The Aharonov-Bohm effect for an exciton

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Abstract

We study theoretically the exciton absorption on a ring threaded by a magnetic flux. For the case when the attraction between electron and hole is short-ranged we get an exact solution of the problem. We demonstrate that, despite the electrical neutrality of the exciton, both the spectral position of the exciton peak in the absorption, and the corresponding oscillator strength oscillate with magnetic flux with a period Φ_0—the universal flux quantum. The origin of the effect is the finite probability for electron and hole, created by a photon at the same point, to tunnel in the opposite directions and meet each other on the opposite side of the ring.

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One of the manifestations of the Aharonov-Bohm (AB) effect [1] in the ring geometry [2,3] is the periodic dependence of the transmission coefficient for an electron traversing the ring on the magnetic flux Φ through the ring [4,5]. The period of oscillations is equal to $\Phi_0 = hc/e$ — the universal flux quantum.

For one-dimensional (1D) continuum interacting quantum systems with translational invariance there is also a periodicity of many-particle states as a functions of flux [6–9]. In 1D lattice systems, the lifting of Galilean invariance allows for various periodicities of the states [6,7]. For the ground state, this behavior can be interpreted, according to the above definition of Φ_0, as a signature of the existence of elementary excitations with multiple — sometimes even fractional — charges [6,10–13]. In the case of strong electron-electron interaction the adequate description of the many-body states is based on excitations of the Wigner-crystal [14,15]. Furthermore, the absence of sensitivity to the flux in such systems is an indication of the onset of the Mott transition [7,16,17]. Similarly, the sensitivity of single-particle energies to the flux [18] can be used as a criterion of the Anderson-type metal-insulator transition in disordered systems [19]. Combined effects of interactions and disorder in 1D have received much attention in the last decade [17,20–22]. Numerical studies of pairing effects for two particles with repulsive interaction in a disordered environment were carried out using the AB setting [23]. Other physical manifestations of the AB effect in the ring geometry considered in the literature include the evolution of electron states for a time-dependent flux [24], and a flux-dependent equilibrium distortion of the lattice caused by electron-phonon interactions [25].

The physical origin of the flux sensitivity of an electron on the ring is its charge which couples to the vector potential. Correspondingly, the coupling to the flux has the opposite sign for an electron and a hole. For this reason an *exciton*, being a bound state of electron and hole and thus a *neutral* entity, should not be sensitive to the flux. However, due to the finite size of the exciton, such a sensitivity will emerge. This effect is demonstrated in the present paper. Below we study the AB-oscillations both in the binding energy and in the oscillator strength of the exciton absorption. We choose as a model a short-range attraction potential between electron and hole, which allows to solve the three-body problem (electron, hole, and a ring) exactly. From this exact solution, we trace the behavior of the AB oscillations when increasing the radius of the ring or the strength of the electron-hole attraction.

Denote with φ_e and φ_h the azimuthal coordinates of the electron and hole, respectively. In the absence of interaction the wave functions of electrons and holes are given by

$$\Psi_N^{(e)}(\varphi_e) = \frac{1}{\sqrt{2\pi}} e^{iN\varphi_e}, \quad \Psi_{N'}^{(h)}(\varphi_h) = \frac{1}{\sqrt{2\pi}} e^{iN'\varphi_h}, \tag{1}$$

where N and N' are integers. The corresponding energies are

$$E_N^{(e)} = \frac{\hbar^2}{2m_e\rho^2}\left(N - \frac{\Phi}{\Phi_0}\right)^2, \quad E_{N'}^{(h)} = \frac{\hbar^2}{2m_h\rho^2}\left(N' + \frac{\Phi}{\Phi_0}\right)^2. \tag{2}$$

Here ρ is the radius of the ring, and m_e, m_h stand for the effective masses of electron and hole, respectively. In the presence of an interaction $V\Big[R(\varphi_e - \varphi_h)\Big]$, where $R(\varphi_e - \varphi_h) = 2\rho\sin(\frac{\varphi_e-\varphi_h}{2})$ is the distance between electron and hole, we search for the wave function of the exciton in the form

$$\Psi(\varphi_e, \varphi_h) = \sum_{N,N'} A_{N,N'} \Psi_N^{(e)}(\varphi_e)\Psi_{N'}^{(h)}(\varphi_h). \tag{3}$$

