Exponents of the localization lengths in the bipartite Anderson model with off-diagonal disorder

Andrzej Eilmes [a,1], Rudolf A. Römer [b,2] and Michael Schreiber [b,3]

[a] *Department of Computational Methods in Chemistry, Jagiellonian University, Ingardena 3, 30-060 Kraków, Poland*

[b] *Institut für Theoretische Physik, Technische Universität, 09107 Chemnitz, Germany*

Abstract

We investigate the scaling properties of the two-dimensional (2D) Anderson model of localization with purely off-diagonal disorder (random hopping). In particular, we show that for small energies the infinite-size localization lengths as computed from transfer-matrix methods together with finite-size scaling diverge with a power-law behavior. The corresponding exponents seem to depend on the strength and the type of disorder chosen.

Key words: Localization, off-diagonal disorder, critical exponents, bipartiteness

1. Introduction

Of paramount importance for the theory of disordered systems and the concept of Anderson localization [1–5] is the scaling theory of localization as proposed in 1979 [6]. Especially in 2D, this theory predicts the absence of a disorder-driven MIT for generic situations such that all states remain localized and the system is an insulator [7–9]. However, already early [10,11] it was suggested that an Anderson model of localization with purely off-diagonal disorder might violate this general statement since non-localized states were found at the band center [12–14]. Further numerical investigations in recent years [15–19] have uncovered additional evidence that the localization properties at $E = 0$ are special. In particular, it was found that the divergence in the density of states DOS is accompanied by a divergence of the localization lengths λ [15,16]. This divergence does not violate the scaling arguments [20], since it can be shown that its scaling properties are compatible with critical states only [16], i.e., there are no truly extended states at $E = 0$. Of importance for the model is a very special symmetry around $E = 0$ which holds in the bipartite case of an even number of sites [20,21]. Then the spectrum is symmetric such that for every eigenenergy $E_i < 0$ there is also a state with energy $E_i > 0$. This situation is connected with a so-called chiral universality class. Furthermore, the model is closely connected to the random flux model studied in the quantum-Hall situation

[1] E-mail: eilmes@chemia.uj.edu.pl
[2] E-mail: r.roemer@physik.tu-chemnitz.de
[3] Corresponding author.
E-mail: schreiber@physik.tu-chemnitz.de

where the off-diagonal disorder is due to a random magnetic flux through the 2D plaquettes.

Thus although we do not have a true MIT, we nevertheless have a transition from localized via delocalized to localized behavior as we sweep the energy through $E = 0$. We consider a single electron on the 2D lattice with N sites described by the Anderson Hamiltonian

$$H = \sum_{i \neq j}^{N} t_{ij} \left| i \right\rangle \left\langle j \right| + \sum_{i}^{N} \epsilon_i \left| i \right\rangle \left\langle i \right| \qquad (1)$$

where $|i\rangle$ denotes the electron at site i. The onsite energies ϵ_i are set to 0 and the off-diagonal disorder is introduced by choosing random hopping elements t_{ij} between nearest neighbor sites.

We test three different distributions of t_{ij}: (i) a rectangular distribution $t_{ij} \in [c - w/2, c + w/2]$ [15], (ii) a Gaussian distribution $P(t_{ij}) = \exp\left[-(t_{ij} - c)^2/2\sigma^2\right]/\sqrt{2\pi\sigma^2}$, and (iii) a rectangular distribution of the logarithm of t_{ij} where $P(\ln t_{ij}/t_0) = 1/w$ if $|\ln t_{ij}/t_0| \leq w/2$ or $P(\ln t_{ij}/t_0) = 0$ otherwise [14]. The logarithmic distribution appears more suited to model actual physical systems [14]. We also note that the logarithmic distribution avoids problems with zero t elements and thus there is no need to introduce an artificial lower cutoff as for the box and Gaussian distributions [15]. Furthermore, the box and Gaussian distributions will usually have negative t values which correspond to a rather artificial phase shift.

In the case of rectangular and normal distributions we set the width w and the standard deviation σ to 1 and change the center c of the distribution. In the case of the logarithmic t distribution $t_0 = 1$ sets the energy scale and we change the disorder width w. Values of the parameters were $c = 0$, 0.25, 0.5, and 1.0 for the rectangular distribution; $c = 0$ and $c = 0.25$ for the Gaussian distribution and $w = 2$, 6, and 10 for the logarithmic t distribution.

