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# Basic Bidding Formats: Characteristics and Differences 

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#### Abstract

In standard auction theory, the 'revenue equivalence theorem' asserts that the outcomes of the elementary allocation methods coincide. However, bidding processes differ fundamentally with regard to the decision situation of the participants: Is it at all imperative to take into consideration the number of competitors ('stochastic’ strategy) or not ('deterministic’ course of action)? Furthermore, established auction theory neglects the operating modes of procurement alternatives under uncertainty. Apart from the lacking knowledge how many rivals have to be beaten, tenderers regularly are ignorant of the buyer's reserve price. Then it is even more tentative to calculate an offer based on probability theory. Consequently, the suppliers' propensity to collude increases.


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## 1 Price Competition for a Given Quantity

From an individual's point of view, making a bargain means: sell expensively, buy cheaply! In case the article of trade is indivisible, the transaction requires specific bidding procedures. Yet, despite all dissimilarities, canonical auction theory assures that the essential methods lead to the same price and are efficient as well, i.e. the most powerful candidate wins. In textbooks, this 'revenue equivalence theorem' is even celebrated as "the biggest result in auction theory" (Rasmusen 2007, p. 403).

The theorem is based on an analysis where alternative auctions are modelled as two-stage games (cf. Harsanyi 1967, 1968a, b). At first, the type of a player is determined by nature (or privately drawn). Accordingly, all contestants form a

[^0](Bayesian) decision function. Then, a (Nash) equilibrium is achieved where every player choses the best response given the best responses of the other participants.

Alas, that description doesn't tell the true story how things happen in real life. The very coexistence of various practices long since provides sufficient reason for doubt about the supposed congruence of outcomes (cf. Lucking-Reiley 1999). Consequently, the usual Bayesian-Nash equilibrium logic has to be overcome by introducing alternative behavioural rules.

The present study probes into the particular characteristics of the basic bidding procedures. For this purpose, it is important to consider separately the possibilities to cede or obtain a specified item for money. The literature, however, usually assumes that sales and purchases represent two sides of the same coin: "The process of procurement via competitive bidding is nothing but an auction, except that in this case the bidders compete for the right to sell their products or services" (Krishna 2010, p. 1). ${ }^{1}$ Correspondingly, established auction theory centres on methods to vend an object. ${ }^{2}$

The following enquiry also starts with such transactions. But the subsequent scrutiny will show that strategic behaviour in the supply of goods suffers from a systematic deficient knowledge vis-a-vis the corresponding demand processes. This fact increases the propensity for agreements among bidders. Finally, proposals for regulating invitations to tender are presented.

## 2 Selling Procedures

### 2.1 The Standard Types

To begin with, allocation arrangements are examined from the auctioneer's perspective who aims to receive the highest price for the article on sale. Each bidder is supposed to possess an individual limit, which indicates the maximum willingness to pay for the object. It is of no importance here whether the bargain hunter ascribes a 'private value' with a particular pecuniary equivalent to, say, a painting, or whether (s)he imputes any 'common value' in money terms, e.g. to the potential exploitation of a particular mine. ${ }^{3}$ Table 1 portrays the two major types of auctions and their specific subcategories (cf. Molho 1997, p. 211).

[^1]Table 1: Bidding procedures for sales.

| Property | Deterministic: no conjecture about the <br> number of competitors required | Stochastic: conjecture about the <br> number of competitors required |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Auction name | English | Vickrey | Dutch | First price |
| Method | Open (multiple) <br> increase | Sealed single bid | Open <br> (continuous) <br> decrease | Sealed single <br> bid |
| Determination <br> of award | Iteratively from <br> minimum price to <br> last bidder | Definitively to <br> highest bidder for <br> the offer of the <br> second-highest <br> bidder | Definitively from <br> highest bid to <br> first bidder | Definitively to <br> highest bidder <br> for highest offer |
| Behaviour | Overbidding to <br> the limit | Bidding the limit | Strategy | Strategy |

