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CHAPTER 6

The Optimal Rotation Period of Renewable Resources: Theoretical Evidence from the Timber Sector

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Wood has always been and still is one of the most important natural substances. It can be used for light and warmth, as a raw material for furniture, and the construction of buildings and boats. Trees cover approximately one third of the earth's surface. About 2 billion tons of timber are harvested per annum, which is more than the yearly output of steel and cement taken together. These figures alone suffice to justify that the economics of forestry is put on the agenda. In the face of discussions on climate change and the vital role of renewable resources, the optimal cutting strategy deserves special attention. For about two centuries, however, this issue has been under investigation. Actually, the answer to the question when to log a tree depends on the specific goal of the decision maker. This fact has not always been stated precisely. The chapter provides a survey of the different approaches and clarifies the conditions for their application.

PRODUCTION AS A TIME CONSUMING PROCESS

The production of a good is a process of varying length. Nevertheless, in the majority of cases, it can be organized in a way that there is a continuous flow of more or less finished products, some just being started and others

completed. Besides that, in special branches, a certain period of time has to elapse in order to allow products to mature. The difference becomes obvious if the production process in an automobile factory is compared to the one in the timber sector.

In industry, the period to which quantities like profit or costs refer is open. The figures are related to the chosen time span that can be a day, a week, a month or a year. Likewise, in a timber company, cultivation will usually be shaped as woodland with a mixed age structure. Trees are cut when they have reached a designated size. Thus, the planning of such a synchronized stock depends on the knowledge when harvesting is most lucrative: Forestry management requires clarity about how long a single tree should grow. This problem of the so-called "optimal rotation period" arises with all renewable resources, not only in tilling the soil, but also in animal farming such as pig fattening.[1]

However, it is astounding that, for the "simple problem" of optimal forestry, several wrong analyses are encountered.[2] As Johansson and Löfgren point out: "Some of the greatest economists have solved the problem incorrectly."[3] It may seem convenient to rely on a widely accepted solution, yet, it is rewarding to look at alternatives carefully to see which specific questions they answer.[4] When we investigate the chosen example, it will become apparent how the determination of the optimal cultivation cycle in forestry serves as a demonstration object to compare different economic calculations. Especially, the investors' maxim can be distinguished from the entrepreneurs' objective.

Consider the following situation. We assume that wood is growing on a piece of land. During harvest, cutting and transport costs are proportional

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[1]Occasionally, the expression *reproducible resources* is used when the regeneration cycle is less than one year; rice or corn cultivation comes to mind. When considering such production processes, labor input is to be optimized and not the production cycle discussed here.

[2]See, for example, Holger Wacker and Jürgen-E. Blank, *Ressourcenökonomik, Band I: Einführung in die Theorie regenerativer natürlicher Ressourcen* (München/Wien: Oldenbourg 1998), p. 105; and Ulrich Hampicke, *Ökologische Ökonomie, Individuum und Natur in der Neoklassik, Natur in der Ökonomischen Theorie*, Teil 4 (Opladen: Westdeutscher Verlag 1992), p. 76.

[3]Per-Olov Johansson and Karl-Gustaf. Löfgren, *The Economics of Forestry and Natural Resources* (Oxford: Blackwell, 1985), p. 74.

[4]See, for instance, Paul A. Samuelson, "Economics of Forestry in an Evolving Society," *Economic Enquiry* 14, no. 4 (1976), pp. 466–492; and Ulrich van Suntum, "Johann Heinrich von Thünen als Kapitaltheoretiker," in *Studien zur Entwicklung der Ökonomischen Theorie XIV, Johann Heinrich von Thünen als Wirtschaftstheoretiker*, edited by Heinz Rieter (Berlin: Dunker & Humblot 1995), pp. 87–113.

to the proceeds. Hence, the net price based on units of quantity (weight or volume) is given and therefore can be used as a numéraire. Due to these assumptions, the physical output is equivalent to its monetary valuation. The revenue of a hectare of trees of age t is specified as follows:

$$f(t) = \frac{1}{30}t^4(15 - t) \tag{6.1}$$

The time t is interpreted as number of years. Exhibit 6.1 depicts the production result depending on the growth period.

The productivity of time is calculated via:

$$f'(t) = \frac{4}{30}t^3(15 - t) - \frac{1}{30}t^4 = \frac{1}{6}t^3(12 - t) \tag{6.2}$$

Setting this equal to zero, the first derivative yields the output maximum at $t_m = 12$. The average output per interval is:

$$\frac{f(t)}{t} = \frac{1}{30}t^3(15 - t) \tag{6.3}$$

For an extremum, it is necessary that:

$$\left(\frac{f(t)}{t}\right)' = \frac{1}{10}t^2(15 - t) - \frac{1}{30}t^3 = \frac{1}{30}t^2(45 - 4t) = 0 \tag{6.4}$$

The average periodical output is maximized at $t_d = 11.25$. Exhibit 6.2 illustrates equations (6.2) and (6.3).

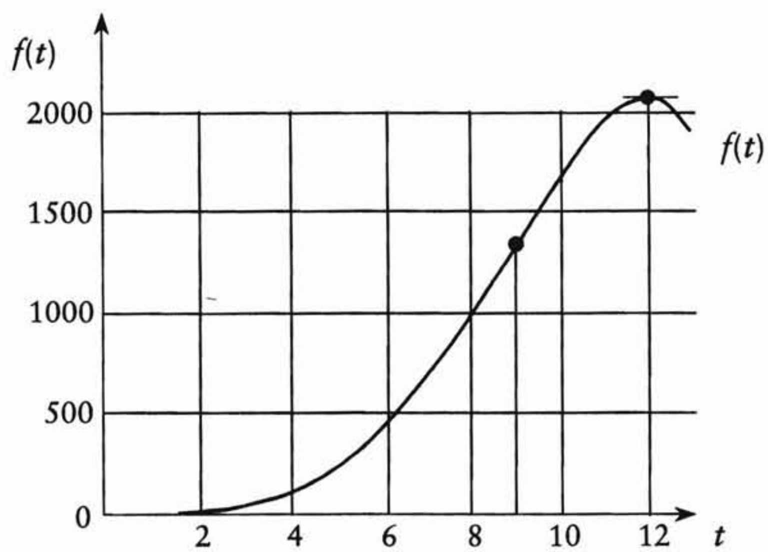

EXHIBIT 6.1 The Production Function
Source: Author.

