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An Axiomatic Characterization of a Generalized Ecological Footprint

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Abstract

The purpose of this paper is to propose an axiomatic characterization of ecological footprint indices. Using an axiomatic approach, we define a set of axioms to represent the properties considered appropriate to ecological footprint measures in general. It can be shown that there exists a generalized index which is unique up to a strictly increasing function of the world land area appropriated to satisfy human needs. As an implication, the well-known compound-based footprint index used by the Global Footprint Network can be characterized as a specification of the generalized index, only if proportionality in world land area use is additionally assumed and the norm is arbitrarily fixed, in terms of *global hectares*. Instead, we find that the applied methodology of normalization and standardization in the aggregation procedure of footprint indices is completely dispensible. In this respect, the proposition of generalized and axiomatically characterized indices for the measurement of the ecological footprint of human activity may be considered as the main contribution of the paper.

JEL classification: Q13, Q41, Q43, C43

Keywords: Ecological Footprint Index, Axiomatic Foundation, Measurement Theory, Overshoot Day, Sustainable Welfare

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1 Introduction

Each year, the Global Footprint Network is announcing the Earth Overshoot Day prominently reported worldwide in the media. However, there is no reason to celebrate this anniversary, nor is it fixed by date. Rather, it marks the date when humanity's demand for ecological resources and services in a given year exceeds what Earth can regenerate in that year (www.overshootday.org). In a sense, from then on the earth is running into an ecological desaster. Ever since the overshoot day has been calculated for the first time, the data show a permanent backward shift in time. It means that the ecosystem services to support consumption activities as well as the absorption of waste products are used up ever earlier from year to year. The respective data are not only available on the global level but also on the national, indicating a much more eco-friendly use of resources in the less developed countries than in the developed ones. For instance, in 2018 the Overshoot Day for Germany has been May 2, compared to August 1 for the world (Global Footprint Network, 2018a, 2018b). What can be observed is the large inequality between developed and less developed countries in terms of the land area appropriated to satisfy human needs. So, while the ecological footprint of industrialized countries is found much higher than the land area available to them the footprint of less developed countries is much lower.

Not surprisingly, there is a serious debate on how the overshoot day exactly should be determined and what kind of measure should be applied. Literally, over the last few decades, various indices have been proposed in the literature to measure the exertion of renewable natural resources and, consequently, the pressure put on the ecosystem by human activity, in particular in form of the land area used. Among the group of Ecological Footprint Indices typically labelled as the 'footprint family' are the sustainable process index (Krotscheck/Narodoslawsky, 1996, Sandholzer/Narodoslawsky, 2007, Stöglehner/Narodoslawsky, 2012) and the component-based ecological footprint (Simmons/ Lewis/Barrett, 2000) as a life-cycle assessment of economic activities, alongside a number of complementary footprint indices related to different environmental media[1] such as the carbon, water or nitrogen footprint (Galli et al., 2012, Čuček et. al., 2012, Fang et. al., 2014, and others). However, over the last years, the focus has been put on one particular index which especially is being applied in statistical cross-country/cross-regional

[1] In this context, the footprint index indicates the pressure exerted by human activity on the respective environment medium or via the respective pollutant.

comparative studies, and which has prevailed: the compound-based ecological footprint (Wackernagel/Monfreda, 2004, Galli et al., 2016), in short referred to as '*the* ecological footprint', together with the biocapacity index as a complement.

Originating from the pioneer works from WACKERNAGEL and REES (i.a. Rees, 1992, Wackernagel/Rees, 1996, Wackernagel et al., 1999), the concept, methodology, and application of ecological footprinting has been continuously refined, especially by the Global Footprint Network (among many others, see Bicknell et al., 1998, Ferguson, 1999, Venetoulis/Talberth, 2008, Wang/Bian, 2008, Galli et al., 2011, 2012, 2016, Borucke et al., 2013, Collins/Flynn, 2015, and Lin et al., 2018). The common methodology is to calculate the supply of and demand for ecosystem services in terms of units of land area appropriated to annually regenerate and provide these services. For this purpose, *global hectares* have been introduced as a common unit of measurement taking into account the specific productivity of various land types (converting world-average hectares into global hectares through *equivalence factors*) as well as the specific productivity across nations with respect to the same land type (converting nation-specific hectares into world-average hectares through *yield factors*).[2]

