Solving the Navier-Stokes equations by multigrid methods using quasi divergence free functions

Gunar Matthies[1] Friedhelm Schieweck[2]

We consider higher order finite element discretisations for solving the Navier–Stokes problem by means of discontinuous elements for the pressure and suitable conforming elements for the velocity such that the global and the local inf-sup condition are satisfied.

Higher order finite element discretisations have proved to be very efficient for the numerical solution of incompressible flow problems. However, the block Gauss-Seidel smoothers of existing coupled multi-level solvers become quite expensive for higher order discretisations, especially in the 3D case.

We will present a new multigrid solver that allows to use a smoother with low computational costs. The idea is to switch inside of the multigrid method to a new velocity basis which leads to a system with a much lower number of unknowns as well as a very small number of couplings between them. We call this new basis quasi divergence free since most of the basis functions are discretely divergence free which implies that they do not have any coupling to the pressure. The quasi divergence free basis functions can be constructed locally during the assembling process of the stiffness matrix by means of local projection operators.

The discrete problem which uses the quasi divergence free basis functions is decomposed into two subproblems. The first one is a problem for the element bubble part of the velocity solution. It can be solved independently from the remaining part in an element-wise way already during the assembling process of the stiffness matrix. The second problem of the decomposition is a reduced problem for the piecewise constant part of the pressure solution and the remaining velocity part. This system has a much smaller dimension than the original discrete Ossen problem. The remaining pressure part can be computed at the end by an element-wise post-processing procedure.

Furthermore, we will show how the multigrid solver constructed for the Oseen equations can be used for solving a discrete problem of a streamline diffusion method inside of a modified Picard-type iteration.

[1]Ruhr-Universität Bochum, Fakultät für Mathematik, Universitätsstr. 150, 44780 Bochum, Germany,
Gunar.Matthies@ruhr-uni-bochum.de
[2]Otto-von-Guericke-Universität Magdeburg, Institut für Analysis und Numerik, PSF 4120, 39016 Magdeburg, Germany,
Friedhelm.Schieweck@mathematik.uni-magdeburg.de