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Critical behavior in the two-dimensional Anderson model of localization with random hopping

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Abstract

We study a variant of the two-dimensional (2D) Anderson model of localization in which the disorder is represented by randomly chosen hopping terms. The density of states reveals an abnormally strong peak in the band center and an analysis of multifractal properties indicates that localization is less strong at $E = 0$ than at $E \neq 0$. A finite-size-scaling analysis of localization lengths as obtained from the transfer-matrix method, shows that the state at $E = 0$ exhibits critical behaviour up to strip width $M = 180$. However, states outside the band center are localized and the critical state vanishes already for very small amounts of onsite potential disorder. Thus there is no violation of the scaling theory of localization.

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In a recent paper [1], we studied a variant of the 2D Anderson model of localization in which the nearest-neighbor hopping matrix elements are randomly distributed. This model may be viewed as a 2D random flux model [2] with phase fixed at zero but random modulus. We found that the density of states shows an unusual feature in the band center $E = 0$. Investigating the participation numbers of eigenstates and also the Lifshitz-Hölder exponents $\alpha(q)$, we showed critical behavior for states at $E = 0$ up to the system size $N = 200 \times 200$ considered. This was confirmed by an analysis of localization lengths in quasi-1D strips with the help of the transfer-matrix method (TMM). However, adding a very small additional onsite potential disorder, we found that these critical states become again localized. In the present work, we will present further evidence that the states in the center of the band exhibit critical behavior.

The 2D Anderson Hamiltonian with random onsite potential energies $\epsilon_i \in [-W/2, W/2]$ and random nearest-neighbor transfer integrals $t_{ij} \in [c - w/2, c + w/2]$ is given as

$$H = \sum_{i}^{N} \epsilon_i |i\rangle\langle i| + \sum_{i \neq j}^{N} t_{ij} |i\rangle\langle j|. \tag{1}$$

The sites $i = (n, m)$ form a square lattice of size $N = L \times L$ and we use periodic boundary conditions. We set the energy scale by $w \equiv 1$, except for the case of pure diagonal disorder with constant hopping ($w = 0$ and $c = 1$). For $c \to \infty$, the off-diagonal disorder width w is negligible compared to its mean, and we get the usual Anderson model; when additionally W remains finite for $c \to \infty$, the system becomes ordered. For $c \leq 0.5$, individual transfer integrals may be zero thus giving a tendency towards localization.

The multifractal analysis of the eigenfunctions $\phi_j(n, m)$, $j = 1, \ldots, N$, used here is based on a parametric expression $\{f(q), \alpha(q)\}$ of the singularity spectrum $f(\alpha)$ as explained in [1]. We also obtain a set of generalized dimensions $D(q) = \{f[\alpha(q)] - q\alpha(q)\}/(1 - q)$. Here $D(0)$ is the Hausdorff dimension of the support (and thus 2 in the 2D case) and $D(2)$ represents the correlation dimension [3]. For a truly extended 2D wave function, $f(\alpha = 2) = 2$ and $D(0) = D(2) = 2$. The more a state becomes localized, the more the $f(\alpha)$ spectrum widens and the $D(q)$ differ from 2. In Fig. 1 we show $f(\alpha)$ and $D(2)$ calculated by (moving) averages