While the biocapacity index depicts the supply side, i.e. the 'ecological budget', the demand human activities place on the ecosystem is measured by the ecological footprint. Comparing annual supply of and demand for ecosystem services finally enables an assessment of a country's or region's ecological overshoot, i.e. the deficit due to overexploitation of resources and accumulation of waste (Wackernagel/Monfreda, 2004). In this respect, the overshoot day is simply defined as the ratio between the footprint index and the biocapacity index times 365 (measured in day units).

With the initiative of the Earth Overshoot Day, as mentioned, the Global Footprint Network has found a very effective means to communicate its results in public. Expressing the framework of an annual time line has struck a chord with large parts of the population in the industrialized world. Given that the date is moving backwards every year, the demand for more sustainable consumer lifestyles and structural change towards a low-carbon global

[2]The most recent methodology even accounts for land productivity differences in time by extending the unit of measure to *constant global hectares* through a *world-average intertemporal yield factor*, expressed with reference to a selected base year. This step is aimed to avoid difficulties of interpretation in intertemporal comparison. (Borucke et al., 2013)

economy becomes all the more compelling (Global Footprint Network, 2018a, 2018b).

However, the ecological footprint concept has also been exposed to severe criticism regarding its applicability, methodology, and policy implications (see van den Bergh/Verbruggen, 1999, van den Bergh/Grazi, 2013, and Galli et al., 2016, among many others). One of the major concerns relates to the identification of the expected characteristics of the ecological footprint and biocapacity measures as well as to the accuracy of the measurement scheme itself (Galli et al., 2016). Indeed, these indices have been designed in a more or less ad-hoc fashion and have not been further analyzed yet with respect to their mathematical foundations, at least from an axiomatic viewpoint - in contrast to the majority of economic indices such like price indices or indices of economic inequality which have been characterized axiomatically. See (Tarabusi/Guarini, 2018) as a recent example.

The lack of an axiomatic foundation of such kind is all the more surprising considering that data on the ecological footprint has been recently collected and utilized in the official statistics of various international organizations. Moreover, ecological footprinting has already been included in those institutions' methodologies on environment statistics as an indicator for sustainable land use (see, for instance, OECD, 2008, UNEP, 2010, UN Statistics Division, 2013, 2018). As a consequence, the family of ecological footprint indices still needs to be reviewed with respect to whether the explicit and implicit properties can be considered as appropriate.

This paper therefore intends to provide an axiomatic approach to the ecological footprint concept. We define a set of fundamental axioms representing general properties any footprint measure should fulfill. The advantage of this approach lies in the proposition of a few stylized facts on which there is a clear understanding of how the index should respond. But, most importantly, once the formula of the index has been determined it can be applied to any real-world situation, irrespective of the particular values the independent variables may take. As a main result, we find that a unique index exists which is fully meeting the proposed axiom system. Subsequently, its features are discussed in detail.

The paper is structured as follows: We begin by setting up the axiom system in section 2. Then, in section 3, we propose an ecological footprint measure derived from the axiom system and give an economic and formal interpretation. A discussion on the empirical implications of the main result of the paper for footprint measures follows in section 4, including the widely used compound-based ecological footprint. The paper is concluded with some final remarks in section 5.

2 The Axiom Set

In this section, we are developing a system of axioms for the characterization of the ecological footprint index. Naturally, the axiom system proposed should comprise properties generally accepted in the literature. In particular, it refers to a specific type of product and a specific type of land area use (with given bio-productivity), thus to a so-called footprint component as a constitutional part of the composite measure.[3] First, we will give a general definition.

