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Subspace-cg-techniques for clinch-problems

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1 Introduction

Subspace-cg-techniques with projection methods are useful for an easy extension of an arbitrary finite element code with error estimation and adaptive strategies (see [2] and [1]) to an algorithm for solving contact problems with additional restrictions such as contact problems, see [3] for details.
In this paper we use the method to apply restrictions in the interior of the domain of an elastic body, not on the boundary.
The so called "clinch connection" is a method to connect two pieces of flat metals like sheet steels.
The connection is done by pressing the clinches in the sheet steels, like in figure 1.

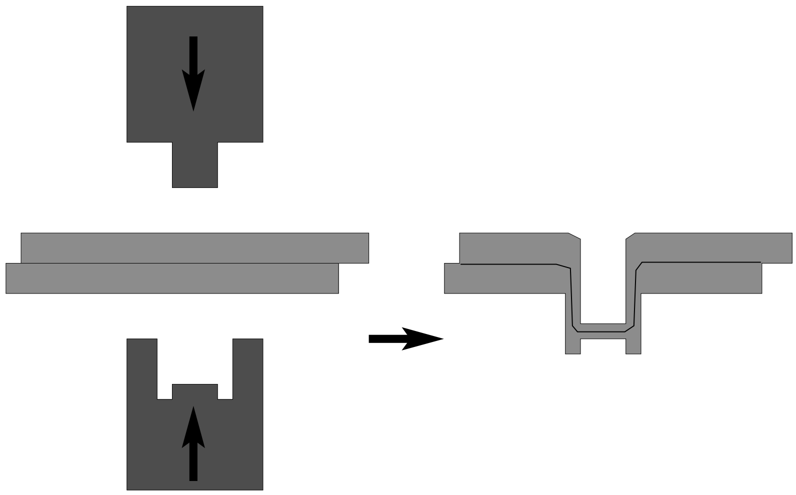

Figure 1: The clinching of sheet steels

Interesting parameters are strain and stress inside the two materials under different distributions of clinches.
In figure 2 you can see a trivial constellation, the sheet on the left hand side is fixed on his left boundary, the sheet on the right hand side is under vertical load on his right boundary.
One can show that the clinchpoint in the middle can not contribute so much to the robustness of the clinch-connection, because the stress is zero in the mid of the sheets (neutral line).

Complicated constructions need the computation of strains and stresses. An optimization of the clinch distribution requires the computation of a series of such problems.

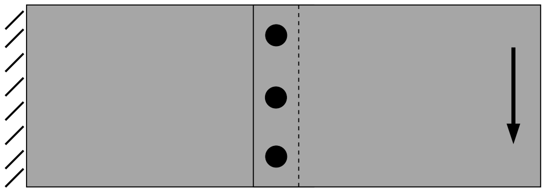

Figure 2: Trivial, not optimal connection

2 Mathematical Formulation

We consider an elasticity problem for 2 (overlapping) elastic bodies Ω_1 and Ω_2 in $2D$ and search for the displacement fields $u^1(x)$, $u^2(x)$ fulfilling the Lamé-equation with Lamé-constants λ, μ and stresstensor σ

$$\begin{aligned} -\mu\mathbf{\Delta}u &- (\lambda+\mu)\mathbf{grad}\ \mathbf{div}\ u = f \\ u(x) &= g_D \quad \text{on} \quad \Gamma_D \\ \sigma(u)\cdot\vec{\mathrm{n}} &= g_N \quad \text{on} \quad \Gamma_N \end{aligned} \tag{1}$$

for 2 domains Ω_1 und Ω_2.
We define the overlapping zone Ω^*

$$\Omega^* := \Omega_1 \cap \Omega_2$$

and the so called clinch zone $\Omega^*_C \subset \Omega^*$
We distinguish between the displacementfield $u^1(x)$ in the subdomain Ω_1 and $u^2(x)$ in the subdomain Ω_2.
The connection of the domains Ω_1 and Ω_2 in the clinchzone leads to the restriction

$$u^1(x) = u^2(x) \qquad \forall x \in \Omega^*_C \tag{2}$$

In practice the clinch zone Ω^*_C consists of nonconnected circular subdomains, the so called clinches.
In the discretised problem this restriction means, that the displacement of nodes with the same coordinates inside a clinchpoint have to be equal.
In the finite element realization the two domains meshed separately, all nodes in the clinch-zone exist two times with equal coordinates.
The variational formulation of (1) and finite element discretization without respecting the restriction (2) leads to a system of linear equations

$$K\underline{u} = \underline{b} \tag{3}$$

which is decoupled.

