An Elasticity Benchmark in Three Dimensions

Calculation via SPC-PM Po 3D (Chemnitz)

April 23, 1996

The problem is to find the solution of the Lamé system of linear elasticity in the Fichera corner

$$\Omega = (-1,1)^3 \setminus [0,1]^3$$

using the modul of elasticity $E = 200\,000$ and the Poisson ratio $\nu = 0.3$. The boundary data is given as follows:

face	u_1	u_2	u_3	t_1	t_2	t_3
$x=-1$				-1000	0	0
$y=-1$				0	0	0
$z=-1$				0	0	0
$x=0$	0	0	0			
$y=0$	0	0	0			
$z=0$	0	0	0			
$x=1$	0				0	0
$y=1$		0		0		0
$z=1$			0	0	0	

The problem was solved by the finite element method on a tetrahedral mesh with 42 user mesh elements and 1 to 5 hierarchical refinements (without adaptivity). The displacement at the boundary faces is given in Figures 1 – 7. The displacement at the line G from $(0,0,0)$ to $(-1,-1,-1)$ is given in Table 1.

The calulations were done using the parallel finite element package SPC-PM Po 3D [1, 2] at the Parsytec GCPP-128 parallel computer and at a HP 9000/819/K200/2 super scalar cluster with two HP–PA 7200 CPUs. The system of equations was solved by the CG method with BPX preconditioner and a coarse grid solver. For details of the algorithm programmed, see [2]. The stopping criterion for the GC was $\varepsilon = 10^{-6}$. Some computation times are given in Tables 2 and 3.

References

[1] Th. Apel. *SPC-PM Po 3D* — User's Manual. Preprint SPC95_33, TU Chemnitz-Zwickau, 1995.

[2] Th. Apel, F. Milde, and M. Thess. *SPC-PM Po 3D* — Programmer's Manual. Preprint SPC95_34, TU Chemnitz-Zwickau, 1995.

$x = y = z$	u_x	u_y	u_z
0.0000	0.000 e+00	0.000 e+00	0.000 e+00
−0.0625	−2.238 e−03	7.479 e−04	7.742 e−04
−0.1250	−3.167 e−03	1.035 e−03	1.049 e−03
−0.1875	−3.880 e−03	1.253 e−03	1.264 e−03
−0.2500	−4.507 e−03	1.453 e−03	1.461 e−03
−0.3125	−5.094 e−03	1.644 e−03	1.652 e−03
−0.3750	−5.655 e−03	1.835 e−03	1.842 e−03
−0.4375	−6.205 e−03	2.026 e−03	2.032 e−03
−0.5000	−6.747 e−03	2.219 e−03	2.224 e−03
−0.5625	−7.281 e−03	2.413 e−03	2.418 e−03
−0.6250	−7.812 e−03	2.609 e−03	2.613 e−03
−0.6875	−8.340 e−03	2.806 e−03	2.809 e−03
−0.7500	−8.864 e−03	3.003 e−03	3.006 e−03
−0.8125	−9.386 e−03	3.201 e−03	3.202 e−03
−0.8750	−9.906 e−03	3.398 e−03	3.399 e−03
−0.9375	−1.042 e−02	3.595 e−03	3.595 e−03
−1.0000	−1.094 e−02	3.791 e−03	3.791 e−03

Table 1: Displacements at G.

			number of processors (GCPP)							
ℓ	N	It-BPX	64	32	16	8	4	2	1	HP
2	1 995	43			1.2	0.9	0.9	0.8	1.0	1.3
3	13 203	49		2.7	2.6	2.6	3.2	4.4		10.6
4	95 523	54		8.2	9.7	13.7				89.8
5	725 571	57								751.2

Table 2: Computation times (in seconds) in the case of linear shape functions.

			number of processors (GCPP)							
ℓ	N	It-BPX	64	32	16	8	4	2	1	HP
2	1 995	50		1.6	1.4	1.1	1.2	1.2	1.7	2.3
3	13 203	53		3.2	3.2	3.6	4.8	7.1		17.7
4	95 523	56		10.5						147.8

Table 3: Computation times (in seconds) in the case of quadratic shape functions.