'Deterministic' models specify the buyer either publicly via a successive (sufficiently small) increase of bids in an 'English-auction' or in a procedure suggested by Vickrey (1961). In this variant contenders submit a covert offer, typically presented on a sheet of paper in an envelope. All participants know that the highest bidder will receive the good for the nearest lower quotation ('second-price-sealed-bid'). According to the principle of indifference (or insufficient reason) we assume that the limits of the relevant $1,2, \ldots, n-1$ contenders are equally distributed between a minimum price $B \geq 0$ and the utmost willingness to pay $L_{\mathrm{n}}>B$. Then the object is awarded to buyer n who has to disburse an amount $G_{D}$, determined by the second highest limit $L_{\mathrm{n}-1}$ :

$$
\begin{equation*}
G_{D}=L_{\mathrm{n}-1}=B+\frac{\mathrm{n}-1}{\mathrm{n}}\left(L_{\mathrm{n}}-B\right) \tag{1}
\end{equation*}
$$

In both deterministic formats, a participant's dominant strategy consists simply in bidding either up to or directly the individual price-cap. The winner's benefit is the difference between his or her maximal willingness to pay and the second highest limit at which the penultimate candidate drops out. ${ }^{4}$ All other attendees do not matter for the result of the bidding process, though they may participate in an open auction at the outset or reveal their willingness to pay in a sealed single bid.

The two 'stochastic' allocation methods are far more complicated because they actually constitute a 'one-shot' game under uncertainty. ${ }^{5}$ In a 'Dutch auction', an

[^2]initial (and obviously) overcharged amount is continuously lowered until the first customer signals acceptance. Then, e.g. a batch of flowers is knocked down for that price. In a 'first-price-sealed-bid', concealed proposals are submitted and the good will be sold for the highest offer.

Contrary to an English- or a Vickrey-design, stochastic formats do not permit a simple 'mechanical' behaviour, i.e. bidding gradually up to or directly the individual limit. Instead, every decision maker faces the dilemma that a high bid increases the probability of success but decreases the advantage from the deal. Thus, an analytic approach is required to make the best of the conflict.

### 2.2 Strategic Behaviour of Demanders

In the just described situations an applicant neither knows how many rivals there are nor their definite offers. In order to cope with this uncertainty, standard auction theory presupposes that the contestants' bids are random numbers drawn from probability and density functions which are common knowledge. Yet, this assumption may be a reasonable substitute for lacking information in theory, alas not in practice. Actually, one thing is for sure: In order to make a bargain, every aspirant will spend no more than the individual appreciation of the item in money terms. Therefore, the selling price definitely won't exceed the highest willingness to pay.

Though every participant naturally hopes to make the victorious bid, only one of them really holds the object on sale in utmost esteem. Let us slip in the shoes of this candidate n with the maximum limit $\left(L_{\mathrm{n}}\right)$. To be sure, ( s )he will make an offer $\left(G_{\mathrm{n}}\right)$ which raises the reserve price $B$ by an initially unspecified fraction $g(0 \leq g<1)$ of the potential price spread $\Delta=L_{\mathrm{n}}-B>0$ :

$$
\begin{equation*}
G_{\mathrm{n}}=B+g\left(L_{\mathrm{n}}-B\right)=B+g \Delta \tag{2}
\end{equation*}
$$

In case of success, the strategist's rent $\left(R_{\mathrm{n}}\right)$ comes to:

$$
\begin{equation*}
R_{\mathrm{n}}=L_{\mathrm{n}}-G_{\mathrm{n}}=L_{\mathrm{n}}-\left(B+g\left(L_{\mathrm{n}}-B\right)\right)=(1-g)\left(L_{\mathrm{n}}-B\right)=(1-g) \Delta \tag{3}
\end{equation*}
$$

Evidently, $g$ coincides with the probability to beat any random bid of a single rival in the applicable range. Yet, for more than one competitor, bidder $n$ has to develop a precise idea of how many applicants - who all hope to win (otherwise they would not participate) - are in the running. It is likely that a relatively high reserve price reduces the estimated number of relevant contenders and vice versa. If the assessment proves to be correct, the strategist receives the article with probability $p_{\mathrm{n}}(g)$ :

$$
\begin{equation*}
p_{\mathrm{n}}(g)=g^{\mathrm{n}-1} \tag{4}
\end{equation*}
$$