$$\begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix} \begin{bmatrix} \underline{u}^1 \\ \underline{u}^2 \end{bmatrix} = \begin{bmatrix} \underline{b}^1 \\ \underline{b}^2 \end{bmatrix}$$

One of the two matrices K_{11}, K_{22} could be singular, if there are no dirichlet type boundary conditions.
The restriction (2) is equivalent to the fact that the solution $\underline{u}$ of system (3) belongs to a subspace $\mathbb{U} \subset \mathbb{R}^N$.
To fulfill this restriction the system is to solve within this matching subspace $\mathbb{U} \subset \mathbb{R}^N$. This can be done by defining a projection on this subspace

$$P : \mathbb{R}^N \longrightarrow \mathbb{U} \quad with \quad P\underline{u} = \underline{u}$$

in an analogous manner to [1].
Then our restriction of problem (3) is equivalent to the equation

$$P^T K P \underline{u} = P^T \underline{b} \quad .$$

with $\underline{u} \in \mathbb{U}$.
The definition of this projector P will be done in the next section.

3 The projector

The definition and implementation of a proper projector for the clinch problem is easy, if both meshes fulfill the condition, that in the overlapping zone all nodes, edges and elements are situated congruent on top of each other, because in this case the definition of edge- and node pairs is possible.
With the restriction of refining each edge together with its partner edge a nodal pairing is guaranteed.
For a node pair (x^r, x^s) an identical displacement for both nodes of the pair is given, if we define a partial projector

$$P^{(r,s)} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \tag{4}$$

mapping the displacements $\underline{u}^r$,$\underline{u}^s$ of the nodes x^r and x^s to the following displacement

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_1^r \\ u_2^r \\ u_1^s \\ u_2^s \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(u_1^r + u_1^s) \\ \frac{1}{2}(u_2^r + u_2^s) \\ \frac{1}{2}(u_1^r + u_1^s) \\ \frac{1}{2}(u_2^r + u_2^s) \end{bmatrix} \tag{5}$$

which is an identical displacement for both nodes.

3.1 Implementational details

For an effective implementation the full projector is defined as a commutative product of the partial projectors.
After a refinement step an update of the projectors and nodal pairings is needed.
Testing all nodes and looking for a partner node within the clinch zone, if a node is in the clinch zone is possible but ineffective, because such an algorithm has quadratical efford in the number of nodes.
To avoid this an edge pairing is helpful, leading to an effective implementation.
We identify edges, which lie one above another and have a node inside the clinch zone Ω_C^*.

The following steps are to do:

1. After each refinement remove all edge pairings for edges which have no nodes in the clinch zone Ω_C^*.

   This step ensures that edges outside the clinch zone after a refinement are not longer linked, see also figure 3

2. Build node pairs for all linked edge pairs. The search for the partner node is only inside the nodes of the partner edge.

   Store the information of the partner node at the node, because this information is needed for the projector application.

3. Search in all new created edges, which come up from the refinement for edges with nodes in the clinch zone and link this node pairs too.

Figure 3: Removing of edge pairings after refinement

All this node pairs will be used in the preconditioning step where the projector, defined in (4) is applied.

4 Numerical examples

4.1 A simple clinchproblem

As the first example, a simple clinch connection of 2 sheets with 2 clinches like in figure 4 is considered.

