ON THE SPATIALLY HOMOGENEOUS BOLTZMANN EQUATION

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We consider the classical spatially homogeneous Boltzmann equation

$$\frac{d\,f}{dt} = Q(f,f)\,, \tag{1}$$

$$Q(f,f)(v) = \int\limits_{\mathbb{R}^3}\int\limits_{\mathbb{S}^2} B(v,w,e)\Big(f(v')f(w') - f(v)f(w)\Big)\,de\,dw\,,$$

$$v' = \tfrac{1}{2}\Big(v + w + |u|\,e\Big), \qquad w' = \tfrac{1}{2}\Big(v + w - |u|\,e\Big)\,, \qquad u := v - w$$

which describes the time evolution of the particle density $f(t,v)\ :\ \mathbb{R}_+\times\mathbb{R}^3 \to \mathbb{R}_+ := [0,+\infty)$ in rarified gas. Here $v, w \in \mathbb{R}^3$ are the pre-collision velocities, $e \in \mathbb{S}^2 \subset \mathbb{R}^3$ is a unit vector, $v', w' \in \mathbb{R}^3$ are the post-collision velocities and $B(v,w,e)$ is the collision kernel. L. Arkeryd show that solution of equation (1) $f \in \mathbb{L}_1(\mathbb{R}^3)$ exists provided the initial data $f_0(v) := f(0,v)$ is in $\mathbb{L}_1(\mathbb{R}^3)$ and conserves the moments

$$\mathbb{X}_s(t,f) := \int\limits_{\mathbb{R}^3} |v|^s f(t,v)\,dv \tag{2}$$

for $s = 0$ (the mass), $s = 2$ (the internal energy) and also the bulk velocity (corresponds to $|v|^s \mapsto v$ in (2)). Existence of solution in $\mathbb{L}_p(\mathbb{R}^3)$ with weight and of all moments for $t > 0$, $s > 2$ was proved under very restrictive angular cut off condition on the kernel. No relevant physical model fulfills this condition.

We prove existence of $\mathbb{L}_p(\mathbb{R}^3)$-solutions, $1 \le p < \infty$ under natural constraints on the kernel and most of physically relevant models fulfil these conditions. Moreover, existence of all moments are proved for $t > 0$ and $s > 2$ for the Variable hard sphere model $B_\lambda(|u|,\mu) = C_\lambda |u|^\lambda$, $0 < \lambda \le 1$.