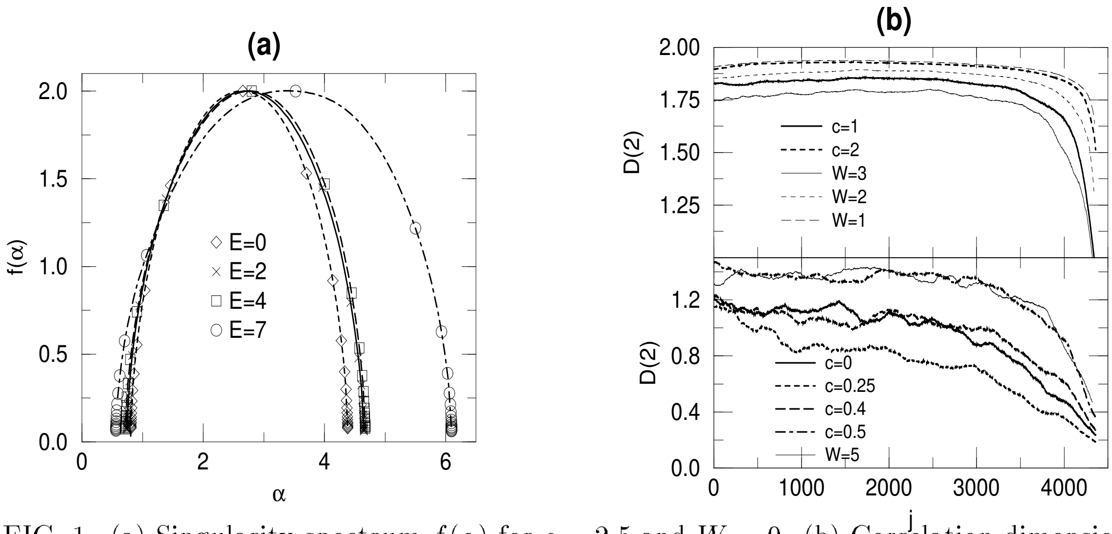

FIG. 1. (a) Singularity spectrum $f(\alpha)$ for $c = 2.5$ and $W = 0$. (b) Correlation dimension $D(2)$ versus the number j of the eigenstate $(0 \leq E_j \leq E_{j+1})$ for $N = 96 \times 96$. Purely off-diagonal disorders are shown by thick lines, purely diagonal disorders by thin lines.

over 250 states. For large energies, the $f(\alpha)$ spectrum widens and the deviations of $D(2)$ from 2 are well pronounced. Close to the band edge, $D(2)$ decreases drastically. Therefore localization of states at the band edge is confirmed by $f(\alpha)$ and $D(2)$ in agreement with the results [1] based on participation numbers and the Lifshitz-Hölder exponents $\alpha(0)$ and $\alpha(1)$. If localization would also be strong at the band center, we would expect a similar deviation of the $D(2)$ values from 2 as at the band edges. However, the differences between the $D(2)$ values for both weak disorders W and c are negligible. For stronger off-diagonal disorder even the opposite tendency can be observed: the $D(2)$ values increase when getting close to the band center, which suggests rather a tendency towards weaker localization.

In [1], we showed that it is essential to look at the scaling properties of the participation numbers $P_N = \left[\sum_{n,m} |\phi_j(n,m)|^4\right]^{-1} \sim N^{\kappa}$, in order to make reliable statements about the localization properties. E.g., for a localized state $\kappa = 0$, whereas for an extended state $\kappa = 1$. The connection to the multifractal properties is given by $\kappa = D(2)/D(0)$ [4]. The values of $D(2)$ at the band center then give $0.5 \leq \kappa \leq 0.95$ for $0 \leq c \leq 2$. Therefore the state at $E = 0$ is neither localized nor extended and has properties similar to critical states, i.e. states at the metal-insulator transition in 3D. This observation is corroborated

[1] by a comparison with typical values of $D(2)/D(0)$ at the MIT in the 3D isotropic and anisotropic Anderson models [3], and also for the Anderson model defined on two bifractals [5]. We emphasize, however, that the non-zero κ for $E = 0$ may be a finite-size effect and the $D(2)$ values may become smaller for larger sizes.

