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From 1849 back to 1788: reconciling the Faustmann formula with the principle of maximum sustainable yield

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Abstract

In 1849, Martin Faustmann published a formula for determining the claim on an insurer if fallow woodland has been definitely destroyed. His evaluation is based on an even-aged plantation and a given logging time. The calculation has later been used to derive an allegedly optimal rotation period which nowadays predominates in forestry economics. In fact, the utilized objective function does not engender the superior harvesting strategy but the highest compensation for the damaged ground. However, Faustmann's approach can be generalized in order to maximize the expected value of a timber company's total assets comprising soil and existing stumpage. The best felling practice turns out to be the principle of maximum sustainable yield already decreed by Austria's emperor Joseph II in 1788. As a result, the difference between profitability and efficiency is resolved.

Keywords Renewable resources · Optimal rotation period · Faustmann formula · Maximum sustainable yield

JEL Classification Q23 · D92

When to cut a tree

What is the optimal strategy to produce timber on a particular area? While apparently simple, this question has proven to be rather tricky. Again and again distinguished scholars have tried to provide a correct answer and suggested several best practices.[1] At present, the so-called Faustmann condition is deemed by most forest economists as state of the art.[2] But alternative proposals are still in circulation.[3] The spectrum of solutions is all the more surprising since the decision seems to be quite clear: Lumber proceeds, the costs for planting, logging and transportation as well as the rate of interest are treated as given data. Furthermore, monetary values can be transformed into physical magnitudes in case the (invariable) price of wood is chosen as numéraire. Then, the material net yield per hectare can be computed for every possible age of the cultivation.

What causes the difficulties in this apparently ideal world? Actually, complications arise because conflicting objectives may be pursued, perhaps directed to a quantifiable surplus, a money value or a rate of return. Besides, the choice of the best cutting plan also depends on the respective time horizon. Does the forester consider just one plantation or an incessant silviculture, i.e., a (nonterminating) sequel of cultivations? Accordingly, diverging instructions on how to act can be found in the literature.

Against this backdrop, it appears inappropriate to criticize supposedly wrong proposals where, in truth, different problems are tackled.[4] The present inquiry centers on profit

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1 Overviews offer Samuelson (1976), van Suntum (1995) and Helmedag (2008). Of course, the cutting policy is straightforward, provided that trees of all ages are available in abundance and the expense for harvesting a tree is constant. Accordingly, it should be cropped when the maximum height has been reached. Then, input per unit of output is lowest.

2 See, e.g. Conrad (2010, p. 136 ff.).

3 For instance a 'best-seller' in microeconomics propagates to maximize the present value of a *single* cultivation: "The optimal time to cut a forest is when its growth rate just equals the interest rate" Varian (2010, p. 211). This recommendation is named 'Jevons/Fisher' rule. See Helmedag (2008, p. 152).

4 Kant (2013) holds the same opinion.

maximization which is regarded as the typical goal of enterprises in capitalism. Consequently, under the presupposed circumstances, the management of a timber company strives after the highest permanent gain per period which entails the greatest present value of the firm's assets.

This investigation starts with a contribution by the German forester Martin Faustmann who estimated the economically justified compensation for devastated woodland. Subsequent researchers used his formula to derive a seemingly optimal cutting instruction in timber production. It will be shown, however, that this nowadays generally accepted harvesting rule provides inferior results. Yet, it is possible to reconcile Faustmann's considerations with a simple and rather old but top performing felling order.

A loss, an assessment, and an optimization

In 1849, Faustmann published his famous article where he determined the value of forest soil which became barren. Such a calculation is needed to claim damages on one's insurance after an area only suitable for silviculture has been definitely destroyed, e.g., by a flood or an infestation with insects. Hence, it is impossible to plant trees on the now infertile ground in the future. Correspondingly, Faustmann posed at the beginning of his treatise the following hypothetical question: "… what is the net annual money yield which bare forest land can provide in perpetuity?"[5] To that effect, Faustmann considered an (imaginary) unlimited chain of production cycles carried out on a particular parcel, e.g., one hectare.