Definition 1. *We consider a set of countries or regions* $\Omega = \{1, \ldots, n\}$ *and a set of possible world ecological states* $\mathcal{W}$.[4] *In a given ecological state* $w \in \mathcal{W}$ *each country* $i \in \Omega$ *consumes*[5] *a certain amount* $C_i(w) \in \mathbb{R}$ *of some type of product, produces a certain quantity* $Y_i(w) \in \mathbb{R}$ *of this product and utilises a certain area* $A_i(w) \in \mathbb{R}$ *in producing this quantity.*
For convenience, the aggregate consumption of a subset of countries $T \subseteq \Omega$ *in a given world state* $w \in \mathcal{W}$ *is defined as* $C(T,w) := \sum_{i \in T} C_i(w)$*, the aggregate production as* $Y(T,w) := \sum_{i \in T} Y_i(w)$*, and the aggregate land use as* $A(T,w) := \sum_{i \in T} A_i(W)$*.* In addition, we call $S_i(w) := (C_i(w), Y_i(w), A_i(w))$ the individual state of country $i \in \Omega$. ■

Then an *ecological footprint* is defined as a mapping from the subsets of countries and the set of possible world states onto the non-negative real numbers

$$F : \mathcal{P}(\Omega) \times \mathcal{W} \mapsto \mathbb{R}_+$$

if it satisfies the following set of axioms.

[3]The issue of characterizing functions for the aggregation of footprint components over the various product types as well as land area types is left to future research.

[4]It is assumed that the set of possible ecological states is unrestricted in the sense that $\{w \in \mathcal{W} | C_i(w) = c_i, A_i(w) = a_i, Y_i(w) = y_i, i \in \Omega\}$ is non-empty for $c_i, a_i, y_i \in \mathbb{R}_+$. This makes the domain of the index as wide as possible.

[5]Consumption is defined as final demand by households, producers, and the state.

Axiom 1 (Additivity)**.** *The footprint of a given set of countries* $T \subseteq \Omega$ *is the sum of the individual footprints of these countries:*

$$F(T,w) = \sum_{i \in T} F(\{i\},w) \quad \textit{for } w \in \mathcal{W}$$

■

Remark 1. The image of a footprint suggest an areal measure, which is by nature additive.

Axiom 2 (Symmetry)**.** *Let two world states* $w, \tilde{w} \in \mathcal{W}$ *differ in that the individual states are interchanged* $S_{\pi(i)}(\tilde{w}) = S_i(w)$ *for all* $i \in \Omega$ *by a permutation* $\pi : \Omega \leftrightarrow \Omega$*. Then the footprint of the set of countries* $\pi(T)$ *with the same states as the original countries* $T \subseteq \Omega$ *is unchanged.*

$$F(\pi(T),\tilde{w}) = F(T,w) \quad \textit{for } w, \tilde{w} \in \mathcal{W}$$

■

Remark 2. This axiom could have been equally well called the axiom of anonymity, since it means that the footprint is indifferent to the name of a country.

Axiom 3 (Monotonicity)**.** *The footprint of a set of countries* $T \subseteq \Omega$ *is monotonically increasing in their consumption,* $C(T,\tilde{w}) > C(T,w)$*, all else equal, i.e.* $A_i(\tilde{w}) = A_i(w)$ *and* $Y_i(\tilde{w}) = Y_i(w)$ *for* $i \in \Omega$*.*

$$F(T,\tilde{w}) > F(T,w) \quad \textit{for } w \in \mathcal{W}$$

■

Remark 3. As a set of countries increases its aggregate consumption, they demand a higher share of the bioresources that were used in production, thus increasing their burden on the ecosystem. This property is evident and perfectly in line with what an ecological footprint should measure from the view of sustainable land area use.

Axiom 4 (Commensurability in Consumption)**.** *The footprint is not changed by an equally proportional increase in consumption,* $C(T,\tilde{w}) = \lambda C(T,w)$*, and world production,* $Y(\Omega,\tilde{w}) = \lambda Y(\Omega,w)$*, with land area use being equal,* $A_i(\tilde{w}) = A_i(w)$ *for* $i \in \Omega$*.*

$$F(T,\tilde{w}) = F(T,w) \quad \textit{for } w \in \mathcal{W}$$

■

Remark 4. Since the same land area is used as before while increasing production, the share of demand on worldwide bioressources is not changed if its consumption demand increases by the same factor.