We also calculated in [1] the localization lengths by means of the TMM [6,7]. The reduced localization lengths $\lambda(M)/M$ for different off-diagonal disorders plus (i) different diagonal disorders $W \neq 0$ at $E = 0$ and (ii) different energies with $W = 0$ scale onto a scaling curve $\lambda(M)/M = \mathcal{F}(\xi/M)$ in both cases. The absolute scale of ξ was obtained by fitting $\lambda/M = \xi/M + b(\xi/M)^2$ for the smallest localization lengths [7]. Only one branch of $\mathcal{F}$ was shown [1] to exist in both cases corresponding to localized behavior [6,7]. We now first turn to the state at $E = 0$ for purely off-diagonal disorder ($W = 0$). As reproduced in Fig. 2 from [1], the reduced localization lengths λ/M are constant vs. $1/M$ and scaling is impossible. This is typical for the critical behavior observed at the MIT in the 3D Anderson model [7]. We next show in Fig. 2 that λ/M for off-diagonal disorder $w = 1$, $0 \leq c \leq 1$ and (i) various small diagonal disorder strengths $W \neq 0$ in the band center $E = 0$ and (ii) various energies $E \neq 0$ but with $W = 0$ can in fact be scaled onto a *single scaling curve* $\mathcal{F}(\xi/M)$. The strip widths were $M = 10, 20, \ldots, 90$. This further confirms the validity of the scaling hypothesis for the present variant of the Anderson model. Moreover, we obtain only one branch of the scaling function corresponding to localization. In Fig. 2 (inset), we show the dependence of the scaling parameter ξ on c. It exhibits a minimum close to $c = 0.25$. This shows in agreement with [1] that the maximum strength of the off-diagonal disorder appears for $c = 0.25$. Thus the presence of the critical state is restricted to $E = 0$ for all off-diagonal disorders and calculations for other energies indicate localized states only.

In summary, we have studied the 2D Anderson Hamiltonian with off-diagonal disorder and shown from the behavior of the correlation dimension $D(2)$ that states in the band center have properties similar to critical states at the MIT. Reanalysing the localization lengths computed in [1], we have shown that they can all be scaled onto a single scaling curve except for purely off-diagonal disorder at $E = 0$, where λ/M behaves similar (up to

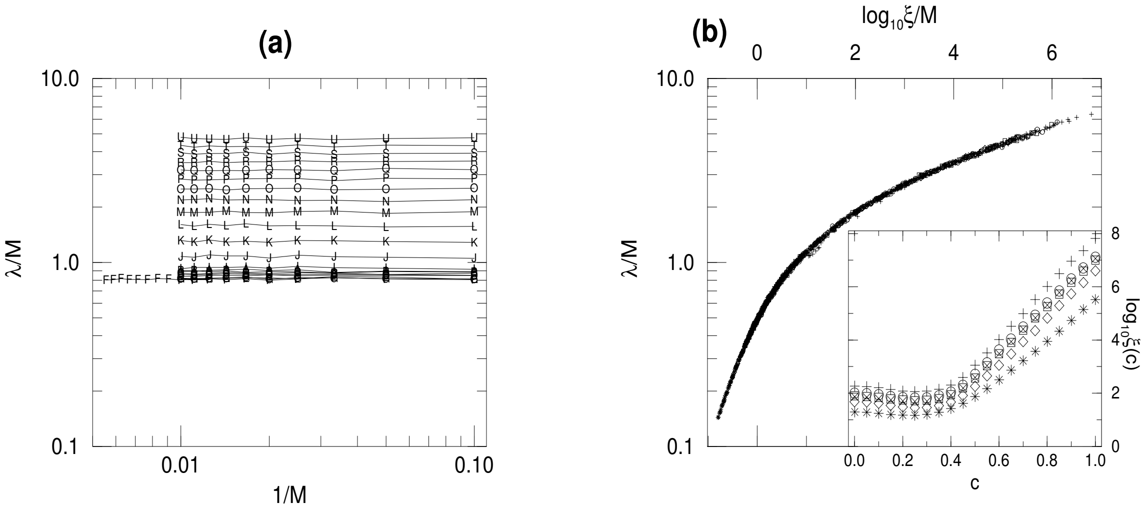

FIG. 2. (a) Reduced localization lengths $\lambda(M)/M$ vs. $1/M$ for purely random hopping ($W=0$) at $E=0$. The off-diagonal disorder strengths $c=0, 0.05, \ldots, 1$ are indicated by A, B, …, U, respectively. (b) Scaling plot of $\lambda(M)/M$ for (i) random hopping at $E=0$ and $W=0.001$ (+), 0.01 ($\times$) and 0.1 ($*$) and (ii) purely random hopping ($W=0$) at $E=0.005$ ($\Box$), $E=0.01$ ($\circ$) and $E=0.1$ ($\Diamond$). Small deviations from scaling are due to data for $W=0.001$ and $M=10$. Inset: Scaling parameters $\xi(c)$.

strip width $M=180$ at 1% accuracy) to reduced localization lengths at the MIT in 3D.