Then, the expected value of the optimizer's rent $E\left(R_{\mathrm{n}}\right)$ is calculated to:

$$
\begin{equation*}
E\left(R_{\mathrm{n}}\right)=p_{\mathrm{n}}(g) R_{\mathrm{n}}=g^{\mathrm{n}-1}(1-g) \Delta \tag{5}
\end{equation*}
$$

A risk-neutral bidder aims to maximize this term. ${ }^{6}$ The necessary condition reads:

$$
\begin{equation*}
\frac{\partial E\left(R_{\mathrm{n}}\right)}{\partial g}=(\mathrm{n}-1) g^{\mathrm{n}-2}(1-\mathrm{g})-\mathrm{g}^{\mathrm{n}-1} \Delta=(\mathrm{n}-\mathrm{ng}-1) g^{\mathrm{n}-2} \Delta=0 \tag{6}
\end{equation*}
$$

Solving gives:

$$
\begin{equation*}
g^{*}=\frac{n-1}{n} \tag{7}
\end{equation*}
$$

At $g^{*}$ the second derivative of expression (5) is negative for $\mathrm{n}>1$, so that the sufficient condition is fulfilled. As a result, the bidder with the supreme limit has found the best surcharge $g^{*} \Delta$ on the minimum price. Obviously, the offer varies with the predicted count of attendees $n .{ }^{7}$

But note the difference: In deterministic procedures the limits of $(n-1)$ partakers were equally distributed between the reserve price $B$ and the maximum willingness to pay $L_{\mathrm{n}}$, i.e. the price margin $\Delta .^{8}$ Thus, the price-setting second highest limit rises with the number of contenders. In a stochastic environment, however, the rivals' bids are taken as random realizations on this line segment. By no means is it thereby assumed that all vying candidates behave haphazardly. Instead, without further knowledge, it appears from the decision maker's point of view as if the assumed other $(n-1)$ bids in the relevant interval are drawn arbitrarily. ${ }^{9}$

After re-substituting the optimal fraction of the price spread (7) into Eq. (4), the probability to beat all other applicants comes to:

$$
\begin{equation*}
p_{\mathrm{n}}\left(g^{*}\right)=\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1} \tag{8}
\end{equation*}
$$

Interestingly, the prospect for success converges towards a positive minimum:

$$
\begin{equation*}
\hat{p}_{\mathrm{n}}\left(g^{*}\right)=\lim _{\mathrm{n} \rightarrow \infty}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}=e^{-1} \approx 0.368 \tag{9}
\end{equation*}
$$

[^3]Even with 'infinitely many' contenders, the player prevails in more than one third of all cases. Remarkably, as shown in Figure 1, already with six competitors the probability to triumph comes close to the lower bound.


Figure 1: Probability of success.

Inserting $g^{*}(7)$ in the bidding function $G_{\mathrm{n}}(2)$ yields the strategist's offer:

$$
\begin{equation*}
G_{\mathrm{n}}=B+\frac{\mathrm{n}-1}{\mathrm{n}}\left(L_{\mathrm{n}}-B\right) \tag{10}
\end{equation*}
$$

The expected surplus for candidate n is:

$$
\begin{equation*}
E\left(R_{\mathrm{n}}\right)=p_{\mathrm{n}}\left(g^{*}\right)\left(1-g^{*}\right) \Delta=\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}\left(\frac{1}{\mathrm{n}}\right) \Delta \tag{11}
\end{equation*}
$$

If, in a first approximation, 'many' potential buyers encompass just $\mathrm{n}=6$ attendees, the optimizer, employing Eq. (10), increases the bottom price $B$ by $83.3 \%$ of the difference to his or her limit and anticipates, by virtue of formula (11), $6.7 \%$ of the margin as rent. But hope can be deceptive not only because the multitude of candidates has been erroneously specified.