Axiom 5 (Commensurability in Production)**.** *The footprint is not changed by an equally proportional change in world production,* $Y(\Omega,\tilde{w}) = \lambda Y(\Omega,w)$*, and world area use,* $A(\Omega,\tilde{w}) = \lambda A(\Omega,w)$*, with aggregate consumption being equal,* $C_i(\tilde{w}) = C_i(w)$ *for* $i \in \Omega$*.*

$$F(T,\tilde{w}) = F(T,w) \quad \textit{for } w \in \mathcal{W}$$

■

Remark 5. World average product yield remaining the same means that a country's consumption requires only the same share of the earth's ecological resources as before, such that the value of its footprint should remain constant.

In the following section, we will derive an index satisfying the entire axiom system.

3 Existence and Uniqueness of the Ecological Footprint Index

Let us now state the following proposition.

Theorem (Existence and Uniqueness)**.** *Axioms 1 through 5 characterize the following unique index up to a strictly increasing function* f*:*

$$F(T,w) := \sum_{i \in T} f\left(\frac{C_i(w)}{Y(\Omega,w)} A(\Omega,w)\right) \quad \textit{for } w \in \mathcal{W}$$

Proof. See Appendix. □

This proposition states that there exists an index which meets the axiom system and that the index is unique up to a strictly increasing function f which can be chosen arbitrarily.

What is the meaning of the index stated above? Interpreted for the case of a country

$i \in \Omega$ the functional form of the index can be read in different equivalent ways. First, it may be interpreted as an index which is strictly increasing in the land area appropriated to provide a country's share of consumption on world production, $C_i(w)/Y(\Omega, w)$

Secondly, the index is strictly increasing in the land area approporiated given world average yield to satisfy a country's consumption needs.

$$F(\{i\}, w) = f(\frac{C_i(w)}{Y(\Omega,w)/A(\Omega,w)})$$

Finally, the index is increasing in a country's consumption share of its production in terms of the national land area used, adjusted by its relative product yield.

$$F(\{i\}, w) = f(A_i(w) \cdot \frac{C_i(w)}{Y_i(\Omega, w)} \cdot \frac{Y_i(\Omega,w)/A_i(\Omega,w)}{Y(\Omega,w)/A(\Omega,w)})$$

Moreover, the trade issue discussed in the literature is resolved since exports and imports can be accounted for as an integrated part from the beginning. Let $C_i(w) = Y_i(w) - XS_i(w)$ with national net exports $XS_i(w)$. Then we get:

$$F(\{i\}, w) = f(\frac{(Y_i(w) - XS_i(w))}{Y(\Omega, w)} A(\Omega, w))$$

Therefore, if a country is a net exporter, i.e. $XS_i > 0$, its ecological footprint is decreasing (increasing) if exports XS_i are going up (down). An equivalent line of reasoning holds if a country is a net importer, i.e. $XS_i < 0$.

4 Implications for Ecological Footprint Indices

What kind of implications follow from the axiomatic foundation of ecological footprint indices in view of empirical application? In fact, it turns out that the index which has been characterized in the last section is a generalization of empirically applied indices. Especially, let us consider the compound-based ecological footprint index given in Wackernagel et al. (1999) (as well as in the subsequent literature referred to in the introduction), and used by the Global Footprint Network.

Formally, the reduced form of this index for the ecological footprint of a single country reads as follows:

$$EF = \sum_{j=1}^{N} FC_j \cdot EQF_j$$

with the footprint component

$$FC_j = \frac{C_j}{\gamma_{world_j}}$$

Here, $j = 1, \ldots, N$ is the index of a given land area type, C_j denotes the annual domestic consumption, γ_{world_j} being the world-average yield, i.e. Y_{world_j}/A_{world_j}, and EQF_j is the equivalence factor, reflecting the world average productivity of different land use types (Global Footprint Network (2019)).

An equivalent formulation is used to account for a country's productivity relative to the world average yield:

$$EF = \sum_{j=1}^{N} \left(C_j / \frac{Y_{n_j}}{A_{n_j}} \right) \cdot \left(\frac{Y_{n_j}}{A_{n_j}} / \frac{Y_{world_j}}{A_{world_j}} \right) \cdot EQF_j$$

with Y_{n_j} denoting the national production, A_{n_j} the national land area appropriated, Y_{world_j} the world production, and A_{world_j} the world land area appropriated.