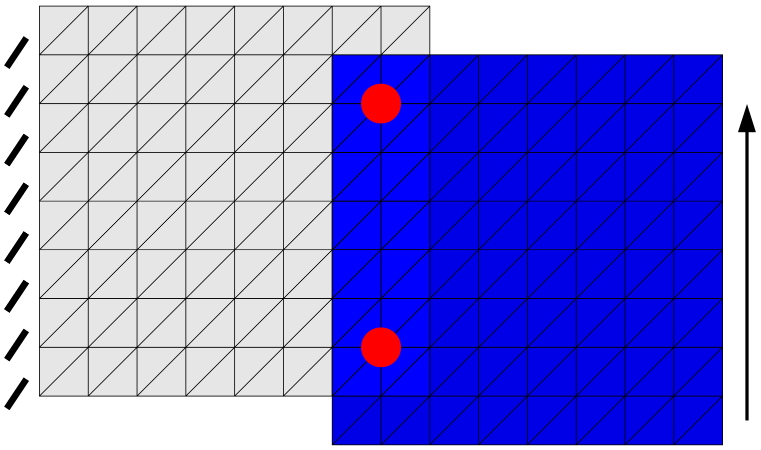

Figure 4: A simple clinchproblem

On the left boundary the displacement is prescribed as zero, on the right boundary as a constant displacement into the x_2-direction.
Both parts are connected with the red marked clinch points, i.e. all nodes in the red marked clinch zone will paired with their partner nodes, and we enforce equal displacements for this node pairs.
In the coarse mesh this restriction is given for only 2 nodes like shown in figure 5. These nodes lie above each other and all other nodes go to their energy minimal position.