### 2.3 The Bids in Comparison

According to the prevailing doctrine, all players behave strategically with the same n in formula (8) since the number of participants is treated as common knowledge (cf. Krishna 2010, p. 12). Consequently, the auction-goer with the superior willingness to pay systematically obtains the good. Thus, efficiency is ensured as in
deterministic designs. Moreover, for an identical n , a comparison of the strategic bid $G_{\mathrm{n}}$ (10) with the victorious offer in deterministic formats $G_{D}$ (1) seems to verify the 'general equivalence theorem': Ostensibly, the canonical bidding formats yield the same outcome.

Yet, the usual account elevates an exception to the rule. Actually, the individual who values the article on sale most may lose against another contestant, even if this person employs the same bidding rule (10) but supposes more applicants. For illustration purposes, consider the following situation. Let the minimum price be zero, i.e. $B=0$. The partaker with the maximum $\operatorname{limit} L_{\mathrm{n}}=1$ assumes five competitors. Consequently, according to formula (10), his or her offer is (4/5) $1=0.8$. The potential purchaser with the second highest price-cap $L_{n-1}=0.9$ uses the same precept but guesses that 10 rivals are in the running. On condition that all other offers are lower, (s)he triumphs with the bid $(9 / 10) \cdot 0.9=0.81$. Clearly, the allocation contradicts efficiency.

To be sure, in a stochastic environment no attendee can predict the exact behavioural conduct of every single rival. Thus, it is obvious to treat the relevant contestants as random bidders. Providing this assumption reflects reality well and the number of candidates has been precisely anticipated by the competitor with the supreme limit, it is nonetheless quite possible that (s)he walks away emptyhanded. Then, an inferior outcome also ensues as in the numerical example above.

In order to calculate the successful contender's mean bid $G_{C}$, let $x$ denote a part of the price spread $\Delta$ in the interval $\frac{\mathrm{n}-1}{\mathrm{n}} \leq x \leq 1$. The probability that $x$ exceeds the evenly distributed other $(\mathrm{n}-2)$ fractions of $\Delta$ is $x^{\mathrm{n}-2}$. Hence the following condition must hold:

$$
\begin{equation*}
\int_{\frac{n-1}{n}}^{1} x \cdot x^{\mathrm{n}-2} d x=g_{C} \int_{\frac{n-1}{n}}^{1} x^{\mathrm{n}-2} d x \tag{12}
\end{equation*}
$$

Equation (12) results in:

$$
\begin{equation*}
g_{C}=\frac{\int_{\frac{n-1}{n}}^{1} x \cdot x^{n-2} d x}{\int_{\frac{n-1}{n}}^{1} x^{n-2} d x}=\left(\frac{n-1}{n}\right)\left(\frac{1-\left(\frac{n-1}{n}\right)^{n}}{1-\left(\frac{n-1}{n}\right)^{n-1}}\right)>g^{*} \tag{13}
\end{equation*}
$$

Of course, an arbitrary guess on the number of candidates can turn out wrong. Therefore, the strategist possibly resorts to the convergence property of the probability to win. Consequently, (s)he inserts e.g. six potential purchasers n in expression (7) and calculates $g^{\star}(6)=0.833$. But it may well be that, according to formula (10), the ensuing bid $G_{\mathrm{n}}$ is lower than $G_{C}$ since Eq. (13) yields $g_{C}(6)=0.927$ :

$$
\begin{equation*}
G_{C}=B+g_{C} \Delta=B+0.927 \Delta>G_{\mathrm{n}}=B+g^{*} \Delta=B+0.833 \Delta \tag{14}
\end{equation*}
$$

Tough the optimizer's bid $G_{\mathrm{n}}$ (10) corresponds to the result in case of a deterministic format $G_{D}(1)$, the item is now not necessarily awarded to the bidder who appreciates it most. Perhaps another individual who holds a lower limit triumphs with a bid $G_{C}$.