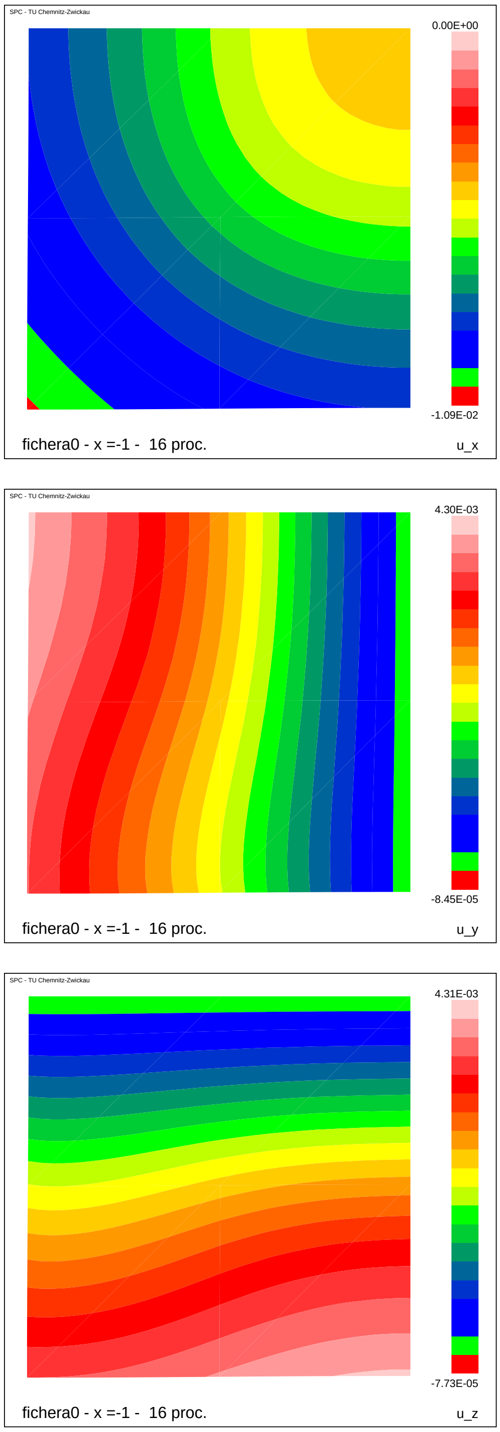

Figure 1: Displacement field at $x = -1$.

