening any other.

Yet, in the present scenario, a Pareto-improvement is quite possible. A look at the last column of the table reveals that the *abandonment* of a charge on market transactions provides a total rent that is $5/7 \approx 71.4$ % higher than the welfare related to price or quantity taxation. In view of the by far larger profit in such a world, it is clear how the state can collect additional yield without impairing the private sector in comparison to a burden on trade: individual incomes represent the appropriate taxable basis to put all participants in a better position than an excise duty. If, e.g. profits, which constitute the only personal earnings in the present model, are taxed at 50 % the monopoly pays half of its surplus to the Treasury but nonetheless receives the comparatively top gain $\left(P_G^* = \frac{1}{2}P^*\right)$. Furthermore, the consumers' surplus quadruples $(CR_G^* = CR^*)$. The tax authorities collect the same amount of revenue $\left(T_G^* = \frac{1}{2}P^* = CR^*\right)$. Then, the maximum welfare W^* is distributed evenly among all agents:

$$P_G^* + CR_G^* + T_G^* = \frac{4(1-c)^2}{32} + \frac{4(1-c)^2}{32} + \frac{4(1-c)^2}{32} = \frac{12(1-c)^2}{32} = W^* \tag{48}$$

Because direct taxes are more powerful than excise duties, the conclusion for fiscal policy emerges that in principle income taxes prove to be superior to levies on trade. However, there are two special situations. If the government wants to restrict the consumption of a demerit good by a certain price increase, ad valorem taxation, which skims off relatively much of profits, makes the business more unattractive compared to a burden on output. In addition, sometimes the tax authorities cannot ascertain actual corporate profits with sufficient accuracy, especially in case of internationally operating enterprises. Then the second-best solution would be to establish levies on proceeds. However, these exceptions do not invalidate the rule: generally, income tax still deserves to be called 'queen of taxes.'

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Declaration of Competing Interest

The author claims that the manuscript is completely original. The author also declares no conflict of interest.

Appendix

To derive the ad valorem tax rate which maximizes fiscal income we refer to equations (35) und (36). Then, the tax revenue T_{pm} can be written as:

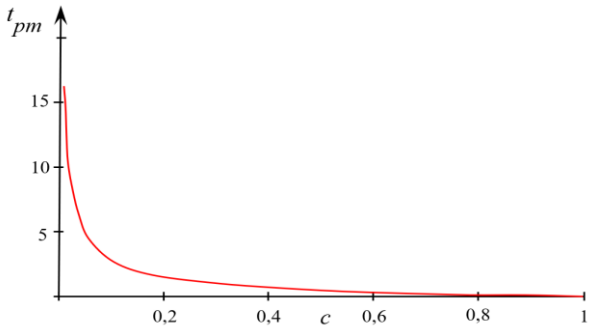
$$T_{pm} = q_p^* p_p^{n*}\left(\theta_p - 1\right) = \frac{1 - c\left(\theta_p\right)}{2}\frac{1 + c\left(\theta_p\right)}{2\left(\theta_p\right)}\left(\theta_p - 1\right) \; with \; \theta_p \equiv 1 + t_{pm} \tag{49}$$

The necessary condition for an optimum leads to a cubic expression from which the relevant root is calculated to:

$$\theta_p^*(c) = \frac{36\left(b(c)\right)^2 + b(c) + 1}{36b(c)} \; with \; b(c) = \left[\frac{1}{4}\left(\frac{1}{c}\sqrt{\frac{1}{c^2} + \frac{1}{27}} + \frac{1}{c^2} + \frac{1}{54}\right)\right]^{\frac{1}{3}} \tag{50}$$

The solution fulfills the sufficient condition. Moreover, Figure A1 demonstrates that the ad valorem tax rate $t_{pm} = \theta_p^*(c) - 1$ for varying marginal costs c is always positive.

Figure A1. Revenue maximizing ad valorem tax rates.