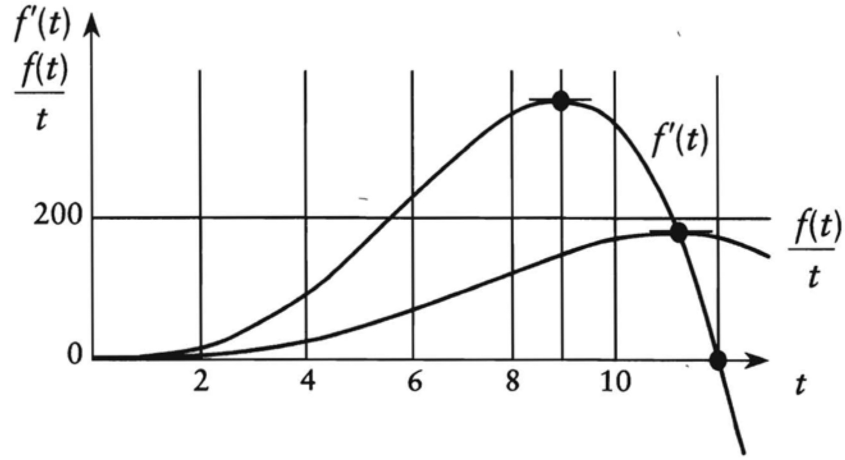

EXHIBIT 6.2 Productivity and Average Output
Source: Author.

To emphasize the underlying objective is what will matter most in analyzing the different alternatives. First, we turn to profit maximization: Which kind of woodland cultivation fulfils this goal? In order to answer this question correctly, one has to take into account the actual situation in which a concrete decision is required.

NET PROCEEDS VERSUS COST RETURN

The Maximum Future Profit

The first scenario considers a cultivator of fallow woodland who borrows the money for planting costs per hectare (L). For simplicity's sake, the bank loan is supposed to be bearing a continuous compounding at an interest rate i and is paid back completely when the stand is cut. The surplus per hectare at time t, also referred to as *profit* from this point on, is positive if the interest on the planting costs is not too high:

$$G(t) = f(t) - Le^{it} > 0 \quad \text{for } 0 \le L < f(t) \quad \text{and} \quad 0 \le i < i_{\max} \tag{6.5}$$

Maximizing the income[5] leads to:

$$G'(t) = f'(t) - iLe^{it} = 0 \tag{6.6}$$

From equation (6.6) follows:

$$it = \ln\left(\frac{f'(t)}{iL}\right) \tag{6.7}$$

[5]We do not state sufficient conditions here and later. Also, we only give solutions that are relevant from an economic point of view.

Hence, we derive the production period:

$$t_G = \frac{1}{i} \cdot \ln\left(\frac{f'(t_G)}{iL}\right) > 0 \quad \text{for } f'(t_G) > iL \tag{6.8}$$

Setting $i = 10\%$ and $L = 100$, we calculate $t_G = 11.883$.

At this point in time, the profit per hectare amounts to $G(t_G) = 1743.517$. But the cultivator has to wait t_G years for this event to occur. However, it is possible to accomplish a backward distribution of the future value. In general, the annuity z which is equivalent to a prospective payoff $E(T)$ at time T can be obtained with:

$$E(T) = \int_0^T z \cdot e^{i(T-t)}\, dt = \int_0^T z \cdot e^{it}\, dt = \left[\frac{z}{i} e^{it}\right]_0^T = \frac{z}{i}\left(e^{iT} - 1\right) \tag{6.9}$$

And, therefore:

$$z = \frac{iE(T)}{e^{iT} - 1} \tag{6.10}$$

Inserting $G(t_G)$ and the other data into equation (6.10) yields:

$$z_G = \frac{1743.517 \cdot 0.1}{e^{0.1 \cdot 11.883} - 1} = 76.424$$

The annuity z_G is equivalent to the present value of the profit accruing in t_G years. Thus, this rent is also suited to characterize the respective lucrativeness.[6]

An Upper Limit to the Interest Rate

While the optimization of future profit plays no role in the literature, the determination of the optimal span of time to invest a sum of money can be found. In this approach, which is often connected with the names Knut Wicksell (1851–1926) and Kenneth E. Boulding (1910–1993), for example, the question arises how long (newly) bought wine is to be kept in the cellar if the development of prices as a function of time is known.[7] Using continuously compounding interest, Wicksell's terminology deals with the

[6]Alternative projects of different lengths are assumed to be executed several times; the minimum time period for comparison is the smallest common multiple of the individual cultivation cycles.

[7]See Knut Wicksell, *Vorlesungen über Nationalökonomie auf Grundlage des Marginalprinzips*, vol. 1 (Jena: Gustav Fischer, 1913), p. 238; and Kenneth E. Boulding, *Economic Analysis*, vol. 2, 4th ed. (New York: Harper and Row, 1966), p. 672.

maximization of the *interest generating energy* r of the capital advanced, subject to the condition that the revenue covers the initial investment including interest:

$$r \rightarrow \text{Max! s.t. } Le^{rt} = f(t) \tag{6.11}$$

Exhibit 6.3 illustrates the graphical solution to the problem: A curve representing a continuously compounded investment progresses in such a way that the production function is just touched upon.