The coefficients $A_{N,N'}$ are to be found from the equation

$$\sum_{N,N'} A_{N,N'}\Big[E_N^{(e)} + E_{N'}^{(h)} - \Delta\Big]\Psi_N^{(e)}(\varphi_e)\Psi_{N'}^{(h)}(\varphi_h) + V\Big[R(\varphi_e - \varphi_h)\Big]\Psi(\varphi_e, \varphi_h) = 0, \tag{4}$$

where Δ is the energy of the exciton. The formal expression for $A_{N,N'}$ follows from Eq. (4) after multiplying it by $\Big[\Psi_N^{(e)}(\varphi_e)\Psi_{N'}^{(h)}(\varphi_h)\Big]^\dagger$ and integrating over φ_e and φ_h

$$A_{N,N'} = -\frac{1}{2\pi}\int_0^{2\pi} d\varphi_e \int_0^{2\pi} d\varphi_h \frac{V\Big[R(\varphi_e - \varphi_h)\Big]\Psi(\varphi_e, \varphi_h)}{E_N^{(e)} + E_{N'}^{(h)} - \Delta} e^{-i(N\varphi_e + N'\varphi_h)}. \tag{5}$$

At this point we make use of the assumption that the potential $V\Big[R(\varphi_e - \varphi_h)\Big]$ is short-ranged. This implies that the integral over φ_h is determined by a narrow interval of φ_h close to φ_e. Then we can replace φ_h by φ_e in the rest of the integrand. As a result, Eq. (5) simplifies to

$$A_{N,N'} = -\frac{V_0}{E_N^{(e)} + E_{N'}^{(h)} - \Delta} \int_0^{2\pi} d\varphi_e \Psi(\varphi_e, \varphi_e) e^{-i(N+N')\varphi_e}, \tag{6}$$

where the constant $V_0 < 0$ is defined as

$$V_0 = \frac{1}{2\pi} \int d\varphi V\Big[R(\varphi)\Big]. \tag{7}$$

Finally we derive a closed equation, which determines the exciton energies. This equation follows from Eqs. (3) and (6) as a self-consistency condition. Indeed, by setting in Eq. (3) $\varphi_e = \varphi_h$, multiplying both sides by $\exp(-iN_0\varphi_e)$, and integrating over φ_e, we obtain

$$\int_0^{2\pi} d\varphi_e \Psi(\varphi_e, \varphi_e) e^{-iN_0\varphi_e} = \sum_N A_{N,N_0-N}. \tag{8}$$

Substituting (6) into (8) we arrive at the desired condition

$$1 + V_0 \sum_N \frac{1}{E_N^{(e)} + E_{N_0-N}^{(h)} - \Delta_{N_0}} = 0. \tag{9}$$

For each integer N_0 the solutions of Eq. (9) form a discrete set, $\Delta_{N_0}^m$. The corresponding (non-normalized) wave functions have the form

$$\Psi_{N_0}^m \propto e^{iN_0\varphi_h} \sum_N \frac{e^{iN(\varphi_e-\varphi_h)}}{E_N^{(e)} + E_{N_0-N}^{(h)} - \Delta_{N_0}^m}. \tag{10}$$

The exponential factor in front of the sum insures that in the dipole approximation only the excitons with $N_0 = 0$ can be created by light. The frequency dependence of the exciton absorption, $\alpha(\omega)$, can be presented as

$$\alpha(\omega) \propto \sum_m F_m \delta(\hbar\omega - E_g - \Delta_0^m), \tag{11}$$

where E_g is the band-gap of the material of the ring; the coefficients F_m stand for the oscillator strengths of the corresponding transitions. A general expression for F_m through the eigenfunction, Ψ_0^m, of the excitonic state reads

$$F_m = \frac{|\int_0^{2\pi} d\varphi_e \int_0^{2\pi} d\varphi_h \Psi_0^m(\varphi_e, \varphi_h)\delta(\varphi_e - \varphi_h)|^2}{\int_0^{2\pi} d\varphi_e \int_0^{2\pi} d\varphi_h |\Psi_0^m(\varphi_e, \varphi_h)|^2}. \tag{12}$$