Notionally, an initial investment of the planting costs (L) is followed by an endless number of harvesting and reforestation processes. Each cohort grows a designated period of time (T) which Faustmann treated as given. Part of the revenue per hectare (net of logging costs) $f(T)$ is spent on the seedlings needed for the next round. In the continuous case with an interest rate (i), the 'land expectation value' (PV_S) amounts to:

$$PV_S(T) = -L + (f(T) - L)\,\mathrm{e}^{-iT} + (f(T) - L)\,\mathrm{e}^{-2iT} + \cdots \quad (1)$$

Rearranging this series will prove worthwhile:

$$PV_S(T) = \left(f(T)\,\mathrm{e}^{-iT} - L\right) + \left(f(T)\mathrm{e}^{-iT} - L\right)\,\mathrm{e}^{-iT} + \left(f(T)\mathrm{e}^{-iT} - L\right)\mathrm{e}^{-2iT} + \cdots \quad (2)$$

Now, the rule for infinite sums can be applied:

$$PV_S(T) = \frac{f(T)\mathrm{e}^{-iT} - L}{1 - \mathrm{e}^{-iT}} = \frac{f(T) - L\mathrm{e}^{iT}}{\mathrm{e}^{iT} - 1} \geq 0 \quad \text{for } f(T) - L\mathrm{e}^{iT} \geq 0 \quad (3)$$

The rate of interest i obviously affects the outcome of the Faustmann *formula* (3). The expression determines a compensatory imbursement which should not be confused with an optimization *condition*. A reasonable result exists, albeit only within a certain range. The upper limit of the rate of interest ensures that the present value of the sterile soil does not become negative: The maximal rate of return ($i_{\max}$) on the costs L is obtained for vanishing numerators of the above fractions. This approach corresponds to the shortest conceivable rotation period and is known as the 'Wicksell/Boulding' solution.[6]

In case an unplanted parcel loses its suitability for silviculture, the Faustmann formula (3) specifies the property owner's insurance benefit. In this way, a potential buyer of a clear-felled but undamaged estate makes no advantageous deal when (s)he pays the imputed price for the plot. By construction, for *each* (fixed) rotation period T and *every* (constant) rate of interest i, the surplus ($f(T) - L$) just suffices to cover the compounded interest charge, i.e., the pure borrowing (or opportunity) costs of the expenditures on the site $PV_S(T)$ and the seedlings L:

$$\begin{aligned}\left(PV_S(T) + L\right)\left(\mathrm{e}^{iT} - 1\right) &= \left(\frac{f(T)\mathrm{e}^{-iT} - L}{1 - \mathrm{e}^{-iT}} + L\right)\left(\mathrm{e}^{iT} - 1\right) \\ &= \left(\frac{f(T)\mathrm{e}^{-iT} - L + L\left(1 - \mathrm{e}^{iT}\right)}{1 - \mathrm{e}^{-iT}}\right)\left(\mathrm{e}^{iT} - 1\right) \\ &= \left(\frac{f(T)\mathrm{e}^{-iT} - L\mathrm{e}^{-iT}}{1 - \mathrm{e}^{-iT}}\right)\left(\mathrm{e}^{iT} - 1\right) = f(T) - L\end{aligned} \quad (4)$$

For a uniform rate of interest, arbitrage possibilities are excluded since investors are indifferent as to financing forestry or putting money into a bank. Therefore, considerations regarding an alternative allocation of resources play no role in the present model. Actually, in order to open up a bargain for the purchaser, woodland has to be traded cheaper than the theoretical compensation à la Faustmann. Consequently, in equilibrium his proposal may be employed to perform a good insurance job but not to profitably acquire forest soil from an entrepreneur's point of view.

So far the rotation period T has been treated as an exogenous magnitude. Nevertheless, the answer to the question 'when to cut a tree' is still lacking. Strangely enough, the predominant suggestion in forestry economics asserts that maximizing the damage claim for an area which became infertile would entail the optimal harvesting time (T_F) in

[5] Faustmann (1968, p. 28). In German: „Welches ist der reine Geldbetrag, den ein jetzt holzleerer Waldboden immerwährend in jährlich gleicher Größe liefert?" Faustmann (1849, p. 442). There are some progenitors, e.g. König (1835). Viitala (2013) even refers to Houghton (1683), who mentioned the opportunity costs of stands and bare land, i.e. forest capital.