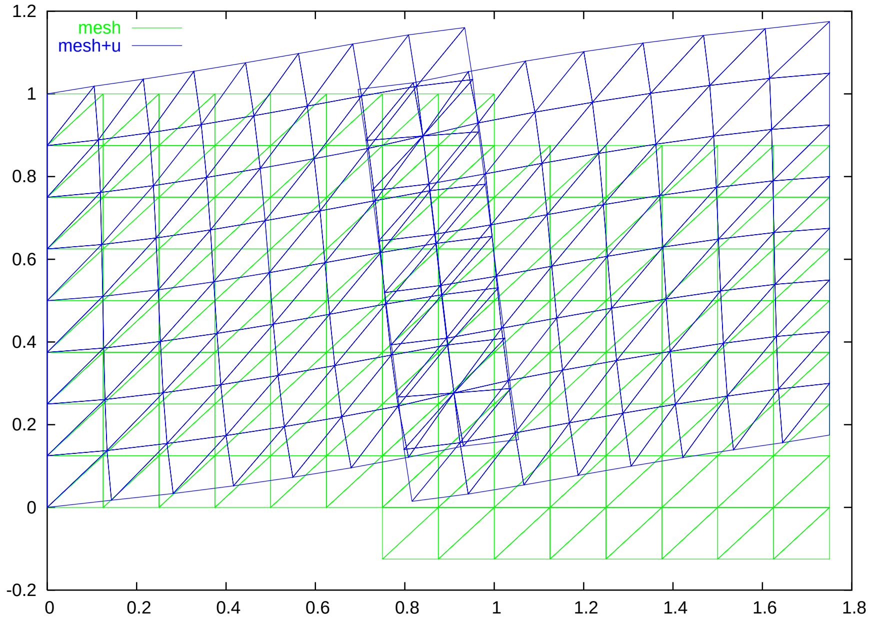

Figure 5: Displacement in the coarse mesh

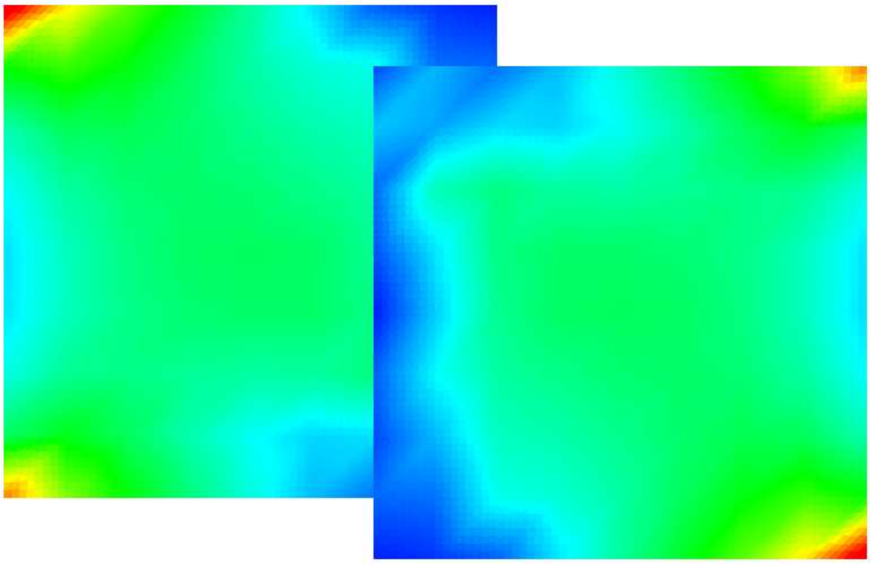

Figure 6: Comparison strain in the coarse mesh (both parts togehter)

In the figures 6 and 7 the comparison strain of both parts together and each part alone is shown.
In figure 8 the linked edges are marked red in further iteration steps this marking will get the shape of the clinchpoints.
After some refinement steps the refinement will go to the clinch zones, which leads to a good approximation of this zone.
In the figures 9 to 12 the same situation is shown after 10 refinement steps.

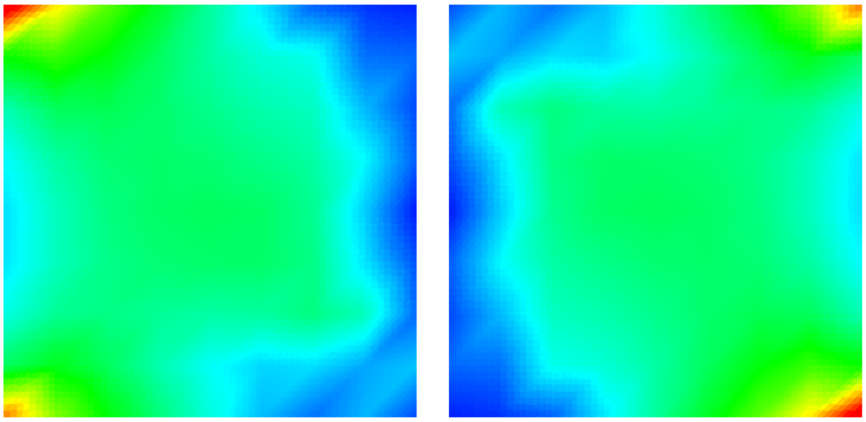

Figure 7: Comparison strain in the coarse mesh (each part for hisself)