To calculate the t-value in question, the constraint in equation (6.11) is logarithmized and solved for r:

$$r = \frac{\ln\left(\frac{f(t)}{L}\right)}{t} \tag{6.12}$$

The first derivative with respect to time reads:

$$\frac{dr}{dt} = \frac{\left(\frac{f'(t)}{f(t)}\right)t - \ln\left(\frac{f(t)}{L}\right)}{t^2} \tag{6.13}$$

We obtain the investment interval t_W by setting the nominator equal to zero:

$$t_W = \frac{f(t_W) \cdot \ln\left(\frac{f(t_W)}{L}\right)}{f'(t_W)} \tag{6.14}$$

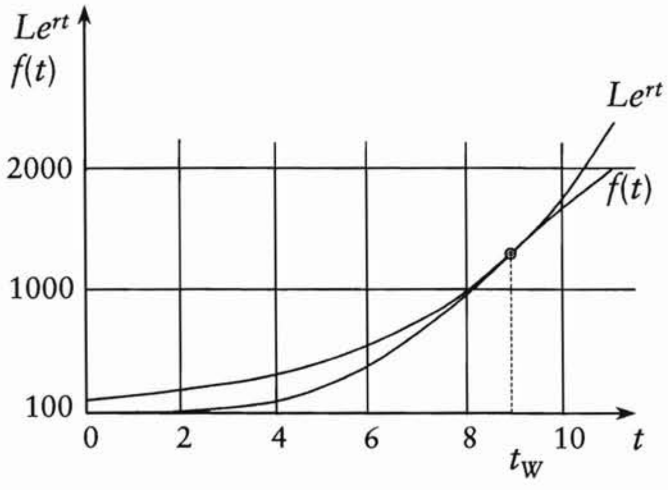

EXHIBIT 6.3 An Upper Limit to the Interest Rate
Source: Author.

In our example, equation (6.14) yields a growth period $t_W = 8.893$. With this (minimum) duration corresponds the highest rate of interest r^* this project is able to yield. At the same time, the critical market interest rate $i_{\max}$ is determined. It must not be exceeded if the investment is to be profitable. The maximum rate of return on the advanced costs amounts to:

$$r^* = \frac{f'(t_W)}{f(t_W)} = i_{\max} = 0.286 \tag{6.15}$$

The future profit is computed with:

$$f(t_W) = Le^{r^* t_W} = 1273.038$$

Calculating the equivalent profit flow over time according to (6.10), one has to take into account that the costs bear an interest rate of $i = 0.1 < r^*$:

$$z_W = \frac{\left(1273.038 - 100 \cdot e^{0.1 \cdot 8.893}\right) \cdot 0.1}{e^{0.1 \cdot 8.893} - 1} = 71.835$$

This annuity is smaller than the one in the previous case. Therefore, the Wicksell-Boulding solution or the maximization of the profit rate alias the return on costs respectively has to be judged as suboptimal.

CAPITAL MANAGEMENT IN FORESTRY

The Stumpage Value

We began our study with a cultivator who borrowed the planting cost L at the interest rate i. It would be interesting to know the limit of a bank loan if the stand is used as a collateral and the profit for repayment. Then, the problem is to find today's top price that a current cultivation could fetch in the future market. We are looking for the maximum capital value of wood (KW_H):

$$KW_H = f(t)e^{-it} - L = \frac{f(t) - Le^{it}}{e^{it}} = \frac{G(t)}{e^{it}} \rightarrow \text{Max!} \tag{6.16}$$

Optimization leads to:

$$\frac{dKW_H}{dt} = f'(t)e^{-it} - f(t)i \cdot e^{-it} = 0 \tag{6.17}$$

Therefore:

$$i = \frac{f'(t)}{f(t)} \tag{6.18}$$

William St. Jevons' (1835–1882) and Irving Fisher's (1867–1947) respective rules enunciate an optimal maturity time of a singular project.[8] If the growth rate of wood drops down to the level of the interest rate, the value of the timber stock reaches its maximum. Thus, increasing the interest rate reduces the rotation period. Equation (6.15), which corresponds to equation (6.18) for $t_W = t$, provides the previously mentioned maximum interest rate $i_{\max}$. For a given market interest rate $i = 10\%$, we obtain as cultivation cycle and capital value: $t_H = 11.140$ and $KW_H = 550.433$.

For comparison purposes, the corresponding cash flow is of interest. Since this time we have a forward distribution of a present value into the future ("capital regain"), we start with:

$$\begin{aligned} KW(0) &= \int_0^T v \cdot \mathrm{e}^{-it} dt = \left[\frac{v}{-i}\mathrm{e}^{-it}\right]_0^T \\ &= -\frac{v}{i}\left(\mathrm{e}^{-iT} - 1\right) = \frac{v}{i}\left(1 - \mathrm{e}^{-iT}\right) \end{aligned} \tag{6.19}$$

Solving for the annuity v gives:

$$v = \frac{KW(0) \cdot i}{1 - \mathrm{e}^{-iT}} \tag{6.20}$$

The concrete result attains the highest value so far:

$$v_H = \frac{550.433 \cdot 0.1}{1 - \mathrm{e}^{-0.1 \cdot 11.140}} = 81.939$$

Thus, the Jevons-Fisher formula seems to deserve priority. After all, the outcome exceeds the maximization of the return on an investment in the Wicksell-Boulding vein. However, the optimal use of forest soil is not a problem of a single investment, but of a continuous silviculture.