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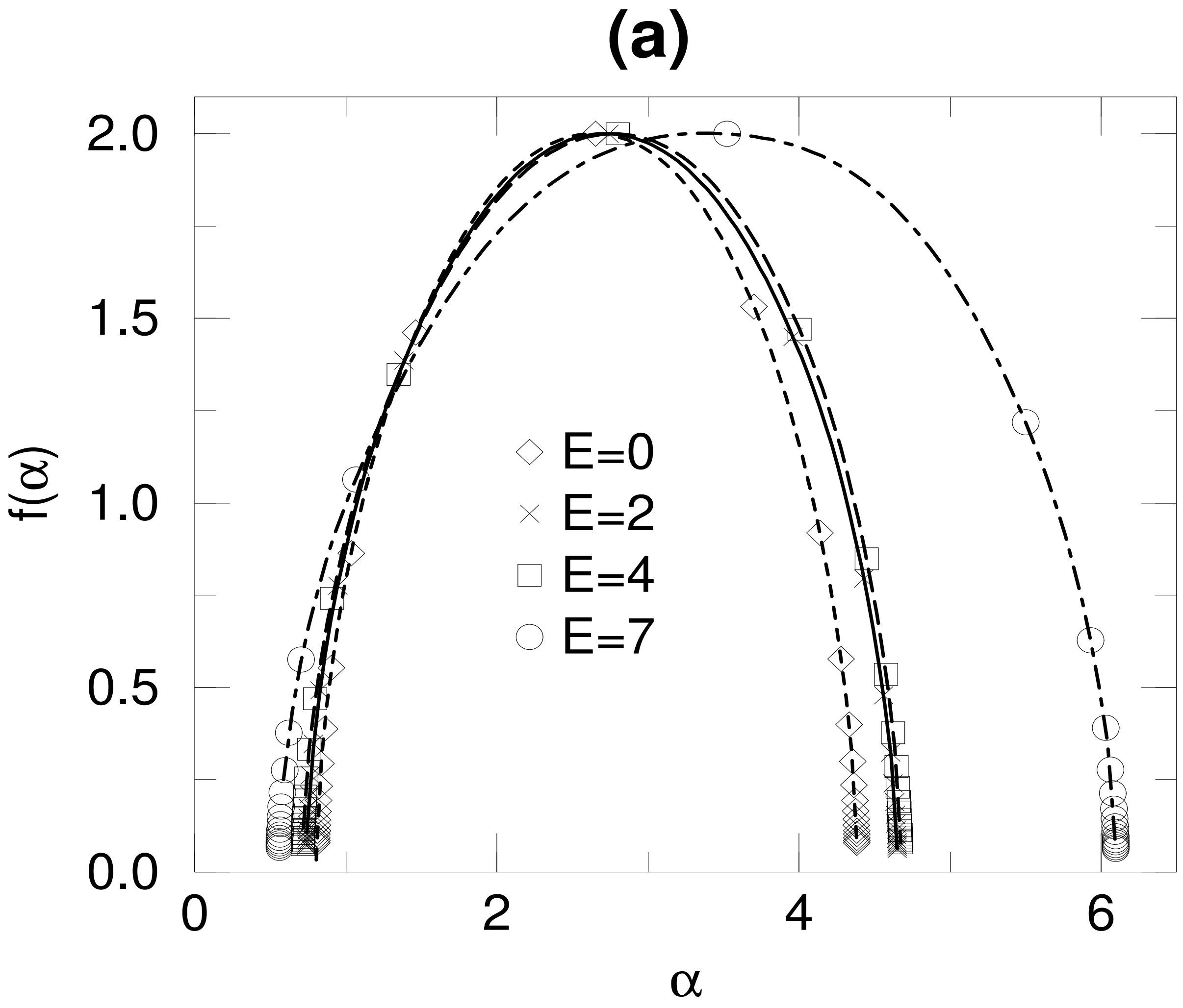

FIG. 1. (a) A. Eilmes *et al.*, “Critical behavior …”.