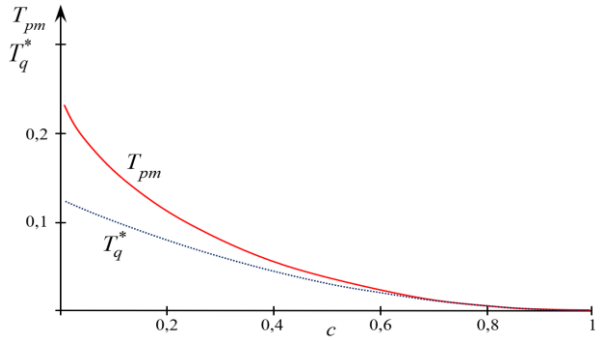


Substituting the tax rates in equations (35), (36), (37) and (43) by t_{pm} provides quantity q_{pm}, net price, profits P_{pm} and maximum tax revenue T_{pm}, respectively. Consumers' surplus CR_{pm} is obtained using the formula for the area of a triangle:

$$CR_{pm} = \frac{1}{2} q_{pm}(1 - p_{pm}) = \frac{1}{2} q_{pm}\left(1 - (1 - q_{pm})\right) = \frac{1}{2}(q_{pm})^2 \tag{51}$$

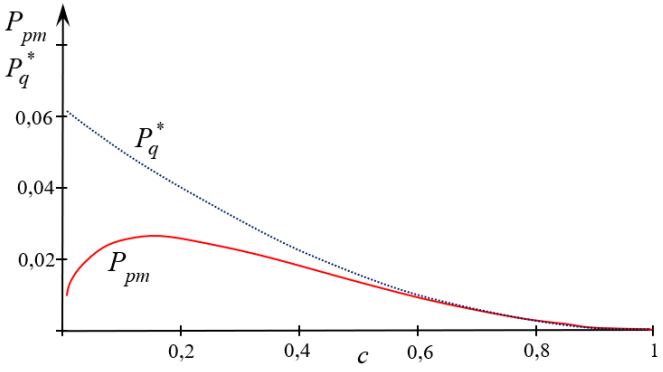
Now we can compare the effects of alternative revenue maximizing practices. Figure A2 illustrates that for any given marginal costs c the attainable fiscal income with ad valorem taxation (49) exceeds the yield in case of unit taxation (26).

Figure A2. Fiscal revenues.



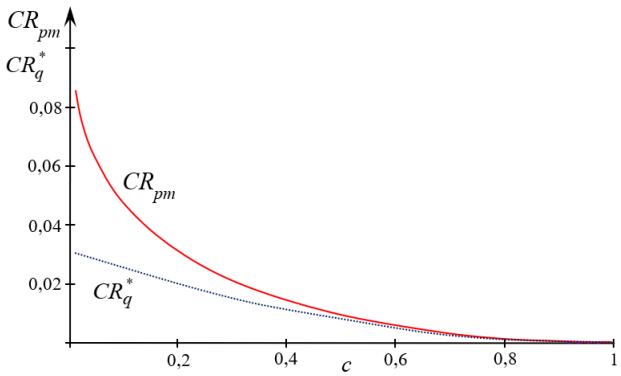
As Figure A3 reveals, levies on quantities entail higher profits than price taxation.

Figure A3. Profits.



According to Figure A4, ad valorem taxation increases consumers' surplus in relation to unit taxation.

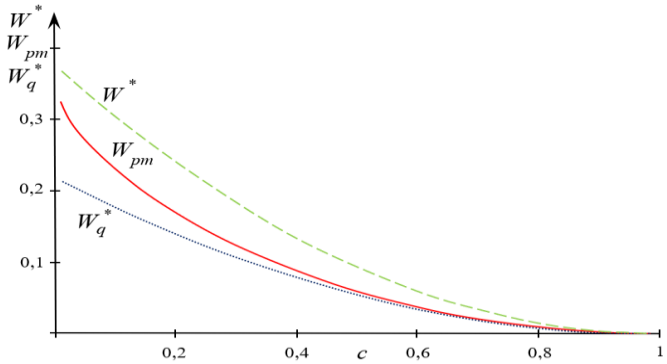
Figure A4. Consumers' surplus.



Obviously, for every level of marginal costs c ad valorem taxation leads to greater fiscal income and consumers' surplus than quantity taxation. Under the latter regime, however, profits are consistently higher. Thus, firms suffer if levies are imposed on prices instead of on units. Consequently, a (uncompensated) transition from quantity to ad valorem taxation cannot represent a Pareto-improvement.

Yet, a hierarchy concerning total welfare exists. As Figure A5 illustrates, total welfare in case of quantity taxation W_q^* is lower than with price taxation W_{pm}.

Figure A5. Total welfare.



In addition, Figure A5 corroborates that the abandonment of commodity taxation engenders the greatest welfare W^*, which is given by equation (11). In this respect, the conclusions presented above also hold true.