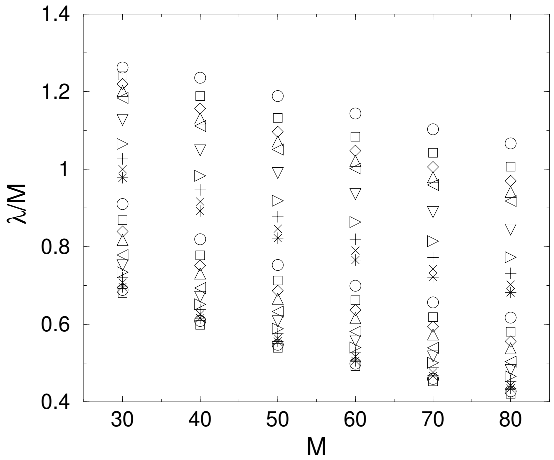

Fig. 1. Reduced localization length λ/M for various system sizes M of a box t distribution with $c = 0$. Symbols indicate different energies ranging from 0.025 (○), 0.0225 (□) to 2×10^{-5} (□).

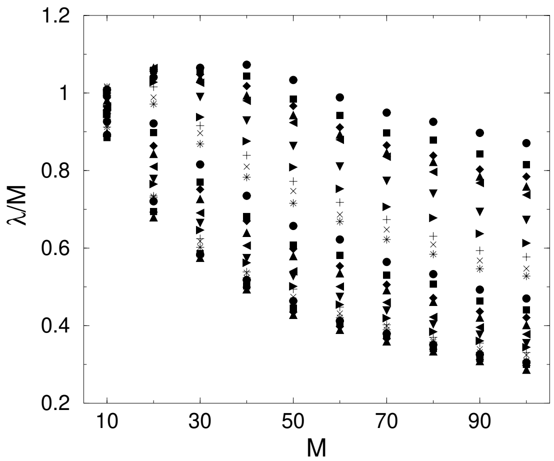

Fig. 2. Reduced localization length λ/M for various system sizes M of a Gaussian t distribution with $c = 0.25$. Symbols indicate different energies ranging from 0.03 (●), 0.0275 (■) to 2×10^{-5} (▲).

2. Computation of the localization lengths at $E \neq 0$

The transfer-matrix method [22,23] was used to compute the localization lengths for strips of various widths M up to $M = 100$ in the energy interval $2 \times 10^{-5} \leq E \leq 0.2048$. In Figs. 1, 2, and 3 we

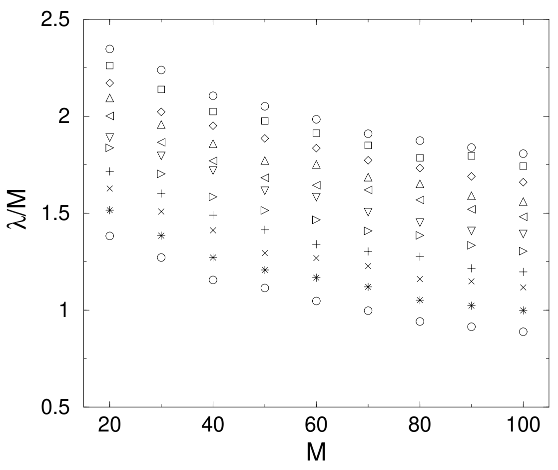

Fig. 3. Reduced localization length λ/M for various system sizes M of a logarithmic t distribution with $w = 2$. Symbols indicate different energies ranging from 0.2048 (○), 0.1024 (□) to 2×10^{-4} (□).

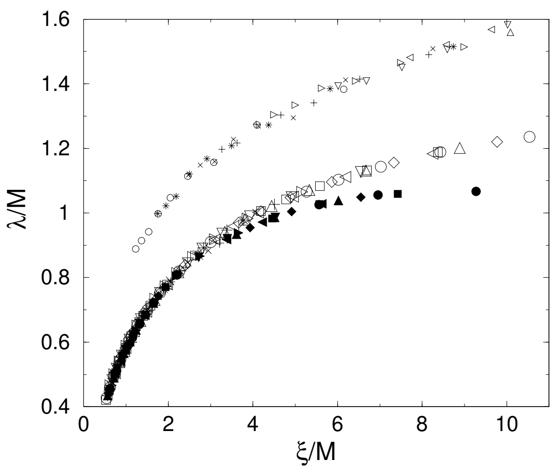

Fig. 4. Finite-size-scaling plots for box ($c = 0$, $M \in [50, 80]$, large open symbols), Gaussian ($c = 0$, $M \in [20, 60]$, filled symbols), and logarithmic ($w = 2$, $M \in [30, 100]$, small open symbols) t distributions.

show the system size dependence for, e.g., special values of c and w and all three disorder distributions. The accuracy of our results was $0.1 - 0.3\%$ or 1% depending on the disorder distribution and the values of parameters, see Table 1 for actual parameter values. Next, the finite-size-scaling analysis of Ref. [23] was applied to the data. The calculated localization lengths usually increase as the energy approaches 0. Only, for small even width values $(10, 20)$ it decreases significantly close to $E = 0$ [19] which makes finite-size scaling impossible. Therefore the smallest system sizes were dropped during the finite-size scaling procedure. Results for the finite-size scaling curves are shown in Fig. 4 for the three different distributions.