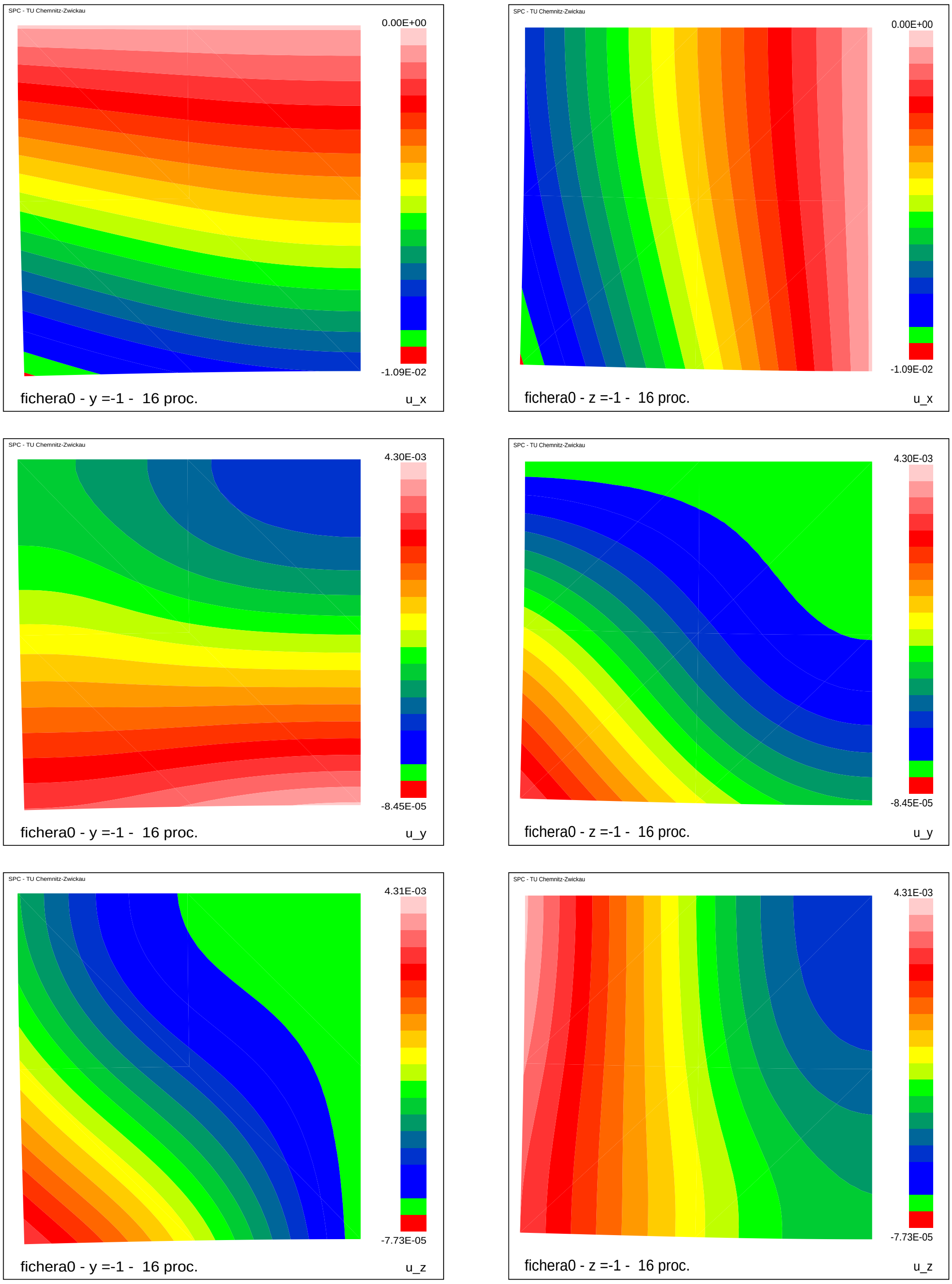

Figure 2: Displacement field at $y = -1$ and $z = -1$.

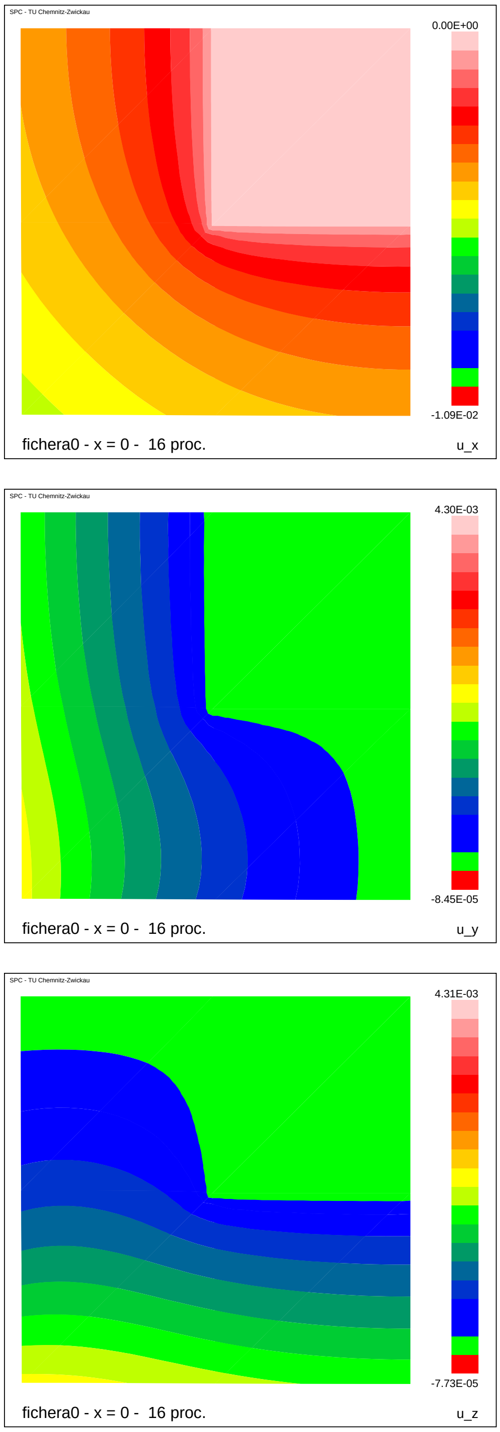

Figure 3: Displacement field at $x = 0$.