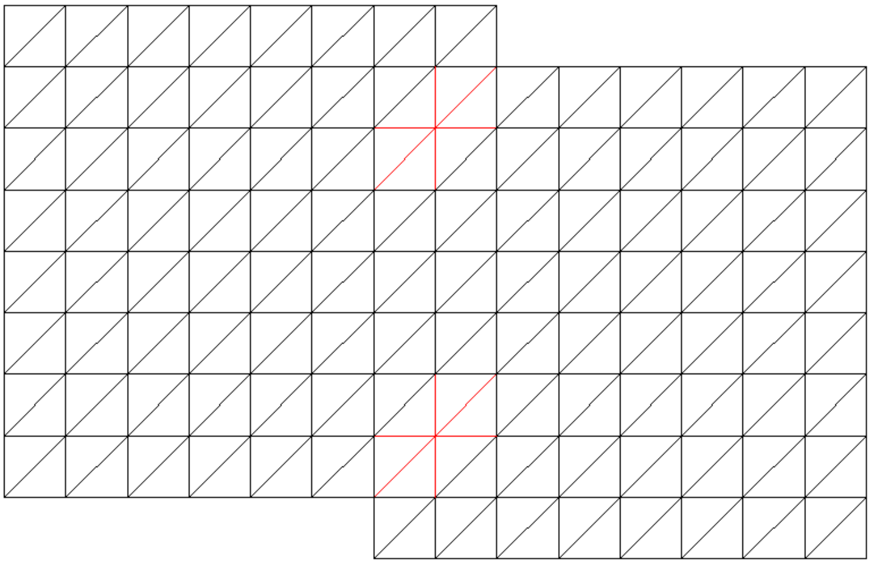

Figure 8: Linked edges in the coarse mesh

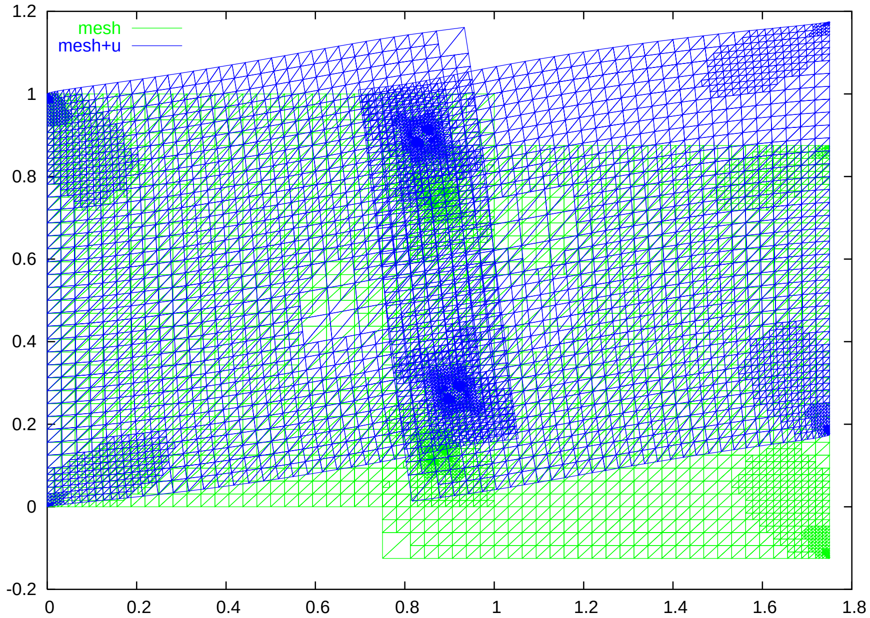

Figure 9: Displacement after 10 refinement steps

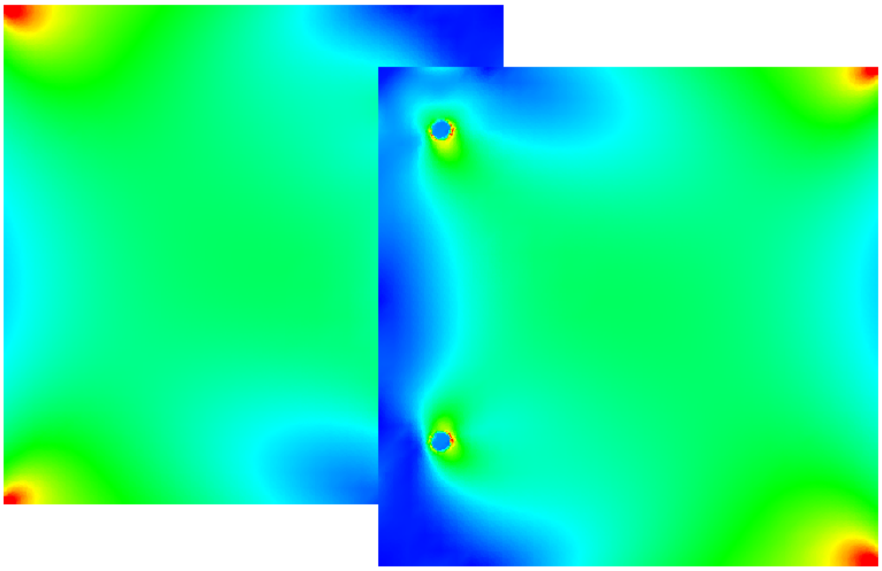

Figure 10: Strain after 10 refinement steps (both parts togehter)