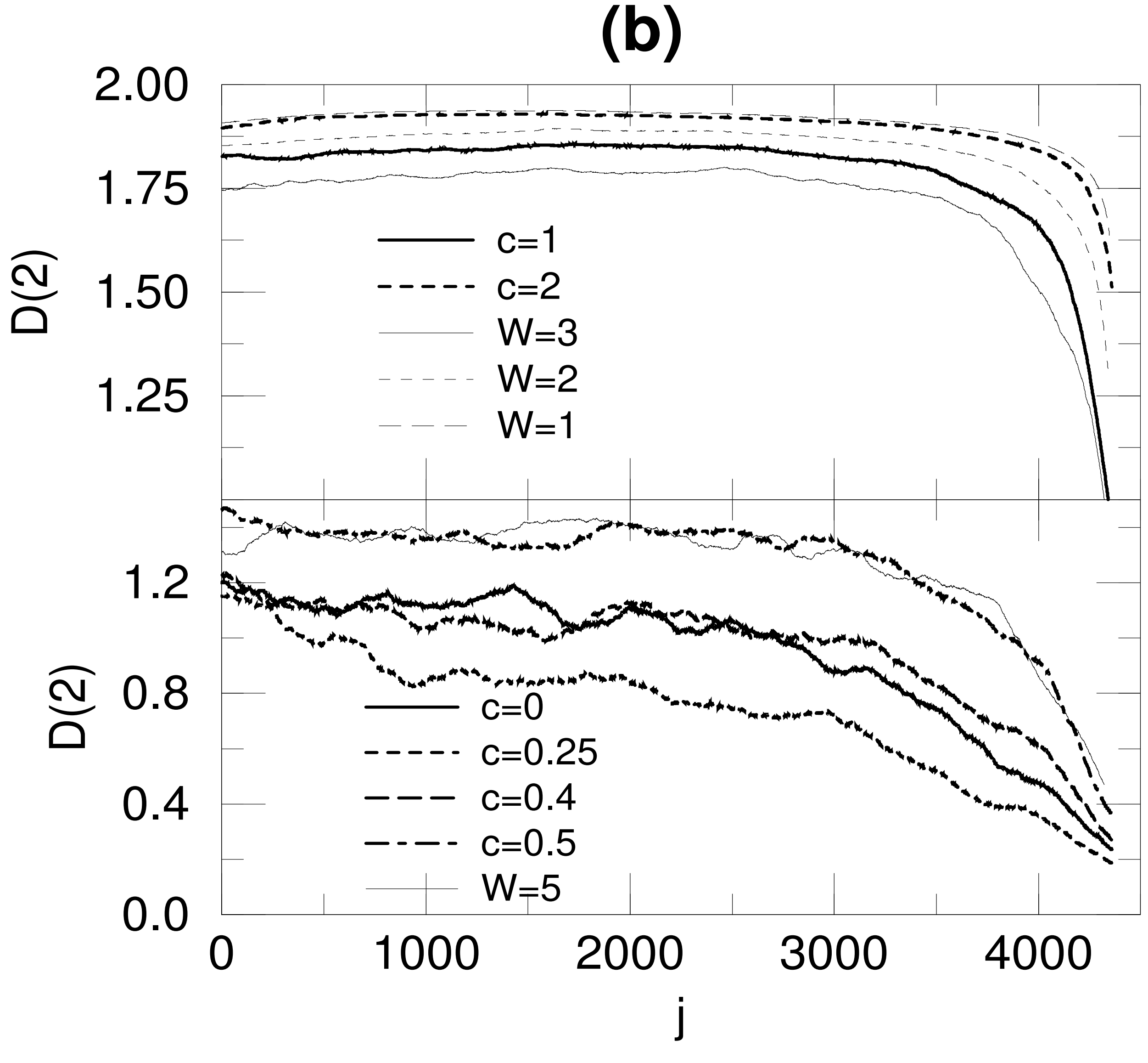

FIG. 1. (b) A. Eilmes *et al.*, "Critical behavior ...".