Upon substituting Eq. (10) into Eq. (12) and making use of Eq. (9), we obtain

$$F_m = \left[V_0^2 \sum_N \frac{1}{(E_N^{(e)} + E_{-N}^{(h)} - \Delta_0^m)^2} \right]^{-1}. \tag{13}$$

The latter expression can be presented in a more compact form by introducing the rate of change of the exciton energy with the interaction parameter V_0. Indeed, taking the differential of Eq. (9), yields

$$F_m = -\frac{\partial \Delta_0^m}{\partial V_0}. \tag{14}$$

We note that the summation in Eq. (9) can be carried out in a closed form by using the identity

$$\sum_{N=-\infty}^{\infty} \frac{1}{(\pi N - a_1)(\pi N - a_2)} = \frac{1}{(a_1 - a_2)} \left(\frac{1}{\tan a_2} - \frac{1}{\tan a_1} \right). \tag{15}$$

For the most interesting case $N_0 = 0$ the parameters a_1, a_2 are equal to

$$a_{1,2} = -\pi \left[\frac{\Phi}{\Phi_0} \pm \left(\frac{\Delta_0^m}{\varepsilon_0} \right)^{1/2} \right], \tag{16}$$

where

$$\varepsilon_0 = \frac{\hbar^2}{2\rho^2} \left(\frac{1}{m_e} + \frac{1}{m_h} \right) = \frac{\hbar^2}{2\mu\rho^2}, \tag{17}$$

and $\mu = m_e m_h/(m_e + m_h)$ denotes the reduced mass of electron and hole. Then the equation (9) for the exciton energies takes the form

$$\left(\frac{\Delta_0^m}{\varepsilon_0} \right)^{1/2} = -\left(\frac{\pi V_0}{\varepsilon_0} \right) \frac{\sin\left(2\pi(\Delta_0^m/\varepsilon_0)^{1/2}\right)}{\cos\left(2\pi(\Delta_0^m/\varepsilon_0)^{1/2}\right) - \cos\left(2\pi(\Phi/\Phi_0)\right)}. \tag{18}$$

This equation is our main result. It is seen from Eq. (18) that the structure of the excitonic spectrum is determined by a dimensionless ratio $|V_0|/\varepsilon_0$. From the definition (7) it follows that, with increasing the radius ρ of the ring, V_0 falls off as $1/\rho$. Thus, $|V_0|/\varepsilon_0$ is proportional to ρ. In the limit of large ρ, when $|V_0| \gg \varepsilon_0$, the spectrum can be found analytically. The ground state corresponds to negative energy and is given by

$$\Delta_0^0 = -\frac{\pi^2 V_0^2}{\varepsilon_0} \left[1 + 4\cos\left(\frac{2\pi\Phi}{\Phi_0}\right) \exp\left(-\frac{2\pi^2 |V_0|}{\varepsilon_0}\right) \right]. \tag{19}$$

We note that the prefactor $\pi^2 V_0^2/\varepsilon_0$ is independent of ρ. It is equal to the binding energy of an exciton on a straight line. It is easy to see that in the limit under consideration we have $|\Delta_0^0| \gg |V_0| \gg \varepsilon_0$.