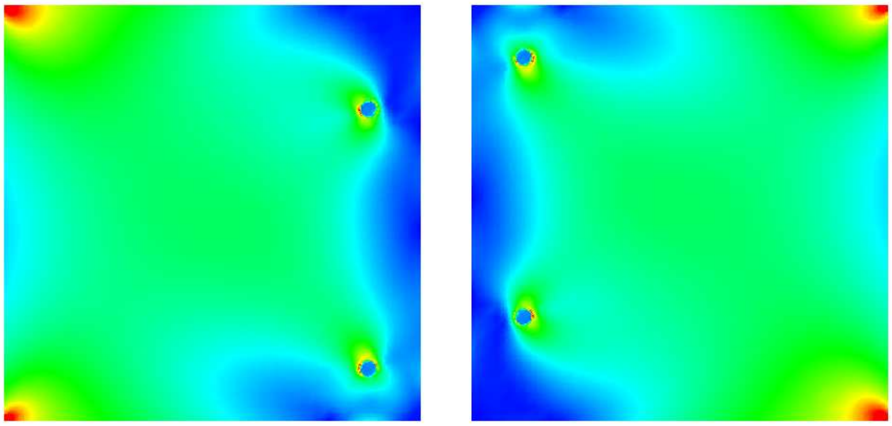

Figure 11: Strain after 10 refinement steps (each part for hisself)

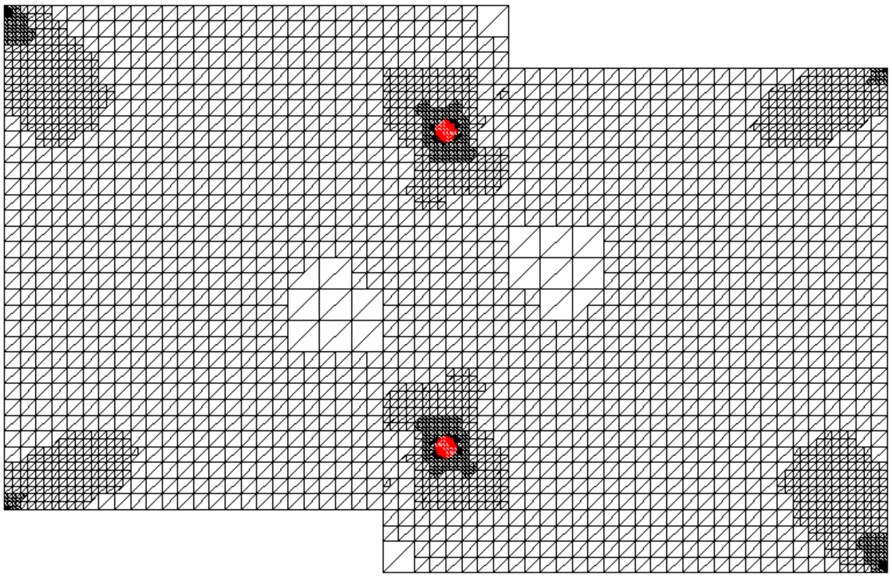

Figure 12: Linked edges after 10 refinement steps

4.2 Clinchproblem with non optimal clinch distribution

As an second example we consider the clinch problem, shown in figure 13.

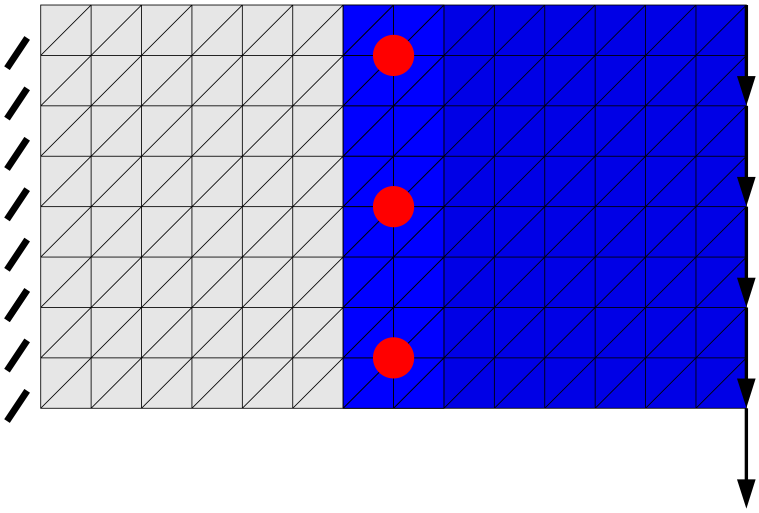

Figure 13: A non optimal clinch distribution

Two parts are connected with 3 clinches, the part on the left hand side is fixed on his left boundary, the part on the right hand side has a constant load in x_2-direction on his right boundary.