3. Critical exponents

One expects that the scaling parameters ξ obtained from finite-size scaling diverge close to $E = 0$ [24]. However, the precise functional form of this divergence is not yet know. In Ref. [24] it has been suggested that for energies $E > E^*$ the divergence can be described by a power law as

$$\xi(E) \propto \left|\frac{E_0}{E}\right|^{\nu} \tag{2}$$

with the critical exponent ν. For even smaller $|E| \ll E^*$, this behavior should then change to

$$\xi(E) \propto \exp\sqrt{\frac{\ln E_0/E}{A}} \tag{3}$$

with constants E_0 and A given by the renormalization group flow [24]. Double-logarithmic plots of ξ vs. E in Figs. 5, 6 and 7 confirm the power-law behavior with reasonable accuracy down to $E \approx 10^{-4}$. For smaller values it has been shown already in Ref. [19] that a new behavior is to be expected.

Table 1 collects the values of the critical exponent obtained for different disorders. In the case of the logarithmic t distribution and $w = 10$ the power-law divergence fails, therefore the exponent was not calculated. From Table 1, it can be easily seen that all calculated values are in the range $0.2 \le \nu \le 0.5$. The exponent is apparently not universal but seems to depend on the kind of disorder and the actual value of parameters; for stronger disorders ν becomes smaller (for the logarithmic t distribution the disorder strength increases with w [14], for the rectangular distribution the strongest disorder appears at $c = 0.25$

Table 1
Estimated values of the exponents of the localization lengths for various disorder strengths and distributions. The error bars represent the standard deviations from the power-law fit and should be increased by at least one order of magnitude for a reliable representation of the actual errors.

disorder distribution	parameters	accuracy in %	sizes used in finite-size scaling	estimated exponent
box	$c = 0$	0.1-0.2	30-80	0.326 ± 0.002
box	$c = 0$	0.1-0.2	25-65	0.325 ± 0.002
box	$c = 0.25$	0.1-0.2	30-70	0.319 ± 0.001
box	$c = 0.5$	0.1-0.2	30-70	0.361 ± 0.001
box	$c = 1.0$	0.1-0.3	30-70	0.444 ± 0.002
Gaussian	$c = 0$	0.1-0.2	30-60	0.314 ± 0.001
Gaussian	$c = 0.25$	1	30-100	0.310 ± 0.001
Gaussian	$c = 0.25$	1	35-95	0.308 ± 0.001
logarithmic	$w = 2$	1	20-100	0.412 ± 0.007
logarithmic	$w = 6$	1	20-100	0.251 ± 0.004
logarithmic	$w = 10$	1	20-100	—

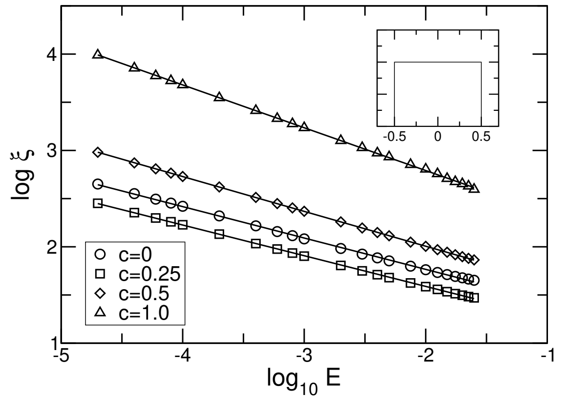

Fig. 5. Variation of the infinite-size localization length ξ with E for box distributions. The inset shows the t distribution for $c = 0$.

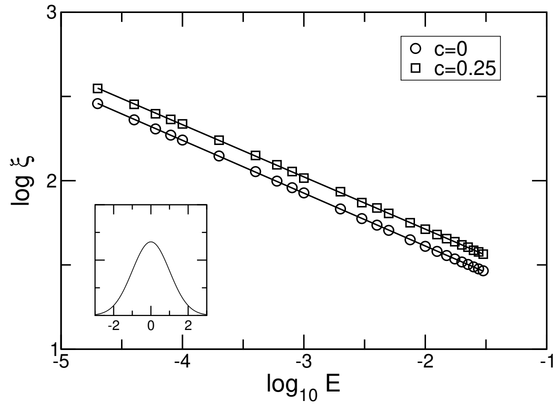

Fig. 6. Variation of the infinite-size localization length ξ with E for Gaussian distributions. The inset shows the t distribution for $c = 0$.