The second term in the brackets of Eq. (19) describes the AB effect for the exciton. In the limit of large ρ its magnitude is exponentially small. The physical meaning of the exponential prefactor can be understood after rewriting it in the form $\exp(-2\pi\rho\gamma)$, where $\gamma = \pi|V_0|\left(2\mu/\hbar^2\varepsilon_0\right)^{1/2}$ is the inverse decay length of the wave function of the internal motion of electron and hole in the limit $\rho \to \infty$. Thus, the magnitude of the AB effect in the limit of large ρ represents the amplitude for bound electron and hole to tunnel in the opposite directions and meet each other "on the opposite side of the ring" (opposite with respect to the point where they were created by a photon). This qualitative consideration allows to specify the condition that the interaction potential is short-ranged. Namely, for Eq. (19) to apply, the radius of potential should be much smaller than γ^{-1}. It is also clear from the above consideration that, within a prefactor, the magnitude of the AB effect is given by $\exp(-2\pi\rho\gamma)$ for arbitrary attractive potential, as long as the decay length γ^{-1} is smaller than the perimeter of the ring. In Fig. 1 we plot the numerical solution of Eq. (18) for various values of Φ together with the asymptotic solution (19) valid in the limit of large $\gamma\rho$. We see that the maximum possible change in exciton energy by threading the ring with a flux $\Phi_0/2$ is 25% of the size-quantization energy ε_0. The asymptotic expression of (19) is good down to $\gamma\rho \approx \pi^{-1}$. In Fig. 2, we show the variation of the exciton energy with Φ within one period. As expected, the AB oscillations are close to sinusoidal for large values of $2\pi\gamma\rho$, whereas for $2\pi\gamma\rho = 1$, unharmonicity is already quite pronounced. The increase of the exciton energy as the flux is switched on has a simple physical interpretation. If the single-electron energy (2) *grows* with Φ then the single-hole energy is *reduced* with Φ and vice versa. This suppresses the electron-hole binding. Fig. 2 illustrates how the amplitudes of the AB oscillations decrease with increasing ring perimeter $2\pi\gamma\rho$ as described by Eq. (19). The AB oscillations in the oscillator strength are plotted in Fig. 3. As expected, the shift is most pronounced for $\Phi = \Phi_0/2$, and the relative magnitude is nearly 80% for the smallest value of $2\pi\gamma\rho$. For larger values of $2\pi\gamma\rho$, the oscillations in $F_0(\Phi)$ become increasingly sinusoidal as can be seen by differentiating Eq. (19) with respect to V_0.

In the consideration above we assumed the width of the ring to be zero. In fact, if the width is finite but smaller than the radius of the exciton, γ^{-1}, it can be taken into account in a similar fashion as in [26] by adding $\hbar^2\pi^2/2m_eW^2$ and $\hbar^2\pi^2/2m_hW^2$ to the single-electron and single-hole energies (2), respectively. Here, W stands for the width of the ring and a hard-wall confinement in the radial direction is assumed. This would leave the AB oscillations unchanged. In the opposite case $W \gg \gamma^{-1}$ the oscillations are suppressed. The precise form of the suppression factor as a function of $(W\gamma)^{-1}$ is unknown and depends on the details of the confinement.

Let us briefly address the excited states of the exciton corresponding $m > 0$. In the limit $|V_0| \gg \varepsilon_0$ for the energies with numbers $m < |V_0|/\varepsilon_0$ we get from Eq. (18)

$$\Delta_0^m = \frac{\varepsilon_0}{4}\Big[m^2 + (-1)^m(m+\frac{1}{2})\frac{\varepsilon_0}{\pi^2 V_0}\cos\Big(\frac{2\pi\Phi}{\Phi_0}\Big)\Big]. \tag{20}$$

In contrast to the ground state as in (19) the AB contribution to the energy Δ_0^m is not exponentially small. Still the AB term is small (in parameter $\varepsilon_0/|V_0| \ll 1$) compared to the level spacing at $\Phi = 0$.

An alternative way to derive Eq. (18) is to follow the Bethe ansatz approach [27]. The intimate relation between Eq. (18) and a Bethe ansatz equation becomes most apparent in the absence of magnetic flux, $\Phi = 0$, when (18) can be rewritten as

$$2\pi\rho k_m = 2\pi m + 2\arctan\Big(\frac{\rho k_m}{c}\Big), \tag{21}$$

where $k_m = (2\Delta_0^m\mu)^{1/2}/\hbar$ is the wave vector and $c = 2\pi\mu V_0\rho^2/\hbar^2$ parameterizes the strength of the attraction analogously to the well-known δ-function gas [28–30]. At finite flux, the structure of the Bethe ansatz equations will be very similar to the equations for a 1D Hubbard model [31] in the presence of a spin flux coupling to the spin-up and spin-down degrees of freedom of the electrons [10,17].