The expected price $E(\varnothing G)$ amounts to the sum of the weighted offers:

$$
\begin{align*}
& E(\varnothing G)=B+\left(p_{\mathrm{n}}\left(g^{*}\right) g^{*}+\left(1-p_{\mathrm{n}}\left(g^{*}\right)\right) g_{C}\right) \Delta=B+\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}} \Delta+ \\
& +\frac{(1-\mathrm{n})\left(\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}-1\right)}{\mathrm{n}} \Delta=B+\frac{\mathrm{n}-1+\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{\mathrm{n}} \Delta>G_{\mathrm{n}} \tag{15}
\end{align*}
$$

From Eq. (15) follows that stochastic formats are on average inefficient, even if the strategist by some amazing fluke correctly estimates the number of competitors. Against this backdrop, it seems sensible to ponder on a statuary prohibition of formats where contenders act under uncertainty. By then the auctioneer will choose that deterministic or stochastic design which promises to fetch the highest price irrespective of allocation issues (cf. Milgrom 1989, p. 10 ff.). Anyway, the possibility to establish the one or the other bidding procedure opens up the scope for action to sellers. But market power increases all the more when it comes to acquire a good.

## 3 Purchasing Procedures

### 3.1 The Standard Types

In the following, the basic types for selecting a supplier will be examined from the perspective of the buyer or customer who specifies the good to be provided. ${ }^{10} \mathrm{By}$ assumption, every competitor knows the minimum price for which (s)he can deliver the ordered article or service. This amount is often based on the estimated costs that an acceptance of a commission entails. Table 2 shows that, similarly to sales procedures, there are four alternatives. Yet, from an economic point of view, they form two major strands.

[^4]Table 2: Bidding procedures for purchases.

| Property | Deterministic: no conjecture about the <br> number of competitors required | Stochastic: conjecture about the <br> number of competitors and <br> maximum willingness to pay <br> required |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Name | Tendering <br> (descending <br> auction) | Vickrey submission <br> (second price) | Dutch tendering | Submission (first <br> price) |
| Method | Open (continuous) <br> decrease | Sealed single bid | Open increase | Sealed single bid |

Ultimately, in deterministic processes the cheapest attendee gets the order. This person is the lowest bidder either in an open descending process starting from an unrealistically high amount or in a sealed Vickrey-submission, where the losing runner-up determines the payment in return for the provision. The advantage for the remaining successful candidate consists in the difference to his or her bottom price. Both mechanical selection techniques lead to the same result and do neither require considerations regarding optimization nor suppositions on the count of contenders. Again, in deterministic environments a dominant strategy for aspirants exists.

But the level of uncertainty in stochastic procurement methods has increased once again in comparison to auctions, where the bidding formula only necessitates to assess how many rivals are involved. For the first or lowest applicant in an open procedure ('Dutch tendering') or a sealed single bid ('submission') an optimal asking price entails additionally to develop an idea about the purchaser's maximum willingness to pay. In this respect, however, a strategically acting tenderer is largely left in the dark, because it is now not his or her own limit that forms an upper bound of an offer, but the buyer's covert reservation expenditure. Before discussing potential consequences for competition policy, it is worthwhile to inquire into the purposeful behaviour of a risk-neutral agent seeking to undercut his or her rivals.

### 3.2 Strategic Behaviour of Suppliers

Initially, in order to detect the formal relationships, a special kind of lottery comes into focus. A participant has the opportunity to demand a premium between zero and one hundred cents. The person earns the declared sum if a previously announced number of ( $n-1$ ) random draws without replacement - representing claims of other contenders - turn out higher. All results are supposed to be located on a line segment with the length of 1 . Contrary to strategic price competition, in the present circumstances the player knows how many times his or her request has to remain unbeaten. The probability $p_{\mathrm{n}}(f)$ that the desired reward $f$ prevails is:

$$
\begin{equation*}
p_{\mathrm{n}}(f)=(1-f)^{\mathrm{n}-1} \tag{16}
\end{equation*}
$$

Since in case of success the benefit $Q$ equals the payment $f$, the expected gain $E\left(Q_{n}\right)$ amounts to:

$$
\begin{equation*}
E\left(Q_{\mathrm{n}}\right)=p_{\mathrm{n}}(f) Q=(1-f)^{\mathrm{n}-1} f \tag{17}
\end{equation*}
$$