The Productive Powers of Woodland

The previous maximization of a capital value referred to trees of the same age. Besides, the question arises which profit potential a piece of fallow land

[8]See William St. Jevons, *The Theory of Political Economy*, 2nd ed. (London: Macmillan, 1879), p. 266; and Irving Fisher, *The Theory of Interest* (New York: Macmillan, 1930), p. 164.

has, whose sole possible exploitation is forestry. Consequently, the capital value of the entire future timber and, therefore, the right price of the plot has to be calculated. This approach was taken by forester Martin Faustmann (1822–1876) in the 19th century: "What is the pure money return bare woodland will continuously yield every year in the same amount from now on?"[9]

The value of real estate (KW_F) reflects a sequence of infinite successions of the same project, taking compound interest rate effects into account. By this, Faustmann was hoping to gain "necessary insight into forest destruction by fire, insects, man."[10] The productive value of the soil—and not the value of wood destroyed—according to Faustmann amounts to:

$$KW_F = -L + (f(t) - L)e^{-it} + (f(t) - L)e^{-2it} + \cdots \tag{6.21}$$

Rearranging yields:

$$KW_F = \left(f(t)e^{-it} - L\right) + \left(f(t)e^{-it} - L\right)e^{-it} + \left(f(t)e^{-it} - L\right)e^{-2it} + \cdots \tag{6.22}$$

Now it is possible to apply the formula for the infinite geometric series:

$$KW_F = \frac{f(t)e^{-it} - L}{1 - e^{-it}} = \frac{KW_H}{1 - e^{-it}} = \frac{G(t)}{e^{it} - 1} \tag{6.23}$$

Of course, the Faustmann capital value—like all profitable investments with infinite lifetime—grows beyond all limits for an interest rate converging to zero. This phenomenon is independent of the future profit $G(t)$. In such a situation, one has to look for a different method with which a precise rotation period can be found. Furthermore, the interest rate must not exceed $i_{\max}$ because otherwise profits $G(t)$ are actually losses and the capital value becomes negative too. Within the admissible range, the latter moves in the opposite direction of changes in the interest rate.

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[9] Martin Faustmann, "Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen," *Allgemeine Forst- und Jagd-Zeitung* (December 1849), pp. 441–455, p. 442. *Note:* Unless otherwise stated, all translations are the author's.

[10] Faustmann, "Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen," p. 441. Obviously, the definitive uselessness of soil for forestry purposes is meant; this raises the problem of an adequate compensation.

The necessary condition for the maximization of the Faustmann value reads:

$$\frac{dKW_F}{dt} = \frac{\left[f'(t)\mathrm{e}^{-it} + f(t)\left(-i\mathrm{e}^{-it}\right)\right]\left(1 - \mathrm{e}^{-it}\right) - \left(f(t)\mathrm{e}^{-it} - L\right)\left(i\mathrm{e}^{-it}\right)}{\left(1 - \mathrm{e}^{-it}\right)^2} = 0 \tag{6.24}$$

Therefore:

$$f'(t)\left(1 - \mathrm{e}^{-it}\right) = i\left(f(t)\left(1 - \mathrm{e}^{-it}\right) + f(t)\mathrm{e}^{-it} - L\right) \tag{6.25}$$

And respectively:

$$f'(t) = \frac{i(f(t) - L)}{1 - \mathrm{e}^{-it}} \tag{6.26}$$

From this, t can be deduced if the interest rate as well as the explicit production function is known. In our example we obtain $t_F = 10.666$ and $KW_F = 828.745$. If this capital value can be realized by selling the land (or leasing it), then the following perpetuity is generated:[11]

$$Z_F = i \cdot KW_F = 0.1 \cdot 828.745 = 82.8745$$

A backward distribution of $G(t_F)$ according to (6.10) entails the same result.[12] Before checking whether the plot actually gets the Faustmann value, we consider a completely different model in the next section.

REVENUES FINANCE EXPENDITURES

Thünen: Over the Top

Up until now, we envisaged to cultivate on our woodland a cohort of trees of the same age that were jointly cut down. In reality, there is an ongoing process of cultivating and harvesting. In our example, this means that

[11]The formula follows from equation (6.20) for $T \to \infty$.

[12]An alternative way of deriving the Faustmann rotation is to insert the future profit $G(t) = f(t) - L\mathrm{e}^{it}$ into equation (6.10) and to optimize z with respect to t. This concurs with the search for a maximum annuity of the hypothetical process chain.

depending on the rotation period t, the t-th portion of a hectare is logged and subsequently reforested.

The topic of a *sustained* instead of *suspended* enterprise is found in the works of Johann Heinrich von Thünen (1783–1850).[13] In contrast to Faustmann, Thünen does not just mention the distinction, but actually applies it by aiming at an income accruing in each time interval. The procedures discussed earlier treated the task as a problem of an investment decision. Thünen however focuses on the *periodical profit* (PG) of the forester. By doing so, he deducts from the proceeds the planting costs L as well as the (forgone) interest on the monetary value of the timber stand. In the continuous case, stumpage equals the integral $F(t)$ over the production function $f(t)$. Thus, Thünen's maximand is:[14]

$$PG_T = \frac{f(t) - if(t) - L}{T} \tag{6.27}$$

The advantage of this approach is to point the objective function right from the outset toward a continuous surplus that therefore directly leads to a synchronized cultivation. The revenues of such a subdivided forest finance the planting costs of new trees that, as a result, cannot give rise to any interest demands. In the Thünen approach, the necessary condition reads:

$$\frac{dPG_T}{dt} = \frac{(f'(t) - if(t))t - f(t) + iF(t) + L}{t^2} = 0 \tag{6.28}$$

According to this procedure, a single tree will reach an age of:

$$t_T = \frac{f(t_T) - iF(t_T) - L}{f'(t_T) - if(t_T)} \tag{6.29}$$

This yields $t_T = 10.453$ and $PG_T = 113.488$.