In figure 14 the displacement field is shown, figures 15 to 18 shows the comparison strain and figure 18 shows the linked edges after 17 refinement steps.

The strain pictures show, that the clinchpoint in the mid do not brings much for the stability of the connection. Without this clinchpoint the strain does not change so much, like shown in figure 17.

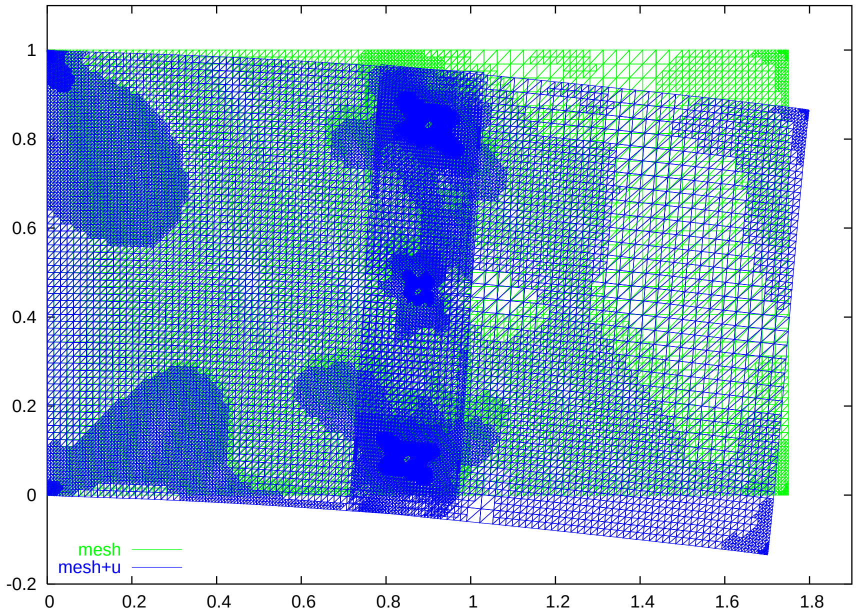

Figure 14: Displacement after 17 refinement steps

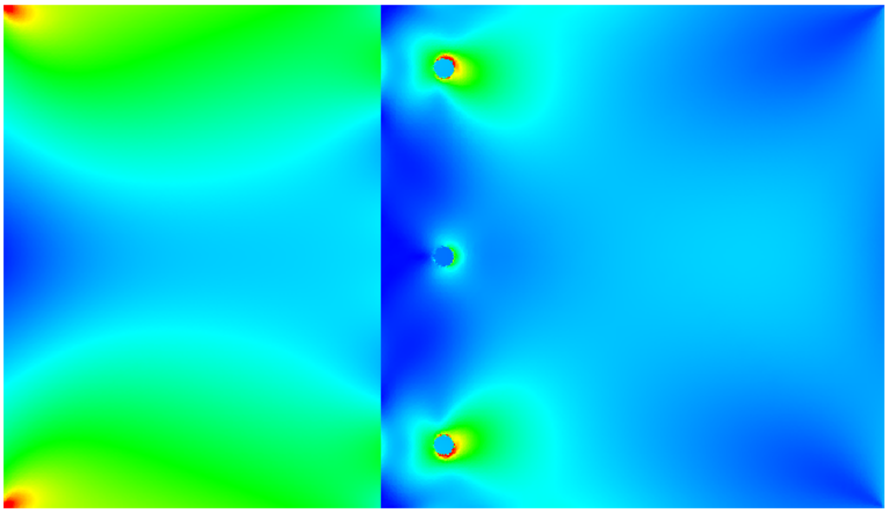

Figure 15: Comparison strain after 17 refinement steps (both parts together)