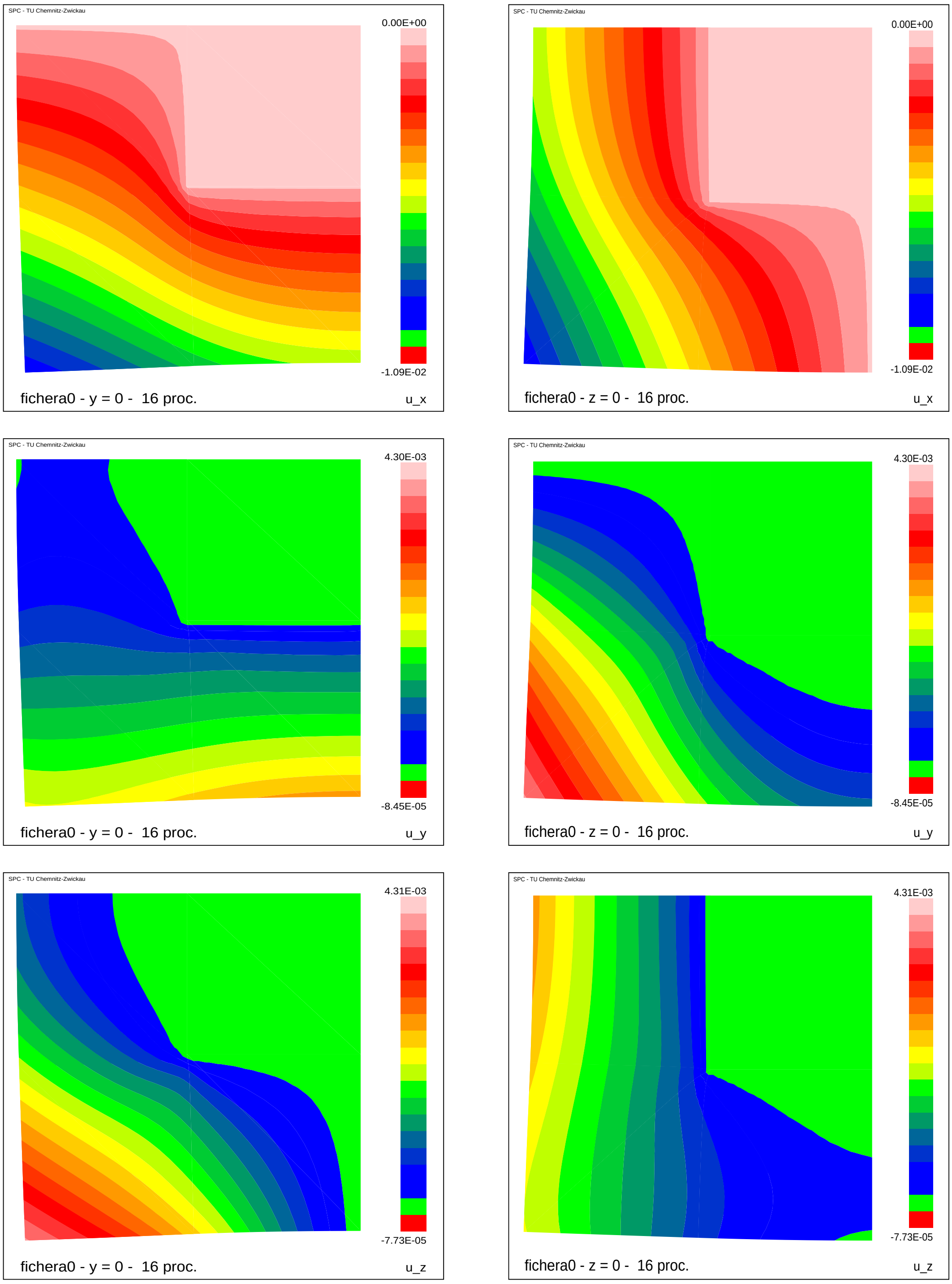

Figure 4: Displacement field at $y = 0$ and $z = 0$.