[6] See Helmedag (2008, p. 148 f.).

practice. Then, the first derivative of (the penultimate fraction in) Faustmann's formula (3) has to vanish:

$$\frac{dPV_S(T)}{\mathrm{d}T} = \frac{\left(f'(T)\mathrm{e}^{-iT} + f(T)\left(-i\mathrm{e}^{-iT}\right)\right)\left(1-\mathrm{e}^{-iT}\right)}{\left(1-\mathrm{e}^{-iT}\right)^2} - \frac{\left(f(T)\mathrm{e}^{-iT} - L\right)\left(i\mathrm{e}^{-iT}\right)}{\left(1-\mathrm{e}^{-iT}\right)^2} = 0 \tag{5}$$

Though Faustmann did not state this requirement, the ensuing *condition* is often named after him[7]:

$$f'(T_F) = \frac{i\left(f(T_F) - L\right)}{1-\mathrm{e}^{-iT_F}} = i\left(\frac{f(T_F)\mathrm{e}^{-iT_F} - L + f(T_F)\left(1-\mathrm{e}^{-iT_F}\right)}{1-\mathrm{e}^{-iT_F}}\right) = i\left(PV_S(T_F) + f(T_F)\right) \tag{6}$$

The maximum indemnity payment for the barren ground is reached once the (fictitious) increase in timber at a certain point in time ($f'(T_F)$) coincides with the (imputed) interest on the corresponding values of soil plus (hypothetical) stumpage. Thus, the right-hand side of formula (6) represents the opportunity costs of (pretended) forest capital at the very moment when harvesting occurs. Furthermore, a rising interest rate would shorten the rotation period and may even lead to over-exploitation. Yet, the Faustmann condition should not be interpreted as a guideline to optimize the gain of an actual timber production. Instead, Eq. (6) is suitable to find the highest value of *vacant* land.

Obviously, a unique and general solution for T_F may not exist. But a specific result can be derived if the output per hectare $f(t)$ and the other data are given. For exposition purposes, the following arbitrary assumptions apply:

$$f(t) = \frac{1}{30}t^4(15-t) \tag{7}$$

$$L = 100 \tag{8}$$

$$i = 10\ \% \tag{9}$$

Based on these details, the (rounded) rotation period amounts to:

$$T_F\ =\ 10.666 \tag{10}$$

Inserting this outcome into Eq. (3) leads to the adequate compensation of a fallow site which has lost its capacity to serve as a habitat for trees:

$$PV_S(10.666) = 828.745 \tag{11}$$

Brought to a bank, the capitalized 'ground rent' per year (R_S) is:

$$R_S = iPV_S(10.666) = 82.8745 \tag{12}$$

To be sure, the maximization of the present value of land on which timber cannot be harvested in the future is not the standard situation in which silviculture operates.

Generalizing Faustmann's approach

Only in exceptional cases, a timber company cares about the appropriate equivalent for the devastation of an uncultivated plot. Normally, the site never lies fallow. In daily business, areas are cropped and the entrepreneur wants to know when reaping is due. Then, irrespective of liabilities, the enterprise's property includes *both* the real estate value *and* the current stand. Contrary to his successors, Faustmann had appreciated this aspect as is noticeable by the title of his classical article: "Calculation of the value which forest land and immature stands possess for forestry."[8] Clearly, an event of damage may also occur when the cultivation is more or less ripe.

Before turning to the sustained production mode with all age classes up to maturity on the cultivated area, an intermittent tillage á la Faustmann is considered where all trees are even-aged. After a designated elapse of time the forest is felled clear. Then, a new round begins. The values of land and lumber depend not only on the rotation period T but also on the stumpage's worth at a certain point in time $t \leq T$. In case of destruction, the insurer's compensation has to cover the forfeiture of soil plus timber.

The formula for the company's *total assets* at a date t ($PV_A(t,T)$) reads:

$$PV_A(t,T) = \left(f(T) + PV_S(T)\right)\mathrm{e}^{-i(T-t)} \text{ with } 0 \leq t \leq T \tag{13}$$

Equation (13) informs about the decent indemnification claim if the possessions of the timber enterprise have been ruined at time t. The potential reimbursement rises continually during the production cycle. At the end, i.e., $t = T$, the compensatory payment has attained its respective maximum. This magnitude corresponds to the notional present value of soil in brackets on the right-hand side of the Faustmann condition (6). Immediately upon harvesting, business assets plunge to the pure land value plus planting expenses. Subsequently, working capital increases anew until the expiration

[7] According to Löfgren (1983), it would be more adequate to refer to Pressler (1859) and Ohlin (1921) as originators of the optimization method. Scorgie and Kennedy (1996) even cite Marshall who anticipated the idea in 1808.

[8] Faustmann (1968).

of the next rotation period is achieved. This permanent up and down movement of holdings recurs again and again.