[13]See Johann H. v. Thünen, *Der isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie, Dritter Theil, Grundsätze zur Bestimmung der Bodenrente, der vorteilhaftesten Umtriebszeit und des Werths der Holzbestände von verschiedenem Alter für Kieferwaldungen* (1863), 3rd edition, edited by H. Schumacher-Zarchlin (Berlin: Wiegant, Hempel & Parey, 1875).

[14]See Ulrich van Suntum, "Johann Heinrich von Thünen als Kapitaltheoretiker," p. 108. For the discrete case see Peter Manz, "Forestry economics in the steady state: the contribution of J. H. von Thünen," *History of Political Economy* 18, no. 2 (1986), pp. 281–290.

The higher surplus of the synchronized production represents an incentive to focus on the maximization of the periodical profit caused by a staggered silviculture as compared to the successive methods treated earlier. But, from an economic point of view, it is questionable to allow the opportunity costs to enter the objective function. Rather, the subsequent comparison with an alternative use of the stumpage serves as a criterion whether the forestry should be continued or not. Because of this, Thünen's thoughts fail to convince.

Back to the Roots: 1788

Nonetheless, there is another cut-down rule which has been discussed among forest economists for some time. According to this guideline, the difference between revenues und planting costs per unit of time (and area)(PG_J) is decisive:

$$PG_J = \frac{f(t) - L}{t} \tag{6.30}$$

Actually, in 1788 such an instruction was decreed by the Royal and Imperial Austrian government during the reign of Emperor Joseph II.[15] This directive equals Thünen's formula for $i = 0$. The optimization requires:

$$\frac{dPG_J}{dt} = \frac{f'(t)t - (f(t) - L)}{t^2} = 0 \tag{6.31}$$

Solving for t leads to:

$$t_J = \frac{f(t_J) - L}{f'(t_J)} \tag{6.32}$$

Interestingly, with interest tending to zero, the Faustmann solution converges to equation (6.32) as well. This follows from applying l'Hospital's rule to (6.26):

$$f'(t) = \lim_{i \to 0} \frac{i(f(t) - L)}{te^{-it}} = \frac{f(t) - L}{t} \tag{6.33}$$

Substituting our revenue function in (6.32) and (6.30) gives $t_J = 11.296$ and $PG_J = 169.108$.

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[15]See F. C. Osmaston, *The Management of Forests* (London: George Allen and Unwin, 1968), p. 188.

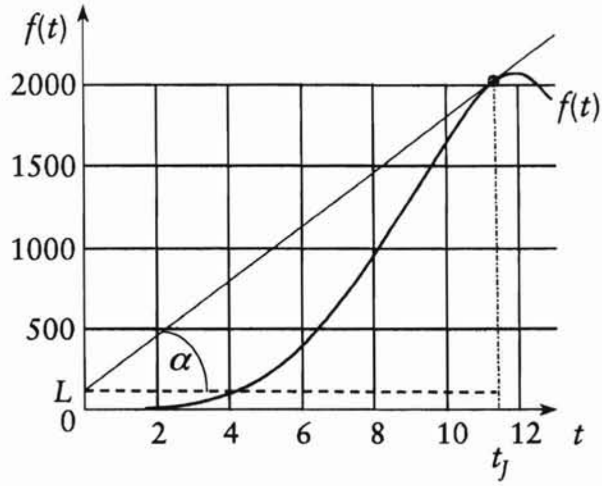

EXHIBIT 6.4 The Maximum Land Rent
Source: Author.

In this case, the optimal rotation period can be easily extracted from a graphic (see Exhibit 6.4).

A tangent from the planting costs towards $f(t)$ is drawn, determining the angle α, which represents the maximum surplus per hectare over time. This is the highest attainable profit stream.

Hence, it becomes clear under which circumstances forestry is no longer worthwhile. When an alternative turns out to be lucrative, stumpage and soil are sold in order to capitalize the sales proceeds (U). Then, the following condition holds:

$$U \cdot i > PG_J \tag{6.34}$$

Putting It to the Test

In an often cited paper, Samuelson discusses the just described maximization of the sustained net yield.[16] According to him, the *Austrian cameral valuation method* is incorrect since it does not take interest rate effects into account.[17] Therefore, the difference per year between timber yield and planting costs appears to him not all that important:

> *This is so absurd as to be almost believable to the layman—up to the moment when the economist breaks the news to the farmer . . . that he can mine the forest by cutting it down without replanting and sell the land, thereafter putting the proceeds into the bank . . . and subsequently earn interest forever.*[18]

[16]Samuelson, "Economics of Forestry in an Evolving Society," p. 477.
[17]Samuelson, "Economics of Forestry in an Evolving Society," p. 489.
[18]Samuelson, "Economics of Forestry in an Evolving Society," p. 474.

With the insights obtained so far, we are able to appreciate Samuelson's criticism. Suppose a forester owning one hectare of Joseph II synchronized wood follows the recommendation and cuts down the stock, sells the timber and even finds afterward a buyer for the land paying Faustmann's value. In total, our ex-forester receives:

$$U = \int_0^{t_J} \frac{1}{t_J} f(t)\, dt + KW_F = 606.401 + 828.745 = 1435.146$$

On the other hand, the periodic profit PG_J computed earlier (interpreted as a perpetuity) represents a present value of:

$$\frac{PG_J}{i} = \frac{169.108}{0.1} = 1691.08$$

Obviously, when seeking advice from Samuelson one takes a loss: The (maximum) company value U is smaller than the amount we calculate for our illustration! Indeed, a basic principle of economic behavior states that the continuous surplus of an enterprise should exceed the interest on the capitalization of the firms' assets. This is not the case here since the clear-cutting condition (6.34) is violated. Therefore, running the business based on the "rule of thumb"[19] from 1788 yields better results than the proposed logging instruction in the given situation.