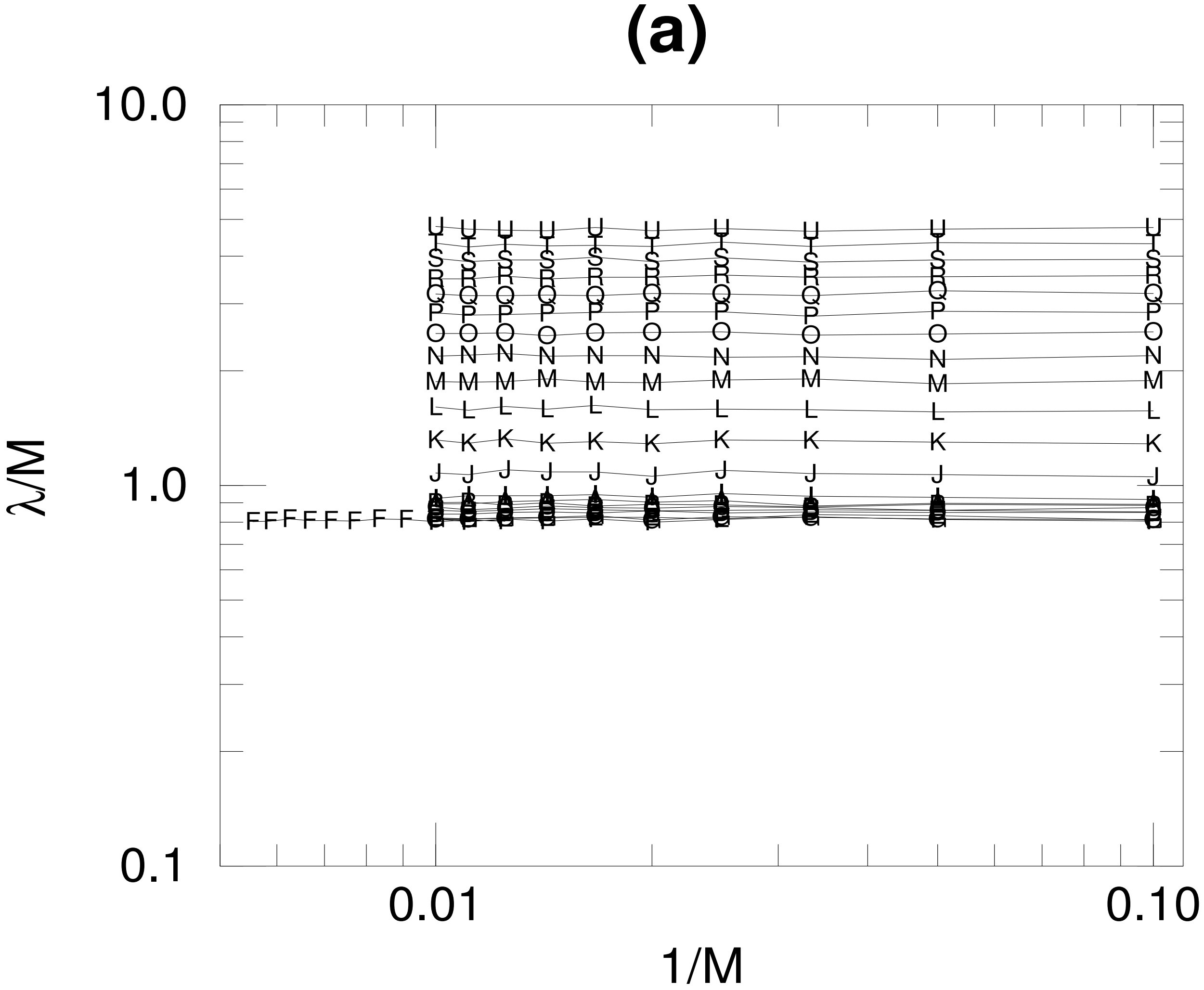

FIG. 2. (a) A. Eilmes *et al.*, "Critical behavior ...".