First experimental studies of the AB effect were carried out on metallic rings [32]. The next generation of rings were based on GaAs/AlGaAs hetereostructures as in Refs. [33] and

[34] and had a circumference of $\sim$ 6000nm and 3000nm, respectively. For such rings the magnitude of the excitonic AB oscillations will be very small. However, quite recently much more compact ring-shaped dots of InAs in GaAs with a circumference of $\sim$ 250nm were demonstrated to exist [35]. This was achieved by modification of a standard growth procedure [36] used for the fabrication of arrays of self-assembled InAs quantum dots in GaAs. Recent light absorption experiments on nano-rings reveal an excitonic structure [37]. However, it is much more advantageous to search for the AB oscillations proposed in the present paper not in absorption, but in luminescence studies. This is because near-field techniques developed in the last decade allow to "see" a single quantum dot and thus avoid the inhomogeneous broadening. This technique was applied to many structures containing ensembles of quantum dots (*e.g.*, GaAs/AlGaAs [38], ZnSe [39]). In particular, extremely narrow and temperature insensitive (up to 50K) luminescence lines from a single InAs quantum dot in GaAs were recorded in [40].

In conclusion, we have demonstrated the AB oscillations for a *neutral* object. This constitutes the main qualitative difference between our paper and previous considerations [41] for two interacting *electrons* on a ring. Lastly, we note that the possibility of the related effect of Aharonov-Casher oscillations for an exciton was considered previously in [42]. The underlying physics in [42] is that even a *zero-size* exciton having zero charge can still have a finite *magnetic moment.*

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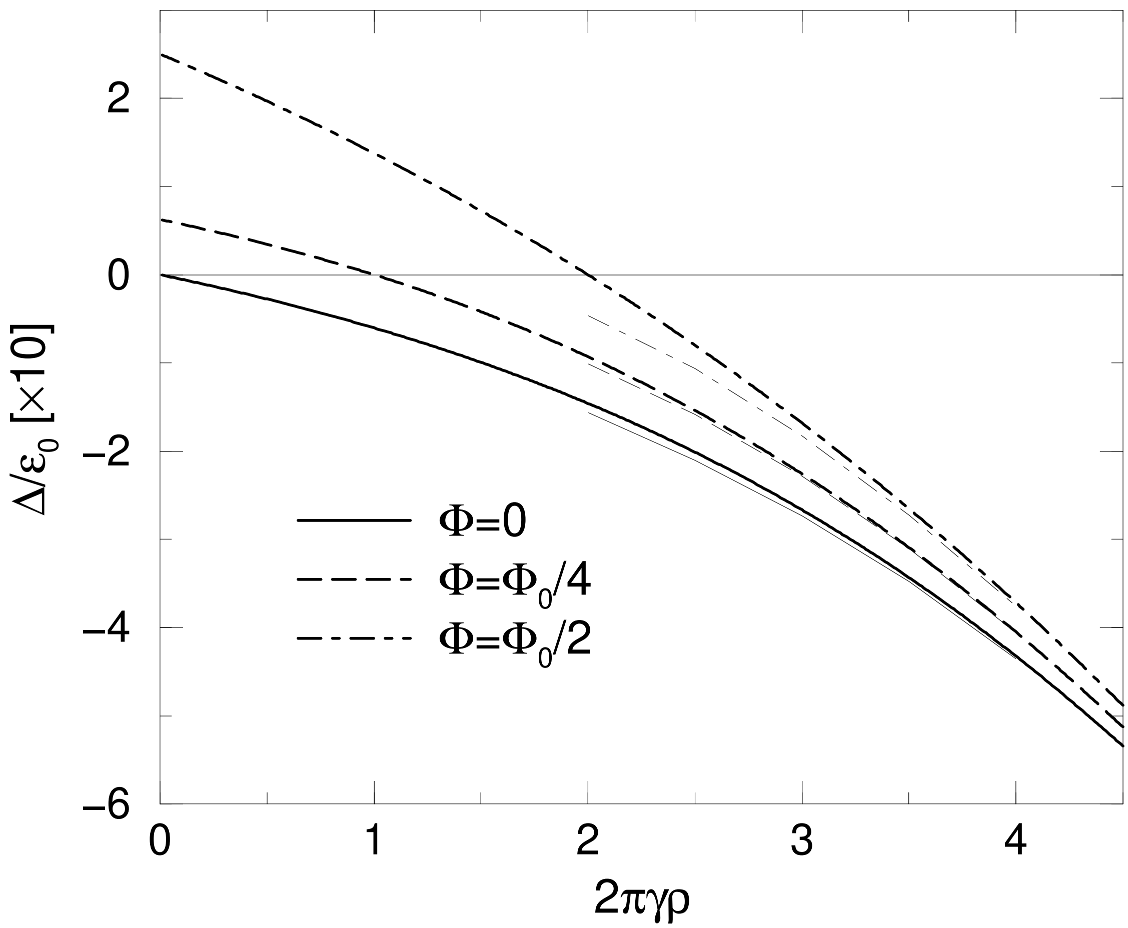

