The Finite Element Method for the Boundary Value Problem with Strong Singularity of Solution

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Mathematical models for a number of physical processes lead to the differential problems in which the strong singularity of solution is caused by the singularity of initial data (the coefficients of the equation, the right hand sides of the equation and boundary conditions). For the boundary value problem of this type, for which the generalized (weak) solution can not be defined, or it does not have necessary regularity, we offered to define the solution as the R_ν-generalized one. Such a concept of solution led to distinction of two classes of boundary value problems: the problems with co-ordinated and non-co-ordinated degeneration of initial data. For these classes of problems this approach allowed to investigate the existence, uniqueness, coercivity, differential properties of the R_ν-generalized solution in the weighted Sobolev spaces. We construct and investigate the h-version and the $h-p$-version of the finite element method for these problems. We introduce the special regularizor and the finite element space which contains the singular functions, having the singularity depending on the space, to which the R_ν-generalized solution of the problem belongs. Using these elements we prove the estimate of the rate of convergence of the approximate solution to the exact R_ν-generalized solution in the norm of the weighted space. Numerical analysis of the modeling problems with singularity was made with elements of parallelizing the process of computing.

Acknowledgement. The work by Russian Foundation for Basic Research (code of project 04-01-97004).

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