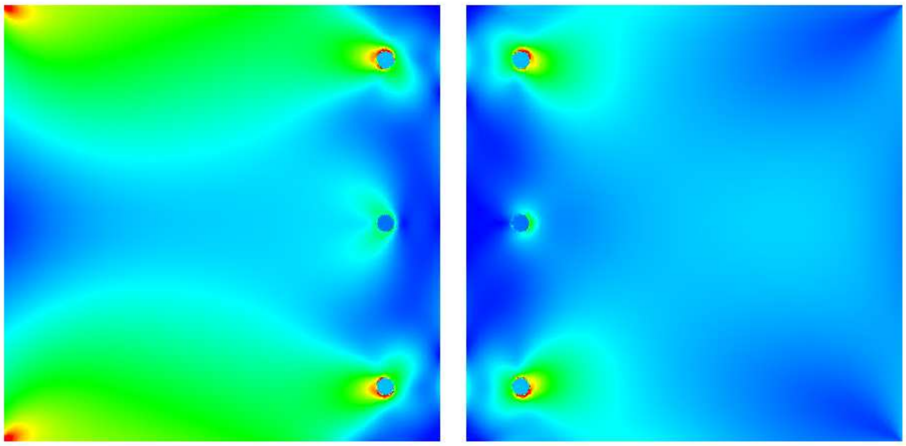

Figure 16: Comparison strain after 17 refinement steps with 3 clinches

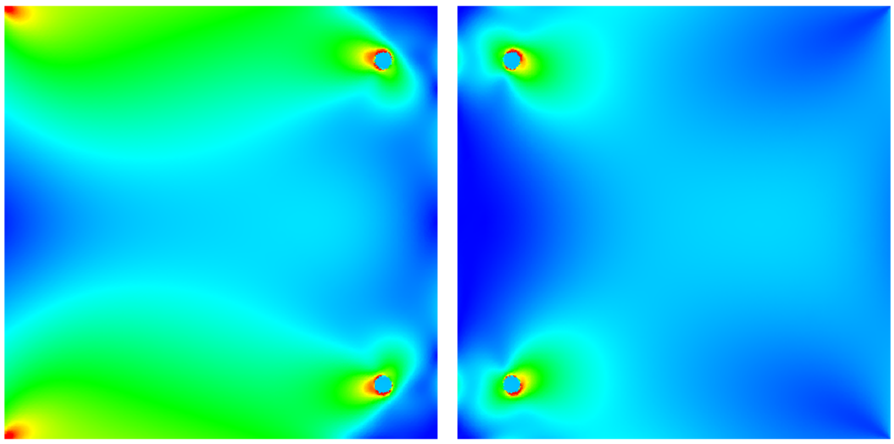

Figure 17: Comparison strain after 17 refinement steps with 2 clinches

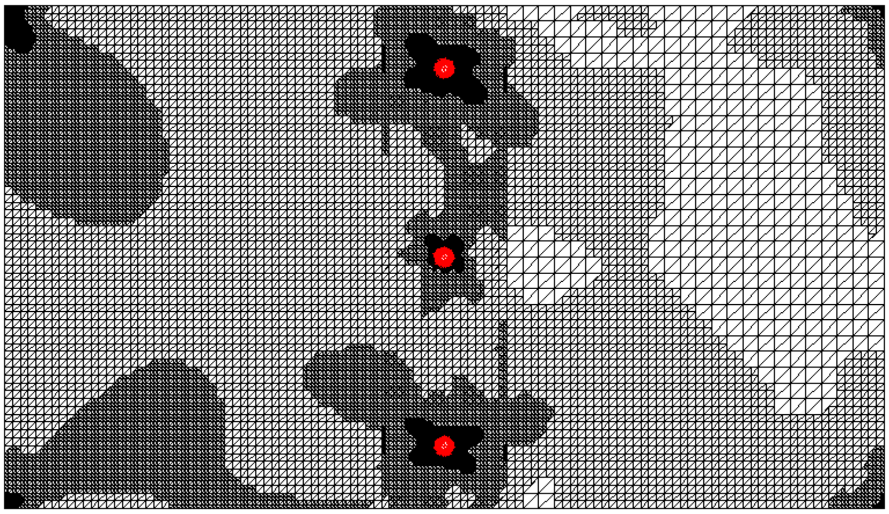

Figure 18: Linked edges after 17 refinement steps

5 Outlook

The handling of the clinch problem with subspace-cg-techniques can be implemented in an easy manner in an existing adaptive finite element code. The computational efford is approximatly the same as solving a usual elasticity problem.
Especially for optimization of the clinch distribution the computation of a series of clinch problems can be done in realistic computing times.

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