Real-space renormalization group approach to the quantum Hall transition

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1. Introduction

The integer quantum Hall (QH) transition is described well in terms of a delocalization-localization transition of the electron wavefunction. In contrast to a usual metal-insulator transition (MIT), the QH transition is characterized by a single extended state located exactly at the center $\epsilon = 0$ of each Landau band.[1)] When approaching $\epsilon = 0$, the localization length ξ of the electron wavefunction diverges according to a power law $\epsilon^{-\nu}$, where ϵ defines the distance to the MIT for a suitable control parameter, e.g., the electron energy. On the theoretical side, the value of ν has been extracted from various numerical simulations, e.g., $\nu = 2.5 \pm 0.5$,[2)] 2.4 ± 0.2,[3)] 2.35 ± 0.03,[4)] and 2.39 ± 0.01.[5)] In experiments $\nu \approx 2.3$ has been obtained, e.g., from the frequency[6)] or the sample size[7)] dependence of the critical behavior of the resistance in the transition region at strong magnetic field.

We study the critical properties of the integer QH transition by employing the real-space renormalization-group (RG) approach to the Chalker-Coddington (CC) network model.[2)] We calculate the critical distribution $P_c(G)$ of the conductance and the critical exponent ν of the QH transition for two different RG units. This allows to demonstrate that the quality of the results crucially depends on the choice of the RG unit.

The CC model describes a single QH transition using a chiral network consisting of electron trajectories along equipotential lines (*links*) and saddle points (SP's) of the potential (*nodes*). Each SP acts as a scatterer and relates the wavefunction amplitudes in two incoming and two outgoing channels. It can be characterized by a 2×2 S matrix, which depends only on the transmission and reflection coefficients t_i and r_i. The links correspond to random phases Φ_j and reflect the randomness of the potential disorder in a sample. We consider two previously studied, different RG units on a regular 2D square lattice as shown in Fig. 1. One is constructed from 4 SP's,[8,9)] the other consists of 5 SP's.[5,10)] The RG unit should be chosen in a way such that the essential properties of the network are taken into account. In the course of our RG approach an RG unit is then mapped onto a new single super-SP using the analytical dependence[5,11)]

$$t' = f\left(\{t_i, r_i\}, \{\Phi_j\}\right) \qquad (1)$$

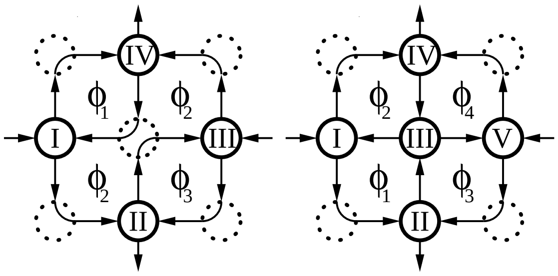

Fig. 1. RG units constructed from 4 SP's (left) and 5 SP's (right) indicated by full circles. Some connectivity is neglected (dotted circles). The phases Φ_j are accumulated by the electron motion (arrows) along contours of the energy potential.

of the transmission coefficient t' of the super-SP on the coefficients t_i and r_i and the phases Φ_j within the RG unit. The effective system size of our samples is doubled in each RG step, which allows to reach very large effective systems. Starting from an initial distribution $P_0(t)$ of the transmission coefficient t of a SP we can now compute $P_1(t')$ of the RG unit and then continue iteratively.[5)] We note that $P(G)$ of the dimensionless conductance G is related to $P(t)$ by $G = t^2$.

2. RG approach for $P(G)$

In order to find the fixed point (FP) conductance distribution $P_c(G)$, we start from a certain initial distribution of transmission coefficients, $P_0(t)$. The distribution is discretized in at least 1000 bins, such that the bin width is typically 0.001 for the interval $t \in [0, 1]$. From $P_0(t)$, we obtain t_i, and substitute them into the RG transformation (1). The phases Φ_j are chosen randomly from the interval $\Phi_j \in [0, 2\pi]$. In this way we calculate at least 10^7 super-transmission coefficients t'. At the next step we repeat the procedure using P_1 as an initial distribution. We assume that the iteration process has converged when the mean-square deviation $\int dt[P_n(t) - P_{n-1}(t)]^2$ of the distribution P_n and its predecessor P_{n-1} deviate by less than 10^{-4}. While we reach convergence in the 4 SP approach only when chosing P_0 very close to the FP distribution, in the 5 SP case any $P_0(G)$ symmetric with respect to $G = 0.5$ is suitable. Our resulting $P_c(G)$ of both RG units is shown in Fig. 2. For the 5 SP unit the FP distribution $P_c(G)$ exhibits a flat minimum around $G = 0.5$, and sharp peaks close to $G = 0$ and $G = 1$. It is symmetric with respect to

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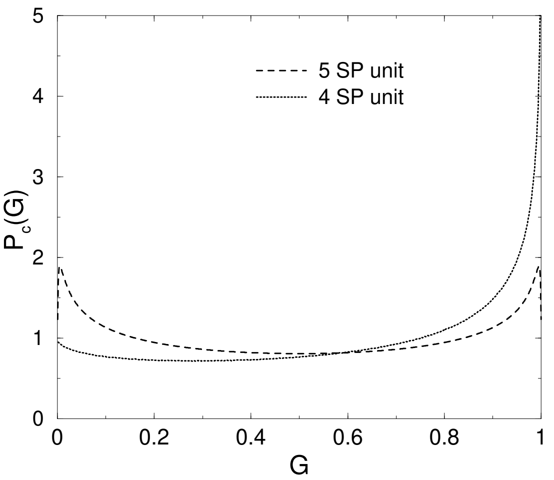

Fig. 2. Comparison of the critical distribution of the conductance $P_c(G)$ at the QH transition obtained using the 5 SP (dashed line) and 4 SP (dotted line) RG unit. The latter clearly deviates from the expected symmetry with respect to $G = 0.5$.