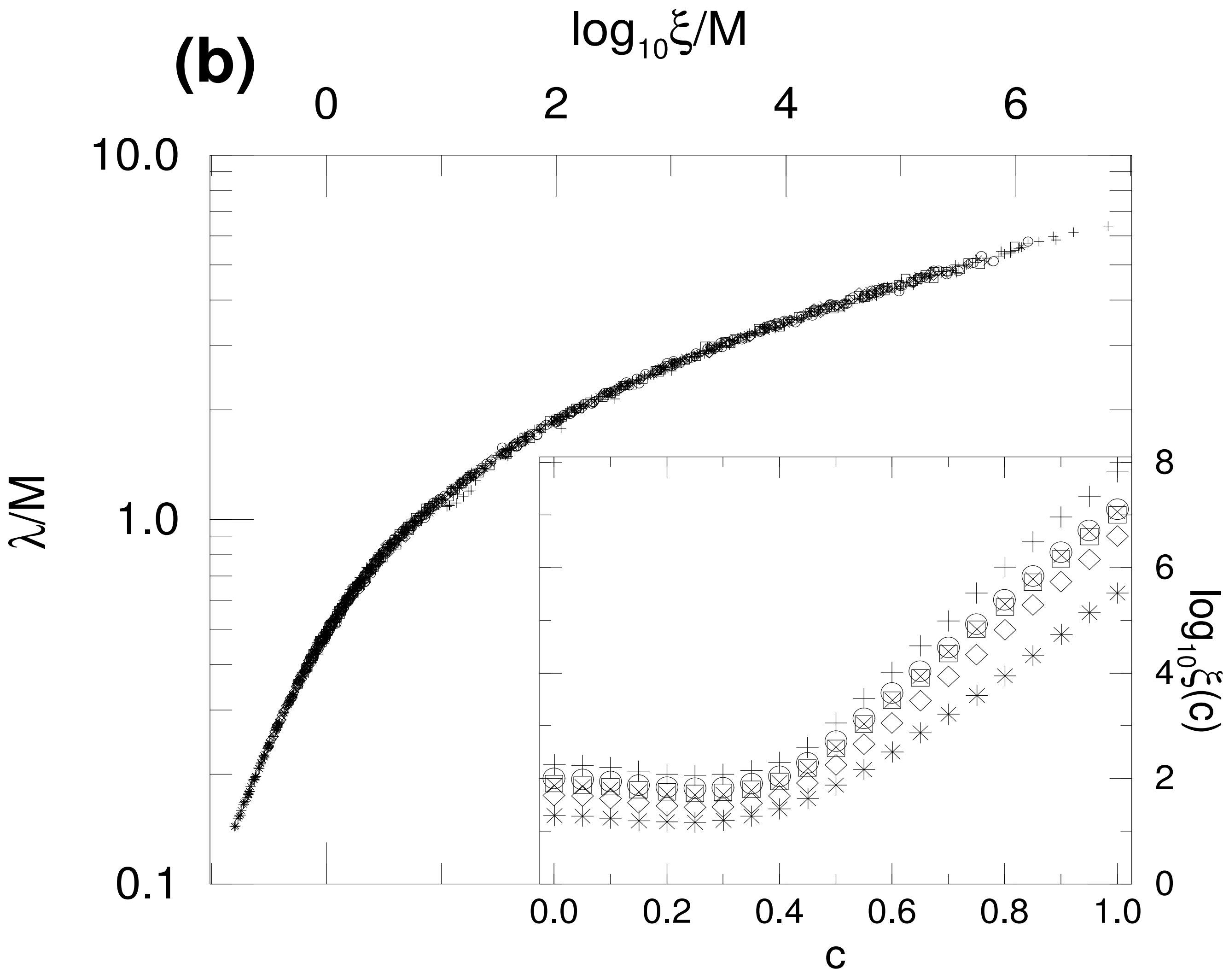

FIG. 2. (b) A. Eilmes *et al.*, “Critical behavior ...”.