References

Anderson, S. P., A. de Palma and B. Kreider (2001), The efficiency of indirect taxes under imperfect competition, *Journal of Public Economics* 81, 231-251. https://doi.org/10.1016/S0047-2727(00)00085-2

Atkinson, A. and J. Stiglitz (1976), The Design of Tax Structure: Direct Versus Indirect Taxation, *Journal of Public Economics* 6, 55-75. https://doi.org/10.1016/0047-2727(76)90041-4

Bundesfinanzhof (2015). Urteil vom 7.7.2015, VII R 65/13. https://openjur.de/u/2246340.html.

Bundesministerium der Finanzen (2021). Steuereinnahmen nach Steuergruppen 2019-2020. https://www.bundesfinanzministerium.de/Content/DE/Standardartikel/Themen/Steuern/Steuerschaetzungen_und_Steuereinnahmen/2021-09-22-steuereinnahmen-nach-steuergruppen-2019-2020.pdf?__blob=publicationFile&v=2.

Engel, E. (1857), Die vorherrschenden Gewerbszweige in den Gerichtsämtern mit Beziehung auf die Productions- und Consumtionsverhältnisse des Königreichs Sachsen, *Zeitschrift des Statistischen Bureaus des Königlich Sächsischen Ministeriums des Innern* 3, Nr. 8/9, 153-184. https://www.econstor.eu/handle/10419/233572

Helmedag, F. (1982), Zur Diskussion und Konstruktion von Gutenbergs doppelt geknickter Preis-Absatzfunktion, *Jahrbücher für Nationalökonomie und Statistik* 197, 545-564. https://doi.org/10.1515/jbnst-1982-0606

Helmedag, F. (2012), Individuelle und kollektive Gewinnmaximierung auf homogenen Märkten, in Oberender, P. (ed.), *Private und öffentliche Kartellrechtsdurchsetzung* (S. 9-38). Berlin Duncker & Humblot. https://www.tu-chemnitz.de/wirtschaft/vwl2/downloads/wettbew/HomogeneMaerkte10.pdf

Helmedag, F. (2018), *Warenproduktion mittels Waren, Zur Rehabilitation des Wertgesetzes*, 3rd ed., Marburg Metropolis.

Helmedag, F. (2019), Marx and Keynes: from exploitation to employment, *European Journal of Economics and Economic Policies: Intervention* 16, 260-271. https://doi.org/10.4337/ejeep.2019.0048

Hoffmann, M. and M. Runkel (2016), A welfare comparison of ad valorem and unit tax regimes, *International Tax & Public Finance* 23, 140-157. https://doi.org/10.1007/s10797-015-9355-2

Li, H. and X. Liu (2021), Ad valorem versus per unit taxation: a perspective from price signaling, *Journal of Economics* 134, 27-47. https://doi.org/10.1007/s00712-021-00736-w

Kiser, E. and S. M. Karceski (2017), Political Economy of Taxation, *Annual Review of Political Science* 20, 75-92. https://doi.org/10.1146/annurev-polisci-052615-025442

Saez, E. (2002), The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes, *Journal of Public Economics* 83, 217-230. https://doi.org/10.1016/S0047-2727(00)00159-6

Schwabe, H. (1879), Das Verhältniß von Miethe und Einkommen in Berlin, Beiträge zu einer Consumtionsstatistik, in Statistisches Bureau der Stadt Berlin (ed.), *Gemeinde-Kalender und städtisches Jahrbuch für 1868* 2, 264 -267.

Skeath, S. and G. Trandel (1994), A Pareto comparison of ad valorem and unit taxes in noncompetitive environments, *Journal of Public Economics* 53, 53-71. https://doi.org/10.1016/0047-2727(94)90013-2

Stiglitz, J. (2000). *Economics of the Public Sector*. 3rd ed., New York / London W. Norton & Company.

Stewart, M. (2022), *Tax and Government in the Twenty-First Century*, Cambridge University Press.

Suits, D. and R. Musgrave (1953), Ad Valorem and Unit Taxes Compared, *Quarterly Journal of Economics* 67, 598-604. https://doi.org/10.2307/1883604

Wicksell, K. (1896), *Finanztheoretische Untersuchungen nebst Darstellung und Kritik des Steuerwesens Schwedens*, Jena Gustav Fischer.