The necessary condition for an optimum requires:

$$
\begin{equation*}
\frac{\partial E\left(Q_{\mathrm{n}}\right)}{\partial f}=(1-f)^{\mathrm{n}-1}-f(\mathrm{n}-1)(1-f)^{\mathrm{n}-2}=-\frac{(f \mathrm{n}-1)(1-f)^{\mathrm{n}}}{(1-f)^{2}}=0 \tag{18}
\end{equation*}
$$

Solving for the best prize money gives:

$$
\begin{equation*}
f^{*}=\frac{1}{\mathrm{n}}=1-g^{*} \tag{19}
\end{equation*}
$$

At $f^{*}$ the sufficient condition is fulfilled for $\mathrm{n}>1$. Therefore, this wish maximizes the average payoff. The probability to succeed is calculated to:

$$
\begin{equation*}
p_{\mathrm{n}}\left(f^{*}\right)=\left(1-\frac{1}{\mathrm{n}}\right)^{\mathrm{n}-1}=\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}=p_{\mathrm{n}}\left(g^{*}\right) \tag{20}
\end{equation*}
$$

As in Eq. (9), expression (20) converges towards a positive minimum:

$$
\begin{equation*}
\hat{p}_{\mathrm{n}}\left(f^{*}\right)=\lim _{\mathrm{n} \rightarrow \infty}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}=\widehat{p}_{\mathrm{n}}\left(g^{*}\right)=e^{-1} \approx 0.368 \tag{21}
\end{equation*}
$$

The strategist also triumphs in more than one third of occasions, and the probability to be victorious quickly approaches the lower bound, as Figure 1 shows. The expected advantage amounts to:

$$
\begin{equation*}
E\left(Q_{\mathrm{n}}\right)=p_{\mathrm{n}}\left(f^{*}\right) f^{*}=\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}\left(\frac{1}{\mathrm{n}}\right) \tag{22}
\end{equation*}
$$

Again, if $\mathrm{n}=6$ is accepted as indicating 'fierce competition', the optimizer demands (rounded down) 16 cents and can hope for $42 \%$ of this sum, i.e. 6.7 cents.

### 3.3 The Results in Comparison

It is possible that the strategist loses against a random call $f_{C}$. Let $y$ denote a request in the interval $0 \leq y \leq \frac{1}{n}$. The probability for $y$ to undercut the evenly distributed other ( $\mathrm{n}-2$ ) asking prizes comes to $(1-y)^{\mathrm{n}-2}$. Then, the ensuing condition must hold:

$$
\begin{equation*}
\int_{0}^{\frac{1}{n}} y \cdot(1-y)^{\mathrm{n}-2} d y=f_{C} \int_{0}^{\frac{1}{n}}(1-y)^{\mathrm{n}-2} d y \tag{23}
\end{equation*}
$$

Equation (23) leads to:

$$
\begin{align*}
& f_{C}=\frac{\int_{0}^{\frac{1}{n}} y \cdot(1-y)^{\mathrm{n}-2} d y}{\int_{0}^{\frac{1}{n}}(1-y)^{\mathrm{n}-2} d y}=\frac{\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}(2 \mathrm{n}-1)+1-\mathrm{n}}{\mathrm{n}\left(\mathrm{n}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}+1-\mathrm{n}\right)}=  \tag{24}\\
& =1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)\left(\frac{1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}}\right)=1-g_{C}
\end{align*}
$$

The average reward $E(\varnothing f)$ amounts to:

$$
\begin{align*}
& E(\varnothing f)=p_{\mathrm{n}}\left(f^{*}\right) f^{*}+\left(1-p_{\mathrm{n}}\left(f^{*}\right)\right) f_{C}=\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1} \frac{1}{\mathrm{n}}+ \\
& +\left(1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}\right)\left(1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)\left(\frac{1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}-1}}\right)\right)=  \tag{25}\\
& =\frac{1-\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{\mathrm{n}}
\end{align*}
$$