Actually, if we add the following proportionality axiom to the axiom system above, we can characterize the index by Wackernagel et al. up to an arbitrary coefficient:

Axiom 6 (Proportionality to Land Area Use)**.** *The footprint of a set of countries $T \subseteq \Omega$ is directly proportional to the land area used in world production $A(\Omega, \tilde{w}) = \lambda A(\Omega, w)$, all else equal, $C_i(\tilde{w}) = C_i(w)$, and $Y_i(\tilde{w}) = Y_i(w)$ for $i \in \Omega$.*

$$F(T, \tilde{w}) = \lambda F(T, w) \quad \textit{for } w \in \mathcal{W}$$

■

Remark 6. This axiom, in fact, is implicitly assuming a fixed proportions production function for bio-resources on the global level. In case of world average yield being unchanged the ecological footprint is proposed to be directly proportional to the world land area appropriated to satisfy the same consumption needs. A comparably weaker axiom would be monotonicity, justified by the fact that bioresources are spatially distributed, such that an increase in the worldwide land area used leads to an increase in the demand on the carrying capacity of the ecological system. But, as it will turn out in the following theorem, the proportionality axiom generates a pretty restrictive functional form of the index.

Then, the following theorem can be stated:

Theorem (Existence and Uniqueness 2)**.** *Axioms 1 through 6 characterize the following unique index up to a strictly positive arbitrary coefficient* $\mu > 0$*:*

$$F(T,w) := \mu \sum_{i \in T} \frac{C_i(w)}{Y(\Omega,w)} A(\Omega,w) \quad \textit{for } w \in \mathcal{W}$$

Proof. See Appendix. □

In the index by the Global Footprint Network the norm is implicitly fixed to $\mu = 1$, such that the unit defined corresponds to the land area unit, which is hectare. Hence, in the the aggregation procedure a common scale, with *global hectares (gha)* as unit, is constructed in an ad-hoc fashion to add up the production and consumption of different countries as well as different land area types, where the equivalence factors provide the weighting scheme. This is the reason for why one might view this procedure as a kind of accounting comparable to the national accounts.

However, one should recall from the axiomatic approach that footprint indices in general do not provide an absolute measure. Instead, a country's ecological footprint of whatever kind is meaningful as a relative measure only. Its information content crucially depends on the feasible transformations of the scale. More precisely, in case of the axiomatically characterized index it depends on the specification of the function f. For example, if this function is taken linear homogenous then the ecological footprint is measured on a ratio scale. If it is defined as a strictly increasing function then the ecological footprint is given as an ordinal measure on the set of countries.

Moreover, the aggregation over different areal entities like regions or countries must not necessarily meet the classical procedure of standardazation and normalization in statistics. Therefore, a common scale and an explicit weighting scheme are fully dispensible.

5 Concluding Remarks

The objective of this paper has been to establish an axiomatic foundation for the concept of the ecological footprint index. We first identified the general properties any footprint index should satisfy. We then proposed five axioms which we considered appropriate for constructing a mathematical formula for footprint indices in general. It has been shown that there exists a unique index. Its functional form has been determined. Furthermore, its informational content is crucially depending on the functional relation to the world land area appropriated which need not necessarily be proportional. Hence, the generalization and characterization of footprint indices for their empirical application in world statistics may be seen as the main contribution of this paper.

Surprisingly, the index found is a generalization of the most widely accepted compound-based ecological footprint index. With respect to empirical applications, this might be considered important because the currently applied indices are still in need of a theoretical foundation. For instance, a hidden conflict lies in the dimension and norm of the scale. The norm, in fact, is implicitly fixed to one, but can take any positive value. Further, the dimension of the scale results in the underlying direct proportionality to the land area use reflecting a fixed proportions production function for bio-resources. However, from an axiomatic point of view the index must not necessarily be measured in natural units like hectares which holds only if the norm is arbitrarily set to 1 and proportionality is given.