[15]). This non-universality is in agreement with the results of Ref. [24].

As the localization lengths calculated for odd and even strip widths may exhibit different behavior [14,19] we repeated the procedure also for some odd-width systems for rectangular and Gaussian distributions. The difference is within error bars, thus for these disorder strengths the effect is negligible.

4. Conclusions

Our results suggest that the localization-delocalization-localization present in the off-diagonal Anderson model of localization in 2D can be described by a set of exponents that model the divergence of the localization lengths ξ at $E = 0$. Note that these exponents are in reasonable agreement with the exponent 0.5 first estimated for the scaling of the participation numbers in Ref. [15]. Down to $E \approx 10^{-4}$ the power-law behavior can model the data reasonably well. Thus we ex-

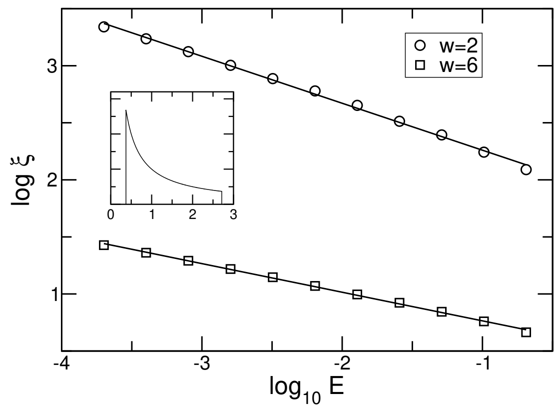

Fig. 7. Variation of the infinite-size localization length ξ with E for logarithmic distributions. The inset shows the t distribution for $w = 2$.

pect the crossover predicted in Ref. [24] to appear at smaller energies. We find that the exponents depend on the strength and distribution of the off-diagonal disorder also in agreement with Ref. [24]. Currently, we are extending these calculations to smaller energies.

We note that it might be interesting to also investigate the situation in honeycomb lattices [25], where the van Hove singularity of the square lattice at $E = 0$ does not interfere with the divergence due to the bipartiteness which is of interest here.

5. Acknowledgments

We thank M. Fabrizio for stimulating discussions. We gratefully acknowledge support by the Deutsche Forschungsgemeinschaft (SFB393) and the SMWK.

References

[1] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

[2] E. N. Economou and M. H. Cohen, Phys. Rev. Lett. **25**, 1445 (1970).

[3] E. N. Economou and M. H. Cohen, Phys. Rev. B **5**, 2931 (1972).

[4] E. N. Economou, Journale de Physique C **3**, 145 (1972).

[5] D. C. Licciardello and E. N. Economou, Sol. State Comm. **15**, 969 (1974).

[6] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[7] P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[8] B. Kramer and A. MacKinnon, Rep. Prog. Phys. **56**, 1469 (1993).

[9] D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[10] E. N. Economou and P. D. Antoniou, Solid State Commun. **21**, 285 (1977).

[11] P. D. Antoniou and E. N. Economou, Phys. Rev. B **16**, 3768 (1977).

[12] C. M. Soukoulis and E. N. Economou, Phys. Rev. B **24**, 5698 (1981).

[13] A. Fertis, A. Andriotis, and E. N. Economou, Phys. Rev. B **24**, 5806 (1981).

[14] C. M. Soukoulis, I. Webman, G. S. Grest, and E. N. Economou, Phys. Rev. B **26**, 1838 (1982).

[15] A. Eilmes, R. A. Römer, and M. Schreiber, Eur. Phys. J. B **1**, 29 (1998).

[16] A. Eilmes, R. A. Römer, and M. Schreiber, phys. stat. sol. (b) **205**, 229 (1998).

[17] P. Cain, Diploma thesis, Technische Universität Chemnitz, 1998.

[18] P. Cain, R. A. Römer, and M. Schreiber, Ann. Phys. (Leipzig) **8**, SI-33 (1999), cond-mat/9908255.

[19] P. Biswas, P. Cain, R. A. Römer, and M. Schreiber, phys. stat. sol. (b) **218**, 205 (2000), cond-mat/0001315.

[20] R. Gade and F. Wegner, Nucl. Phys. B **360**, 213 (1991).

[21] R. Gade, Nucl. Phys. B **398**, 499 (1993).

[22] A. MacKinnon and B. Kramer, Phys. Rev. Lett. **47**, 1546 (1981).

[23] A. MacKinnon and B. Kramer, Z. Phys. B **53**, 1 (1983).

[24] M. Fabrizio and C. Castellani, (2000), cond-mat/0002328.

[25] M. Schreiber and M. Ottomeier, J. Phys.: Condens. Matter **4**, 1959 (1992).