Moreover, this time it is possible to calculate the expected rent of the winning competitor since it is equivalent to the weighted remuneration $f_{C}$ :

$$
\begin{equation*}
E\left(Q_{C}\right)=\left(1-p_{\mathrm{n}}\left(f^{*}\right)\right) f_{C}=\frac{\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}(1-2 \mathrm{n})+\mathrm{n}-1}{\mathrm{n}(\mathrm{n}-1)} \tag{26}
\end{equation*}
$$

With Eqs. (22) and (26), the strategist's anticipated share in the overall surplus can be ascertained:

$$
\begin{equation*}
\frac{E\left(Q_{\mathrm{n}}\right)}{E\left(Q_{\mathrm{n}}\right)+E\left(Q_{C}\right)}=\frac{\mathrm{n}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{(1-\mathrm{n})\left(\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}-1\right)} \tag{27}
\end{equation*}
$$

Just as the probability to come first, the proportion converges for an increasing n to a positive limit:

$$
\begin{equation*}
\lim _{\mathrm{n} \rightarrow \infty} \frac{E\left(Q_{\mathrm{n}}\right)}{E\left(Q_{\mathrm{n}}\right)+E\left(Q_{C}\right)}=\lim _{\mathrm{n} \rightarrow \infty} \frac{\mathrm{n}\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}}{(1-\mathrm{n})\left(\left(\frac{\mathrm{n}-1}{\mathrm{n}}\right)^{\mathrm{n}}-1\right)}=\frac{1}{e-1} \approx 0.582 \tag{28}
\end{equation*}
$$

Thus, even with a large number of rivals, the strategist can attract on average nearly $60 \%$ of the (decreasing) payment. But this statement relates to a particular decision-making situation, where, as in the raffle discussed above, the prize offered as reward is known. De facto, however, invitations to tender hardly ever announce the available budget for the procurement. This information deficit impedes the applicant to calculate (and maximize) the expected advantage from the commission. In auctions, on the other hand, the potential purchaser is able to confine possible outcomes to the range between the minimum price and the person's own willingness to pay. Thus, probability theory is applicable. The transfer of analogous considerations to the behaviour of suppliers in stochastic formats requires that the costumer is obliged to publicly declare his or her reservation price. ${ }^{11}$ Without this notification, the prerequisite to optimize a tender is missing.

## 4 Rules for Bids

The highest possible profit a purveyor $i$ can earn equals the difference between the ensuing costs and the client's revealed maximal budget ( $M$ ). In order to adapt the previously reviewed lottery to the situation now prevailing, costs $\left(C_{i}\right)$ are raised by the fraction $f^{*}=1 / \mathrm{n}$ times the maximal rent $\left(M-C_{i}\right)$, which was set to unity in the sweepstake above. Then one gets the quotation $\left(F_{i}\right)$ of a strategically operating tenderer:

$$
\begin{equation*}
F_{i}=C_{i}+\frac{1}{\mathrm{n}}\left(M-C_{i}\right) \tag{29}
\end{equation*}
$$

Even in this milieu, however, the bidding formula does not at all guarantee the triumph of the most economical attendee. Of course, a victorious lower asking

[^5]price is not always just a fluky shot. Instead, it can also be based on precept (29), when both costs and the number of contenders were estimated higher. ${ }^{12}$

Whatever the reason may be: If the most potent candidate fails, the allocation proves to be inefficient. Yet, this is of no significance to those demanders who want to purchase as cheap as possible. Hence, similar to auctions, the chosen bidding method to obtain a good also reflects the economic power relation between the organizing principal and the procuring agents.

Obviously, the deterministic techniques applied to receive or provide an indivisible object entail advantages for contestants. In contrast to a stochastic course of action, aspirants do not have to worry about the multitude of rivals. Entrants simply continue to participate in the bidding process as long as their willingness to pay or their minimal claim has not been reached yet. At the same time, these routines correspond to the Pareto criterion: The superior applicant wins, while the runner-up, though dropping out, fixes the price.

