Adaptive Finite Element Algorithms of Optimal Complexity for the Stokes Problem.

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Nowadays adaptive finite element algorithms are recognized as powerful techniques for solving PDEs. The general structure of the loop of an adaptive algorithm is Solve - Estimate - Refine, Derefine. Especially the analysis of the last step of this loop is important for showing the optimality of the method. In Ädaptive Finite Element Methods with Convergence Rates" [Numer. Math., 97,(2004), pp.219-268], Binev, Dahmen and DeVore and in Än Optimal Adaptive Finite Element Method" [to appear in SIAM J. Numer. Anal.], Stevenson showed optimality of adaptive FEM algorithms for elliptic problems.

Concerning the solution of mixed variational problems, the situation is more complecated, and we are not aware of any theoretical study of optimality of finite element algorithms.

In Än Adaptive Uzawa FEM for the Stokes Problem: Convergence without the Inf-Sup Condition" [SIAM J. Numer. Anal., 40, (2002), pp. 1207-1229], Bänch, Morin and Nochetto introduced an adaptive FEM algorithm for the Stokes problem. Although they proved convergence of the algorithm, and numerical experiments showed (quasi-)optimal triangulations for some values of the parameters, a theoretical analysis whether the algorithm is optimal is missing.

In this talk, we present a detailed design of adaptive FEM algorithms for the Stokes problem, and an analysis of their computational complexity. We apply a fixed point iteration to an infinite dimensional Schur complement operator, where to approximate the inverse of the elliptic operator we use a convergent adaptive finite element method. Further, we apply a Chebyshev acceleration of this fixed point iteration, and show that the overall method has optimal computational complexity.

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