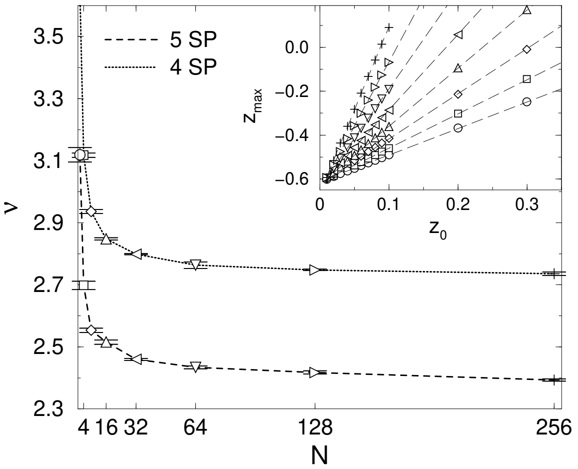

Fig. 3. The critical exponent ν as function of the effective system size $N = 2^n$ for 4 SP (dotted line) and 5 SP unit (dashed line). Inset: Maximum $z_{\max}$ of $Q(z)$ vs. initial shift z_0 for 8 RG iterations (symbols) using 4 SP. Dashed lines indicate linear fits.

$G \approx 0.5$. The 4 SP unit yields differing results. While $P_c(G)$ is still rather flat it is clearly asymmetric, which already indicates that the 4 SP unit can not describe all of the underlying symmetry of the CC network.

3. RG approach for ν

We now turn to the critical exponent ν. We introduce the dimensionless SP height z_i. It is related to the transmission coefficient t_i at $\epsilon = 0$ by $t_i = (e^{z_i} + 1)^{-1/2}$, which allows to translate transformation (1) into the language of SP heights. Correspondingly, the distribution $P(G)$ determines a distribution $Q(z)$ of SP heights via $Q(z) = P(G)dG/dz$. Suppose that the RG procedure starts with an initial distribution, $Q_0(z) = Q_c(z - z_0)$, shifted from the critical distribution, $Q_c(z)$, by a small $z_0 \propto \epsilon$. As a result of the instability of the FP, an initial shift z_0 results in a further drift of the maximum position, $z_{\max,n}$, away from $z = 0$ after each RG step where $z_{\max,n}$ depends linearly on z_0 for small z_0.[5] From the corresponding slope the critical exponent can be calculated for each RG step n using

$$\nu = \frac{\ln 2^n}{\ln\left(\frac{z_{\max,n}}{z_0}\right)}. \tag{2}$$

Figure 3 shows the behavior of ν as function of n for the 4 and 5 SP RG units. Both curves approach convergence monotonously from larger values of ν. During all iteration steps, ν for the 4 SP differs from the 5 SP result by an almost constant positive shift. After 8 iterations, which equals an increase of system size by a factor of 256, we find $\nu = 2.39 \pm 0.01$ (5 SP) and $\nu = 2.74 \pm 0.02$ (4 SP). The error describes a confidence interval of 95% as obtained from the fit to a linear behavior. The result for ν of the 4 SP RG unit deviates clearly from the 5 SP result and the values obtained by other methods.[2–4]

4. Conclusion

The real-space RG approach allows only a crude approximation of the network, since links of the network are cut and the full connectivity of the network is not preserved. Therefore for the determination of $P_c(G)$ and ν, it is essential to construct the RG unit in such a way that it includes the symmetry of the network and the corresponding physical problem. We have shown that a simple 4 SP RG unit already provides a rough picture of the critical properties of the QH transition. A slightly larger 5 SP unit yields surprisingly accurate results. It seems obvious that the accuracy of the RG approximation should further enhance when increasing the size of the RG unit.[8] However from a numerical point of view one benefits from the analytic form of Eq. (1), which is not known for large RG units.

We remark that the RG approach can be performed also for the phases of the RG unit.[12] This allows to calculate the energy level statistics which is a further characteristic of the critical properties at the QH transition and examplifies the strength of the RG approach.

1) B. Huckestein, Rev. Mod. Phys. **67** (1995) 357.
2) J. T. Chalker and P. D Coddington: J. Phys.: Condens. Matter **21** (1988) 2665.
3) D.-H. Lee, Z. Wang, and S. Kivelson: Phys. Rev. Lett. **70** (1993) 4130.
4) B. Huckestein: Europhys. Lett. **20** (1992) 451.
5) P. Cain, R. A. Römer, M. Schreiber, and M. E. Raikh: Phys. Rev. B, **64** (2001) 235326.
6) F. Hohls, U. Zeitler, and R. J. Haug: Phys. Rev. Lett. **86** (2001) 5124.
7) S. Koch *et al.*: Phys. Rev. Lett. **67** (1991) 883.
8) A. Weymer and M. Janssen: Ann. Phys. (Leipzig) **7** (1998) 159.
9) U. Zülicke and E. Shimshoni: Phys. Rev. B **63** (2001) 241301.
10) D. P. Arovas, M. Janssen, and B. Shapiro: Phys. Rev. B **56**, (1997) 4751.
11) A. G. Galstyan and M. E. Raikh: Phys. Rev. B **56** (1997) 1422.
12) P. Cain, R. A. Römer, and M. E. Raikh: Preprint SFB393/02-10.