However, it needs to be taken into account that the direct comparison between the theories is improper: Faustmann is (yet) located on empty land and looks for its value, whereas Joseph II continuously wants to make as much profit as possible out of his already existing trees.[20] This discrepancy is also of importance to the modern forester. Furthermore, institutional changes need to be taken into account.

[19]See Philip A. Neher, "Forests," in *The New Palgrave*, vol. 2 (London, New York, and Tokyo: Macmillan/Stockton Press/Maruzen, 1994), pp. 412–414.

[20]In science, there has been a long-term conflict between these opposing schools. For the history of this quarrel, see Cristof Wagner, *Lehrbuch der theoretischen Forsteinrichtung* (Berlin: Parey, 1928), who summarizes: "Hence, we have sustainability against profitability, Prussia versus Saxony" (p. 199). However, this confrontation misses the point. A sustainable production is also possible with the Faustmann rotation period, but leads to suboptimal earnings. Inserting in equation (6.30) t_F from our illustration, we gain: $PG_F = 165.920 < PG_J = 169.108$.

FROM FEUDALISM TO CAPITALISM

The Accumulation Phase

In what follows, an entrepreneur seeking to maximize his profit is introduced as an idealized economic agent. This person is not constrained by the capital he is able to invest in a project, but by the demand side. His main task is to supply goods at prices no less than unit costs. For analytical reasons, we assume that the entrepreneur does not resort to own money—his reputation or a convincing business concept will grant him a loan. From this standpoint, rates on return—figures providing a relationship of surplus to capital advanced—are not suitable as an indication for economic success. The limits of woodland prices and forest values first and foremost reflect the concerns of investors, not those of entrepreneurs striving for profit maximization.

In our model world, at least one of the entrepreneur's problems is not that difficult. The revenues of wood production are known; sales do not pose a problem. Once all costs including the use of forest soil have to be incurred, which rotation period proves to be optimal?

In a first step, the woodland shall only be available for purchase by foresters accumulating land piece by piece at the Faustmann value. Period after period, depending on a given growth phase T, an additional part $1/T$ of a hectare is acquired. Additionally, there are planting costs. Hence, the following debt has amassed until the (yet unknown) optimal rotation period T is reached:

$$D_{\mathrm{Buy}}(T) = \int_0^T \frac{1}{T}(KW_F + L)\mathrm{e}^{i(T-t)}dt = (KW_F + L)\left(\frac{\mathrm{e}^{iT} - 1}{iT}\right) \qquad (6.35)$$

Substituting in this expression the Faustmann rotation t_F as well as the other data, we get:

$$D_{\mathrm{Buy}}(t_F) = (828.745 + 100)\left(\frac{\mathrm{e}^{0.1 \cdot 10.666} - 1}{0.1 \cdot 10.666}\right) = 1659.20$$

The interest accumulated with this debt during the construction of the silviculture just equals the periodic surplus from the synchronized cultivation:

$$PG_F = \frac{f(t_F) - L}{t_F} = i \cdot D_{\mathrm{Buy}}(t_F) = 165.920$$

This equation characterizes the nature of Faustmann's woodland value from a buyer's point of view: It represents the maximum amount of money an entrepreneur without means is able to pay for additional land in order to create a staggered forest while time elapses.[21] If the purchaser wants to make profit, the actual price for the additional woodland bought year after year must be *lower* than the Faustmann value. It is therefore a constraint for forestry business just as the maximum interest rate $i_{\max}$ is.

In order to prepare for the next section, let us now take a short look at the alternative to woodland acquisition: The entrepreneur disburses a rent R per period and hectare. While the necessary expenses for the planting still sum up until the desired ages of the trees are reached, the formula for the accumulated debt from rent payments looks different. The total loan at time T amounts to

$$\begin{aligned} D_{\text{Rent}}(T) &= \int_0^T t\frac{1}{T}\mathrm{R}e^{i(T-t)}\,dt + \int_0^T \frac{1}{T}\mathrm{L}e^{i(T-t)}\,dt \\ &= \frac{R\left(e^{iT} - iT - 1\right) + Li\left(e^{iT} - 1\right)}{i^2 T} \end{aligned} \tag{6.36}$$

For the entrepreneur's profits after completion of the synchronized silviculture, we obtain:

$$PG(T) = \frac{f(T) - L}{T} - R - i \cdot D_{\text{Rent}}(T) \tag{6.37}$$

With a given rent R, our model forester chooses the appropriate T. Depending on the assumptions about the variables and data, he can pay off his debt sooner or later. Hence, provided the undertaking is crowned with success, there will be no more borrowing costs one fine day—the same situation as with the purchase of forest soil. Then, the following equation holds:

$$PG(t) = \frac{f(t) - L}{t} - R \tag{6.38}$$

Let us take a closer look at the long run and the determination of the rent.

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[21] "Herr Faustmann must have reasoned along lines somewhat as follows: If I were to start planning a forest from scratch, how much could I afford to pay for bare land?" G. Robinson Gregory, *Forest Resource Economics* (New York: John Wiley & Sons, 1972), pp. 286. In Appendix A of this chapter, it is proven that the accumulated debt from the purchase of land at the Faustmann price and the subsequent planting of the whole area cannot be made up by revenues since they merely suffice to pay for interest.

Land Rent in Competition

From a modern stance, the 1788 regulation has one disadvantage. It served as a maxim in feudalism: Forestry was performed by the proprietor himself, who at times possessed giant estates. He acted in personal union as a landowner as well as a timber producer without consideration of rent payments. Furthermore, Nature herself took care of the first cultivation free of charge. Under these circumstances, the landowner was geared to the entire continuous stream of income from his property.

