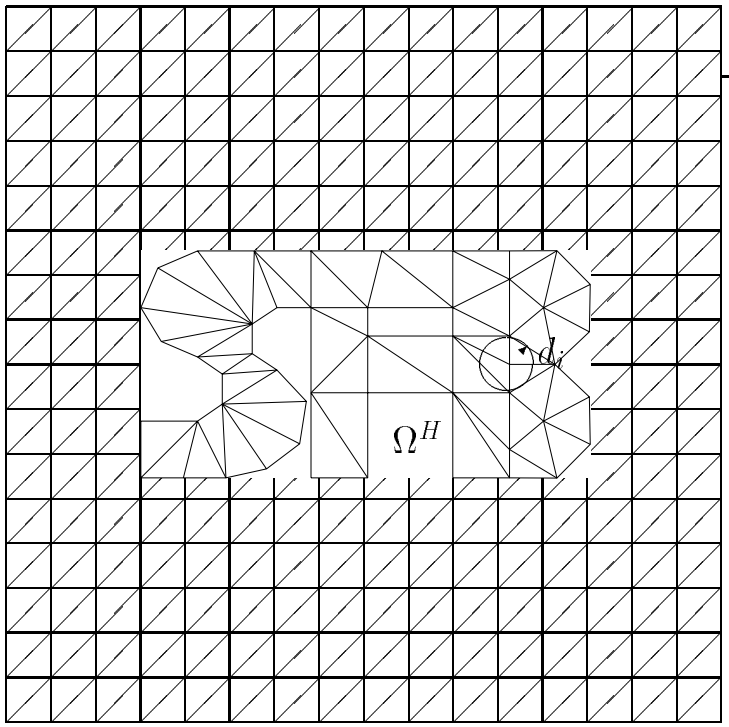
The hierarchical preconditioning having unstructured grids

$$\text{HPCCG} \rightarrow \quad K_H \underline{u}_H = \underline{f}_H, \quad K_H = K_H^T > 0$$

In engineering often **only** N nodal coordinates and elements:



Problem:

$\mathcal{L}u = f$ in Ω

$l\,u = g$ auf $\partial\Omega$

$\rightarrow K_H \underline{u}_H = \underline{f}_H$

Mapping:

$$\Omega^H \underset{\text{R}}{\overset{\text{T}}{\rightleftarrows}} \Pi^h$$

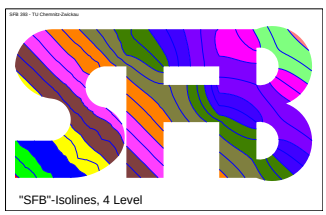
$$h := \frac{1}{2\sqrt{2}} \min(d_i)$$

Fig.: Embedding the unstructured grid Ω^H into the auxiliary grid Π^h.

Fictitious Space Lemma (S.V. Nepomnyaschikh):

$$c_1(K_H^{-1}\underline{u}_H, \underline{u}_H) \leq (C_{\Pi^h}^{-1}\mathbf{R}^* \underline{u}_H, \mathbf{R}^* \underline{u}_H) \leq c_2(K_H^{-1}\underline{u}_H, \underline{u}_H)$$

$\Rightarrow$ Artificial Yserentant (artYS) and BPX (artBPX) prec., resp.:



$N(\Omega^H)$	$N(\Pi^h)$	HPCCG	artYs	artBPX
536	257×257	22, 0.02	20, 3.56	15, 3.59
1954	513×513	28, 0.08	27, 17.69	20, 24.06
7430	1025×1025	33, 0.64	36, 96.79	24, 80.67
28942	2049×2049	38, 3.70	56, 458.50	27, 476.38
114206	4097×4097	41, 19.71	73, 2176.6	mem. ex.

Numerics on GCPP (#it,CPU in sec): hier. prec. vs. artYS, artBPX