A similar line of reasoning in principle holds for the Overshoot Day. Since that measure is given by the ratio of the footprint index and the biocapacity index, the Overshoot Day should be invariant to feasible transformations of both of the scales. A formal analysis is the scope for future research. Another remaining issue would be the axiomatic characterization of composite measures over different land area types. So far, the axiomatically characterized index corresponds to a footprint component of the compound-based measure. In this respect, we find that the empirically applied prodecure of standardization and weighted aggregation by using a common scale is completely dispensable.

Appendix

Proof of Existence and Uniqueness. First, it can be easily seen that $G(T,w) := \sum_{i \in T} f(\frac{C_i(w)}{Y(\Omega,w)} A(\Omega,w))$, with a strictly monotonically increasing function $f : \mathbb{R}_+ \mapsto \mathbb{R}$ satisfies axioms 1 to 5. This is shown by a straightforward calculation after substituting G for F in the axioms.

It remains to show the uniqueness of the index. For this we will show that any index F that satisfies axioms 1 to 6 will have the stated form. In the following, let $Y(w) := Y(\Omega, w)$ be the worldwide production and $A(w) := A(\Omega, w)$ the world land use.

Consider a country $i \in \Omega$ and an ecological world state $w^0 \in \mathcal{W}$. Let w^1 be an auxillary state with $C_j(w^1) = 1$, for all $j \in \Omega$, $Y(w^1) = \frac{Y(w^0)}{C_i(w^0)}$, and $A_i(w^1) = A_i(w^0)$. Using Axiom 4 with $\lambda = C_i(w^0)$ and substituting $\tilde{w} = w^0$ and $w = w^1$ we get

$$F(\{i\}, w^0) = F(\{i\}, w^1)$$

Continuing with $w^2 \in \mathcal{W}$ so that $C_j(w^2) = 1$, for all $j \in \Omega$, $Y(w^2) = 1$, and $A(w^2) = \frac{C_i(w^0)}{Y(w^0)} A(w^0)$, we apply axiom 5 with $\tilde{W} = w^2$, $w = w^1$, and $\lambda = \frac{C_i(w^0)}{Y(w^0)}$, we have

$$F(\{i\}, w^1) = F(\{i\}, w^2)$$

Since w^2 can be chosen to be the same for every $w \in \mathcal{W}$ with $\frac{C_i(w)}{Y(w)} A(w) = \frac{C_i(w^0)}{Y(w^0)} A(w^0)$, we will have $F(\{i\}, \tilde{w}) = F(\{i\}, w)$ whenever $\frac{C_i(\tilde{w})}{Y(\tilde{w})} A(\tilde{w}) = \frac{C_i(w)}{Y(w)} A(w)$, in particular $f_i := \{(\frac{C_i(w)}{Y(w)} A(w), F(i,w)) \mid w \in \mathcal{W}\}$ will be a function with $f_i(\frac{C_i(w)}{Y(w)} A(w)) = F(\{i\}, w)$ for all $w \in \mathcal{W}$.

Finally, symmetry implies that $f_i = f_j =: f$ for all $i, j \in \Omega$ and monotonicity that f is a strictly increasing function.

Thus axioms 1 through 5 define an unique index up to a strictly monotone function f. □

Proof of Existence and Uniqueness 2. If we continue in the previous proof from w^2 and choose w^3 with $A(w^3) = 1$, $Y(w^3) = 1$,$C_j(w^3) = 1$, we obtain with axiom 6 by substituting

$\tilde{w} = w^2$, $W = w^3$, and $\lambda = \frac{C_i(w^0)}{Y(w^0)} A(w^0)$

$$f(\{i\}, w^2) = \frac{C_i(w^0)}{Y(w^0)} A(w^0) f(\{i\}, w^3)$$

Since w^3 can be chosen independently of w^0 we finally have

$$f(\{i\}, w^0) = \frac{C_i(w^0)}{Y(w^0)} A(w^0) k_i$$

with some constant k_i.

Symmetry implies that $k := k_i = k_j$ for all $i, j \in \Omega$ and monotonicity that $k > 0$.

Thus axioms 1 through 6 define an unique index up to a multiplicative factor K $\square$

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