In modern capitalism, separation serves the purpose of realizing something important: The function of an entrepreneur and a resource provider has to be distinguished. Otherwise, the income categories *profit of wood production* and *rent for land lease* cannot be isolated. In the following, we consider the long-term situation where investment expenses to create a synchronized production structure has already been paid off. Let us assume that the use of a hectare requires payment of rent $R \geq 0$. Moreover, in every period there is one hectare ready for harvesting. Equation (6.38) provides the hectare profit (HG):

$$HG = t \cdot PG(t) = f(t) - L - R \cdot t \tag{6.39}$$

The last term on the right side of equation (6.39) states the total rent due. Differentiating yields:

$$HG' = f'(t) - R \tag{6.40}$$

Thus, the necessary condition for an optimum reads:

$$f'(t) = R \tag{6.41}$$

This result makes sense from an economic point of view: In equilibrium, productivity of time equals rent. If land is available at no costs, harvesting occurs at the maximum return $f(t_m)$ independent of the interest rate and planting costs. As expected, the earnings per hectare are smaller than those under the 1788 regime:

$$\frac{HG_m}{t_m} = \frac{f(t_m) - L}{t_m} - R = \frac{2073.6 - 100}{12} - 0 = 164.466$$

Nevertheless, total profit is higher since more soil is cultivated for free:

$$HG_m = 164.466 \cdot 12 = 1973.6 > PG_J \cdot t_J = 169.108 \cdot 11.296 = 1910.24$$

In such a situation, consumption of land does not need to be taken into account. Rather, it seems now reasonable to choose the same amount of costs L which also may be a wage bill as a reference point for a comparison between different logging strategies. Equation (6.41) informs us when to terminate the trees' growth. Minimizing the costs per unit of timber then proves to be the crucial criterion for the choice of technique in forestry.

If rent has to be paid, then the life cycle of the trees is shortened compared to their maximum size. To boot, if profits vanish in consequence of the competition for scarce plots of the same fertility, HG in equation (6.39) will tend to zero. Consequently, the landowner can pocket the maximum profit per hectare in the form of rent:

$$R_{\max} = \frac{f(t) - L}{t} \tag{6.42}$$

This is equivalent to the Joseph II case: There are no entrepreneurial foresters, but only proprietors who maximize their income per area unit; the economy shows signs of feudalism.[22] For modern times, David Ricardo's (1772–1823) exploration of capitalism is appropriate. He considered an expanding economy where land of decreasing quality is taken under the plough. Then, the farmers do not pay any rent for the least cultivated plot. Nevertheless, the superior ground receives a premium depending on its fertility.

LOOKING BACK AND AROUND

Initially, there was a forester who sought advice from an economist but received inadequate counsel. And this happened in a field where established knowledge is supposed to be solid: The analysis of a clearly structured microeconomic decision situation.

The question arises why the academic tenet led to wrong conclusions. The answer is that different problem-solving approaches were mixed up. Computing the maximum interest rate on the costs advanced or the determination of (interest rate dependent) capital values of timber or land respectively may be significant within the investment calculus, but this does not provide an optimal cut-down strategy from an entrepreneurial point of view in long-term forestry.

[22]In Appendix B of this chapter it is demonstrated, what the owner of a plot of soil must do to realize a synchronized planting.

Moreover, the widely accepted Faustmann approach suffers from a discrepancy between theory and practice which has to be explained. Without doubt, the choice of the interest rate used in calculations is more or less arbitrary. Remarkably, however, often unrealistically low interest rates are chosen for the purpose of obtaining desired results. Many years ago, a silviculture interest rate of 3.5% had been proposed.[23] But convincing arguments for such conventions are still lacking.[24] In fact, the use of a forestry interest rate makes the Faustmann formula compatible to real behavior. Thus, the observable forest management reconciles something that is felt to be right with a supposedly correct course of action, which unfortunately does not quite fit the plan. Consequently, demands from scientists for an allegedly necessary deforestation policy are ignored.[25]

When a forester pays a rent for someone's land, he compensates differences in fertility—and who has nothing special to offer will earn nothing in return. The choice of the profit maximizing rotation period depends on these rent rates. Harvesting takes place when the increase in the value of wood has decreased toward the payment for a part of the earth's surface. If land is free of charge, trees grow up until the maximum return $f(t_m)$, which is equivalent to the minimization of cultivation costs. Production efficiency on fertile land, on the other hand, includes a compensation for Nature's extra powers. The optimal time for logging can be observed between t_J—Joseph II's interval maximizing rent per hectare—and t_m, the span of time until tree growth has peaked.

The deliberations above reveal why fallow field generally is quite cheap: The price of soil as a capitalized rent merely reflects differences in fertility. Against this background, the significance of the Faustmann formula fades away, even when its proper purpose is considered, namely the determination of pure land value. The productive power of soil merely provides an extreme solution never attained in practice.

Demand dictates the price of real estate when supply cannot be increased. Contrary to timber, the available ground and its quality is a fixed

[23]See Max Robert Pressler, *Der Rationelle Waldwirth und sein Waldbau des höchsten Ertrages, Zweites (selbstständiges) Buch, Die forstliche Finanzrechnung mit Anwendung auf Wald-Werthschätzung und -Wirthschaftsbetrieb* (Dresden: Tuerk, 1859), p. 10.
[24]See the reflections of Wolfgang Sagl, *Bewertung in Forstbetrieben* (Berlin and Wien: Blackwell Wissenschafts-Verlag, 1995), p. 59.
[25]Swiss foresters have to deal with the following instruction: "The cultivation cycle within Switzerland needs to be reduced by one third . . . and the average wood supply is to be reduced to 50 per cent of today's value." Peter Manz, *Die Kapitalintensität der schweizerischen Holzproduktion, Eine theoretische und empirische Untersuchung* (Bern: Paul Haupt, 1987), p. 189.

quantity. Hence, the rent requested for its use reflects scarcity. In capitalism, these circumstances determine the value of landed property.