If bidding is executed via the Internet instead of relying on older means of communication, the arranger can hope for a better result, since, with more potential buyers (suppliers), the deciding second-best limit probably increases (decreases). Alas, the mechanical line of action also has its possible pitfalls, because it is not at all safeguarded against manipulation attempts. According to rumours front men intervene every now and then in bidding events to move prices up or down so that the good is sold dearer or bought cheaper. To be sure, these endeavours should not be carried too far because a missing deal with a third party could be the consequence.

Contrary to the established opinion, stochastic formats sometimes turn out to be inefficient. Thus, from the vantage point of competition policy, it seems worth considering whether such methods should be banned. ${ }^{13}$ At any rate, there is an indisputable need to regulate at least acquisitions via first-price-sealed-bids. This inquiry has shown that strategic bidding practices exhibit a fundamental asymmetry between sales and purchases. In the case of auctions, every attendee knows his or her limit. This enables the person, once the number of contenders has been assessed, to optimize the expected gain from the transaction.

In contrast, to date the customer, when inviting for tenders, keeps a low profile regarding the maximum expenditure. Prospective purveyors, therefore, literally

[^6]have no basis to apply probability theory, since they possess little or no information about the highest price the opposite party is ready to pay. This uncertainty furthers the tendency of potential suppliers to engage in prohibited agreements; behaviour that e.g. in Germany constitutes a criminal offense by now. On the other hand, the country's 'Gesetz gegen Wettbewerbsbeschränkungen' (Act Against Restraints of Competition) specifies in Section 97 that at least public procurement should be operated by 'transparent procedures'. In order to comply with this provision, authorities should, in future, reveal their willingness to pay for a specific good, possibly derived from a preliminary calculation (cf. Helmedag 2004). If the contract is conferred on a contender who receives a (perhaps just slightly) lower remuneration, the allegation of overcharging becomes invalid because a reasonable case for damages can no longer be made.

Nevertheless, it appears to be even better that, instead of a first-price-sealedbid, a Vickrey-submission became the norm. This creates the incentive to disclose the true minimal claims, since tenderers, including the winning and most costeffective competitor, are less concerned about accepting a commission which eventually turns out to be a loss-making deal.

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[^1]:    1 This assertion can also be found in the textbook by Monezes and Monteiro (2005, p. 11, fn. 2).
    2 Leitzinger (1988) constitutes an exception to established practice by thoroughly addressing alternative options to find a purveyor.
    3 The 'Winner’s Curse' (cf. Kagel and Levin 1986), which is not addressed in the present paper, rests upon an allegedly too optimistic calculation of the revenue that an acquired asset yields.

[^2]:    4 Depending on the stipulated minimum increment of the bids, the hammer price in an open cry format is (somewhat) higher than in a Vickrey design.
    5 Secondary markets, resale options and (re-)negotiation possibilities are disregarded.

[^3]:    6 Matthews (1987) enquires into the behavior of risk adverse demanders.
    7 Levin and Ozdenoren (2004) investigate situations in which participants assign different probabilities to the respective number of competitors.
    8 Bose et al. (2006) analyze how the design of auctions is affected by ambiguity about the distribution of valuations.
    9 Put in the terminology of level-k-reasoning (see e.g. Crawford and Iriberri 2007) the bidder holds a first-order belief, thinking "... that others select a number at random, and he chooses his best response to this belief" (Nagel 1995, p. 1313).

[^4]:    10 In German, such procurements were formerly often referred to as 'Lizitation', a term that has now fallen into oblivion. In English, however, all bidding formats, both for sales and purchases, are called 'auction'. To avoid misunderstanding, the words 'tendering' or 'submission' are used to denote alternative possibilities to find a purveyor.

[^5]:    11 In order to guarantee the announcement's credibility, an obligation to buy or a liability for damages in case of suspending the purchase should be implemented.

[^6]:    12 If n is the actual count of competitors with their costs evenly distributed in the interval $\left[C_{i}, M[\right.$, then the mechanical price determination coincides with the strategic offer.
    13 "Indeed, some hold the view that one essential role of government is to declare that the rules of certain social 'games' must be changed whenever it is inherent in the game situation that the players, in pursuing their own ends, will be forced into a socially undesirable position" (Luce and Raiffa 1957, p. 97).