Of course, the date of an accidental demolition is unknown in advance. Yet, in order to balance the incessant change of wealth, it is possible to ascertain the medium aggregate extent of damage which coincides with the expected amends in the event of loss.[9] Firstly, the weighted amount of all stands from the beginning to the termination of the growth phase including the corresponding land value ($\Sigma PV_A(T)$) is computed:

$$\Sigma PV_A(T) = \int_0^T \left(f(T) + \frac{f(T)\mathrm{e}^{-iT} - L}{1 - \mathrm{e}^{-iT}} \right) \mathrm{e}^{-i(T-t)} \mathrm{d}t = \frac{f(T) - L}{i} \tag{14}$$

Secondly, the division of (14) by the rotation period T leads to a 'generalized' Faustmann formula reflecting the mean operating assets ($\emptyset PV_A(T)$)[10]:

$$\emptyset PV_A(T) = \frac{\Sigma PV_A(T)}{T} = \frac{f(T) - L}{iT} \tag{15}$$

Remarkably, now for all rotations periods T opportunity costs $i \cdot \emptyset PV_A(T)$ are independent from the rate of interest since it cancels out. Maximizing the enterprises' average capital requires:

$$\frac{\mathrm{d}\left(\frac{f(T)-L}{iT}\right)}{\mathrm{d}T} = 0 \tag{16}$$

Thus, the condition for the best lifetime of trees (T^*) generating the highest medium total forest property reads:

$$f'(T^*) = \frac{f(T^*) - L}{T^*} = i \cdot \emptyset PV_A(T^*) \tag{17}$$

Interestingly enough, the rate of interest is also of no significance for determining the optimal rotation period. Merely revenues and costs (expressed as physical quantities) are decisive. From a formal point of view, however, the interpretation of Eq. (17) is in accordance with that of the Faustmann condition (6): The highest earning power is reached when the increase in timber $f'(T^*)$ coincides with the mid-level opportunity costs $i \cdot \emptyset PV_A(T^*)$. In the example, reaping takes place at:

$$T^* = 11.296 \tag{18}$$

This result ensues from a numerical evaluation of condition (17) resting on a specified production function, viz. Equation (7), which is normally unidentified in the real world. Yet, it is not at all necessary to recourse to higher mathematics in order to discover the optimal harvesting time. Due to an alternative interpretation of the problem, it suffices to employ a simple graphical method. Furthermore, this approach makes it possible to easily compare the profitability of particular cutting strategies.

In a continuous production process, soil de facto *never* lies unexploited as it is presupposed in the original Faustmann setting. Instead, a timber supplier aims to establish a *normal* forest where the available ground is split up in pieces of equal size each covered with trees of a certain vintage from zero to maturity T.[11] Evidently, the division of the whole area by the rotation period gives $1/T$ plots with different stands. Then, due to the appropriately staggered age structure the same amount of lumber can be logged perpetually.[12] With regard to the whole estate, so to speak, a sustained operation structure is implemented, whereas on a specific section an intermittent setup is realized. Therefore, the optimization of a steady income should be directed at a normal cultivation's net yield often called 'forest rent.' Consequently, the return on the initial investment for a uniform plantation of the entire hectare (4) has to be divided by the number of parcels T each bearing a different age class:

$$\frac{\left(PV_S(T) + L\right)\left(\mathrm{e}^{iT} - 1\right)}{T} = \frac{f(T) - L}{T} \tag{19}$$

For every subdivision of the company's woodland into T portions, a forest management according to Eq. (19) complies with sustainability since the annual crop equals the yearly increase in timber. Of course, the optimization now refers to the particular stretch which is harvested per annum instead of taking the overall site into account. Fortunately, when output per hectare depending on the age of the stand and planting costs are depicted in a diagram, the optimal lifetime of a tree (T_{J})—according to a decree by Austria's emperor Joseph II in 1788[13]—and thus the number of subareas can straightforwardly be found.[14] Figure 1 shows a tangent drawn from the planting costs L to the production function $f(T)$ forming an angle α which determines the largest mean surplus over time ($(f(T_{\mathrm{J}}) - L)/T_{\mathrm{J}}$). This 'mercantilist' cutting order ensures to reap the highest permanent

[9] Johansson and Löfgren (1985, p. 86) present an analogous expression to Eq. (13) but without calculating the average value of land and standing timber as a point of reference.

[10] Chang (1998) proposed another 'generalized Faustmann formula' where the stumpage price, timber yield, regeneration cost, and interest rate are allowed to vary.

[11] See Amacher et al. (2009, p. 3).

[12] Helmedag (2008, p. 165 f.) tackles the best tilling practice, starting from scratch, to attain the desired synchronized stumpage configuration in year T.

[13] See Osmaston (1968, p. 188).