FIG. 1. The exciton energy Δ/ε_0 at flux $\Phi = 0$ (solid lines), $\Phi_0/4$ (dashed line), and $\Phi_0/2$ (dot-dashed line) through the ring are plotted versus the dimensionless perimeter of the ring $2\pi\gamma\rho$. The thick and thin lines represent the exact solution of Eq. (18) and the asymptotic result of Eq. (19), respectively.

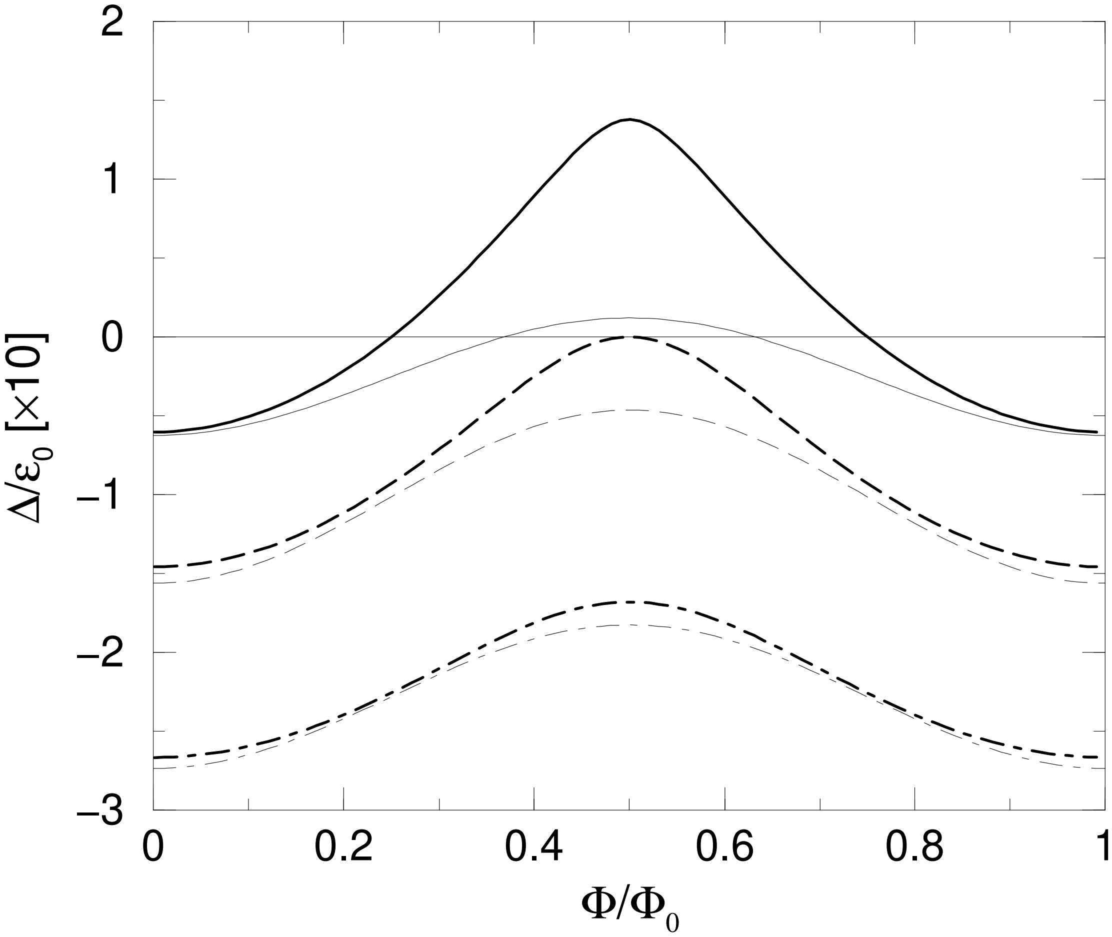

FIG. 2. The Aharonov-Bohm oscillations of the exciton energy is shown for three values of the dimensionless ring perimeter $2\pi\gamma\rho = 1$ (solid lines), 2 (dashed lines) and 3 (dot-dashed lines). As in Fig. 1, the thick and thin lines are drawn from Eq. (18) and Eq. (19), respectively.