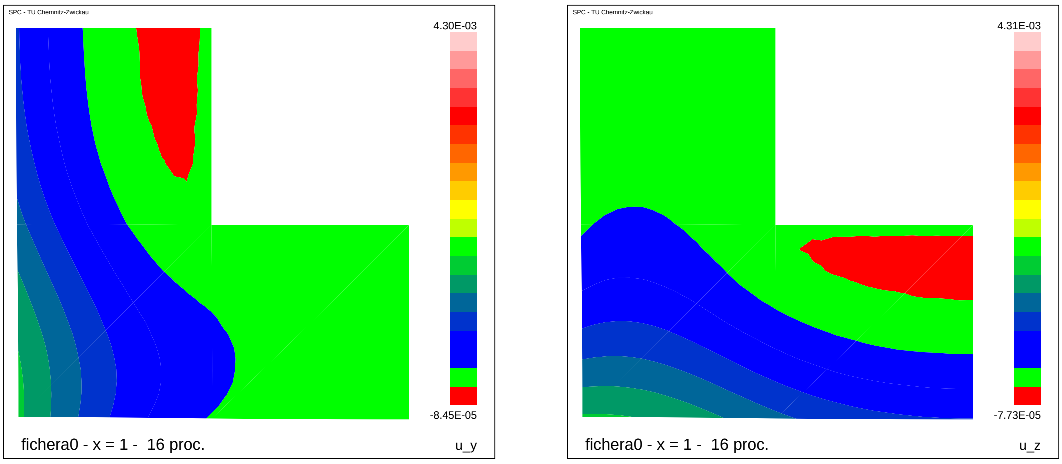

Figure 5: Displacement field at $x = 1$ ($u_x \equiv 0$).

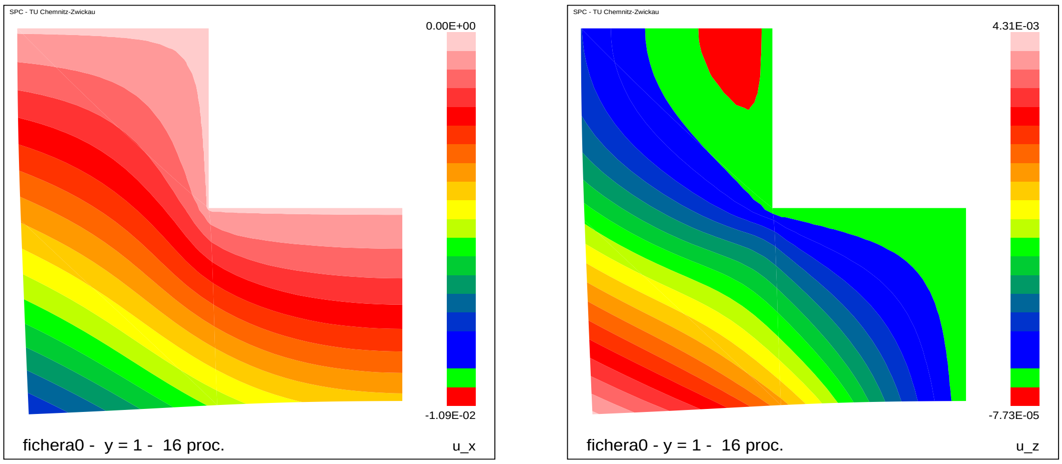

Figure 6: Displacement field at $y = 1$ ($u_y \equiv 0$).

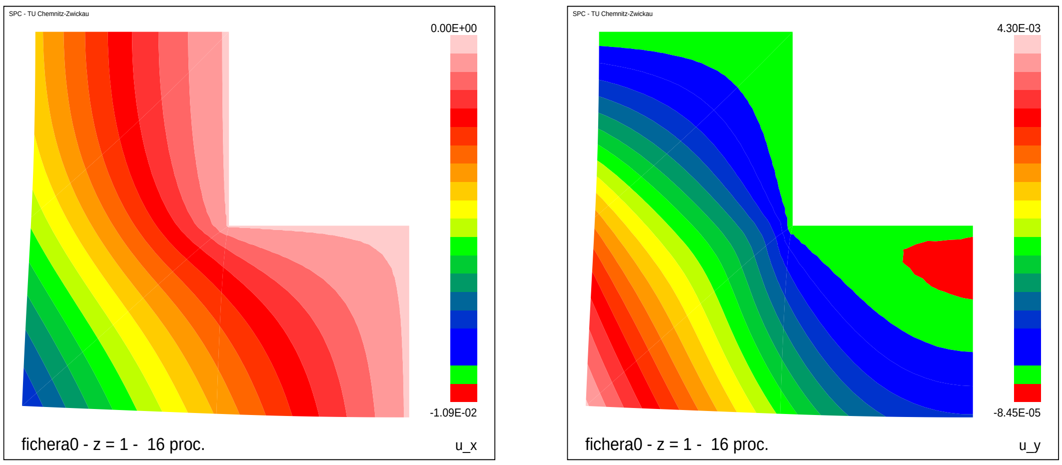

Figure 7: Displacement field at $z = 1$ ($u_z \equiv 0$).