Yet, forestry comes up with another peculiarity: In Central Europe, there is almost no stock of trees currently available for rent. The rather long gestation periods require contracts with a legal force over several generations. According to § 594 b BGB, German law provides that rent contracts signed for more than thirty years have a period of notice of just one year after this time. The only alternative would be to sign the contract for the lifetime of the renter or the landowner which also does not guarantee the necessary long-term planning certainty. Hence, forestry is performed almost exclusively by the landowner.

CONCLUSION

In closing, we will point out the capital theoretic implications of the preceding analysis. Provided that the forests possess a perfectly adjusted age spectrum, the interest rate is of no special significance. Though the return of a single tree depends on its maturity, there is a quasi-physically determined way of generating the maximum surplus. It is the task of the accumulation process to install the optimal production structure efficiently.

Just as with the continuous and circular production in the industrial sector, one has to free oneself from the concrete product and the time until its completion in order to consider the flows as a whole. In any case, it is misleading to interpret the interest on the costs during a production period as profit, instead of paying attention to the difference between revenue and costs.[26] The production process in general is not organized successively but synchronized. Hence, the result of this investigation fits into a uniform and elementary theory of the choice of technique, which offers more explanatory power than other endeavors to treat the subject.

APPENDIX A

In footnote 21, it has been remarked that the accumulated debt from the purchase of land at a price equal to the Faustmann value (6.23) and the

[26]See in detail Fritz Helmedag, "Warenproduktion mittels Arbeit oder die Neueröffnung der Debatte," in *Nach der Wertdiskussion*, edited by Kai Eicker-Wolf, Torsten Niechoj, and Dorothee Wolf (Forschungsgruppe Politische Ökonomie: Marburg, 1999), pp. 67–91.

subsequent planting of the whole area cannot be retired out of revenues. The share of interest in total debt amounts to

$$\left(\frac{f(t)\mathrm{e}^{-it} - L}{1 - \mathrm{e}^{-it}} + L\right)\left(\mathrm{e}^{it} - 1\right) = f(t) - L \tag{A6.1}$$

Thus, net revenues are just enough to pay interest.

If land is purchased successively, production also covers merely interest. During the gestation period of the staggered silviculture, the entrepreneur has to buy additional plots step by step. Substituting in the formula for the debts from the gradual acquisition of land (6.35) the Faustmann value (6.23), leads to the interest charge at time t:

$$i \cdot D_{\mathrm{Buy}} = i(KW_F + L)\left(\frac{\mathrm{e}^{it} - 1}{it}\right) = i\left(\frac{f(t)\mathrm{e}^{-it} - L}{1 - \mathrm{e}^{-it}} + L\right)\left(\frac{\mathrm{e}^{it} - 1}{it}\right) \tag{A6.2}$$

This expression boils down to:

$$i \cdot D_{\mathrm{Buy}} = \frac{f(t) - L}{t} \tag{A6.3}$$

Obviously, the successive sale of soil at the Faustmann value entails a synchronized production. This is the only way to pay the burden of interest with the proceeds. Redemption, let alone profit, is out of the question. Actually, the Faustmann capital value sets an upper limit to the price of a piece of land.

APPENDIX B

Now we fulfil the promise given in footnote 22, namely to illustrate in some detail how the planting of a synchronized forest comes about. Consider a forester who owns a plot of soil. Investments are financed by loans. It is to clarify whether the agent will be free from the "fetters of interest" at the end of the construction period. Then, he can continuously pocket profits according to the Joseph II rule.

One might think that—notwithstanding the intention to create finally a staggered forest—the whole area is planted in the first instance. In the years to come $(1/t)$-th of the stock is sold and replanted respectively. This procedure, however, has the disadvantage that trees are cut which do not refund their compounded planting costs during the start-up period. Such

loss-making deals must be excluded. The critical minimum growth time t_K results from:

$$f(t_K) = Le^{it_K} \tag{B6.1}$$

Inserting the data of the example gives:

$$t_K = 4.63 \tag{B6.2}$$

At the beginning of the project the open space in percent is:

$$\frac{1}{t_J} \cdot t_K = 40.99\% \tag{B6.3}$$

Thus, initially the forester cultivates approximately 60% of the soil. Let us first calculate the cumulated costs up to the minimum age t_K. The first planting amounts to:

$$FP = 0.5901 \cdot 100 \cdot e^{0.1 \cdot 4.63} = 93.757 \tag{B6.4}$$

Besides, the costs of the succeeding seedlings have to be taken into account:

$$SP = \int_0^{4.63} \frac{1}{t_J} \cdot 100 \cdot e^{0.1(4.63-t)} dt = 52.127 \tag{B6.5}$$

After 4.63 years, the forester faces a totally wooded area and a mountain of debt to the tune of:

$$\mathrm{D}(\mathrm{t_K}) = FP + SP = 93.757 + 52.127 = 145.884 \tag{B6.6}$$

Short of knowing how the repayment is stipulated, we charge interest until t_J:

$$D(t_J) = 145.884 \cdot e^{0.1(t_J-4.63)} = 284.124 \tag{B6.7}$$

But from t_K onwards there are net revenues that are brought to a bank in order to yield interest:

$$\mathrm{N}(t_J) = \int_{4.63}^{t_J} \frac{1}{t_J}(f(t) - 100)e^{0.1(t_J-t)} dt = 665.408 \tag{B6.8}$$

Balancing gives:

$$V(t_J) = \mathrm{N}(t_J) - D(t_J) = 665.408 - 284.124 = 381.284 \tag{B6.9}$$

Apparently, once the forester has created a synchronized silviculture, he possesses not only a fortune of $V(t_J) = 381.284$ but he also receives the maximum profit $PG_J = 169.108$ per hectare and year from this time on.