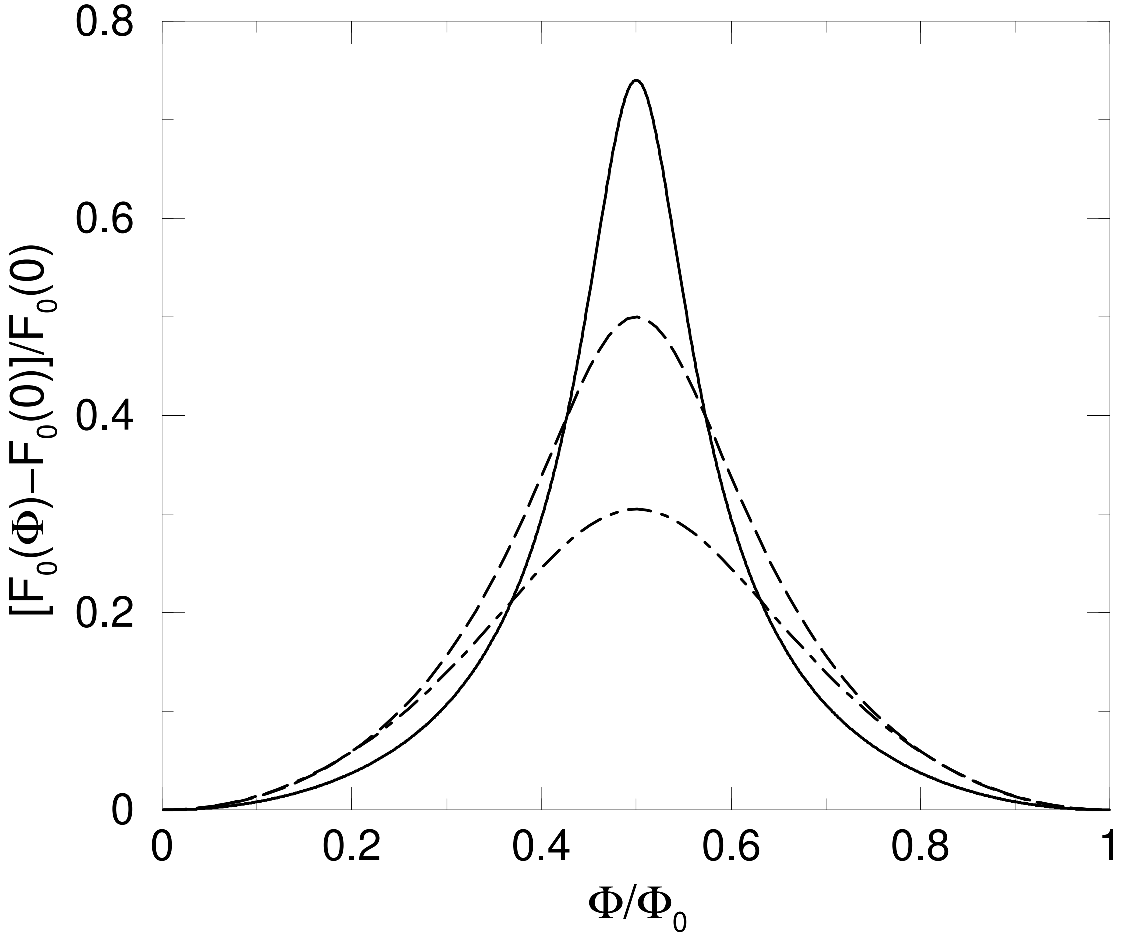

FIG. 3. The Aharonov-Bohm oscillations of the oscillator strength for the three values of the dimensionless ring perimeter $2\pi\gamma\rho = 1$ (solid line), 2 (dashed line) and 3 (dot-dashed line).