[14] Incidentally, Faustmann (1856) invented a 'mirror hypsometer' to measure the height of trees.

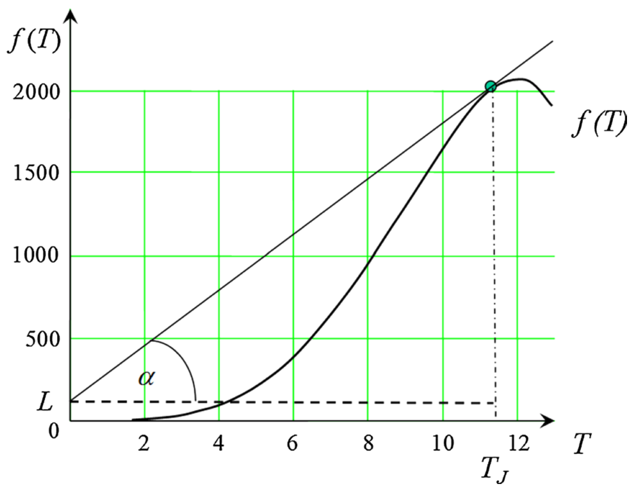

Fig. 1 The rule of 1788

sustainable yield resulting from the equality of marginal and average revenues.

The (diagrammatic) ascertainment of condition (17) leads to the greatest annual surplus if the terrain is correspondingly partitioned. The derived instruction also ensures the timber firm's maximal average value of total wealth (15) comprising land as well as stumpage. Of course, in contrast to forest rent as a flow magnitude, the enterprises' present value is affected by the rate of interest since it serves to discount future profits. As it were, synchronized and successive processes come eventually to the same result because T^* and T_J coincide with regard to the organization of either time or space.

Profitability meets efficiency

Most remarkably, the rotation periods observed in practice exceed by far the hypothetical right ones.[15] Apparently, professional experience advises woodmen to diverge considerably from the Faustmann condition. Moreover, specific silviculture interest rates have been suggested long since, e.g., at the level of approximately 3%.[16] It may also be due to intuition-based judgements that radical deforestation proposals,[17] allegedly in order to enhance gainfulness, have found no favor with the addressees. But the difference between theory and practice can be resolved.

The recourse to moderate interest rates in forestry provides another approach to detect the optimal rotation period in timber production. What happens when the rate of interest vanishes? Applying the rule of de l'Hospital, i.e., differentiating numerator and denominator of the first fraction in Eq. (6) separately with respect to i, gives:

$$f'(T) = \lim_{i \to 0} \left(\frac{i(f(T) - L)}{1 - e^{-iT}} \right) = \lim_{i \to 0} \left(\frac{f(T) - L}{Te^{-iT}} \right) = \frac{f(T) - L}{T} \quad (20)$$

Now, a normal forest emerges directly from the Faustmann condition (6). As a consequence, the old monarchist felling order will prevail: $T_F(i = 0) = T_J$. But for positive rates of interest, the rotation period $T_F(i > 0)$ proves suboptimal. In the example, the sustainable yield ($SY(T_F)$) comes to:

$$\mathrm{SY}(T_F) = \frac{f(T_F) - L}{T_F} = \frac{f(10.666) - 100}{10.666} = 165.920 \quad (21)$$

Although this permanent income is higher than the ground rent R_S reported in Eq. (12), it falls short of the production mode complying with the decree of Joseph II ($SY(T_J)$):

$$\mathrm{SY}(T_J) = \frac{f(T_J) - L}{T_J} = \frac{f(T^*) - L}{T^*} = \frac{f(11.296) - 100}{11.296} = 169.108 \quad (22)$$

Obviously, the enterprise which deviates from the principle of maximum yield dispenses with profit year by year. Therefore, the firm is run uneconomically. In contrast, once the respective age structure of the cultivation is established, the rule of 1788 proves superior since it generates the greatest periodical net gain and ipso facto the highest present value of total assets. Thus, the appropriate specification of a timber company's typical business interest leads back to the beginnings: the most efficient crop growing of renewable resources. But regardless of whether the plot under consideration is cultivated in a sustained or a suspended mode of production, the optimal rotation period can either be inferred graphically from Fig. 1 or analytically from Eq. (17). Consequently, the generalization of Faustmann's approach ensures the reconciliation between profitability and efficiency in forestry.

[15] See Moog and Borchert (2001).

[16] See Pressler (1859, p. 10), Endres (1919, p. 11 f.) and Sagl (1995, p. 59 f.).

[17] See Manz (1987, p. 189).

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