Aharonov-Bohm oscillations in the exciton luminescence from a semiconductor nanoring

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Abstract

Magnetoluminescence of an exciton confined within a semiconductor ring is studied theoretically. We demonstrate that, despite the net electrical neutrality of the exciton, both the spectral position and the intensity of photoluminescence peak exhibit periodic oscillations as a function of magnetic flux threading the ring. The period of oscillations is Φ_0—the universal flux quantum. The origin of the effect is the finite probability for bound electron and hole to tunnel in the opposite directions and meet each other on the opposite side of the ring.

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I. INTRODUCTION

In the past decade much attention was devoted to experimental studies of the optical properties of objects with reduced dimensionality [1–13]. These studies were enabled by two recent technological advances. Firstly, the technique for fabrication of nanometer size quantum dots was developed [14]. These dots represent islands of $InGaAs$ self-assembled in the course of epitaxial growth in a $GaAs$ matrix. Together with generic quantum dots formed by inhomogeneities in the width of $GaAs$ quantum wells [1–5,8,9], self-assembled quantum dots are able to confine photoexcited carriers (electrons and holes) in all directions making their energies truly discrete. Secondly, a near-field optical technique was developed, which allowes to detect exciton luminescence from a *single* quantum dot [1–13], and, thus, to avoid ambiguity in interpretation of the spectra, resulting from inhomogeneous broadening.

The luminescence spectrum from an isolated dot represents a sequence of very sharp lines. The widths of these lines are of the order of 0.05meV (or smaller, being limited by the spectrometer resolution [9]). It was also demonstrated [11] that these widths remain unchanged with increasing temperature up to $T = 50K$. This indicates that dephasing processes are ineffective, so that the widths are limited by the radiative lifetimes. The sharpness of exciton lines opens up the possibility to register very small changes in their positions and intensities caused by a change in the external parameters (like magnetic field).

Recently, further progress in self-assembly techniques [15–17] allowed to fabricate a very different type of nanostructures. Namely, modification of the growth process [16,17] led to a drastic change in the shape of the quantum dots from lens-like to ring-like. By now, the optical properties of this novel type of nanostructures were studied by means of infrared spectroscopy [15]. It was demonstrated [15] that the *electronic* energy levels in nanorings exhibit sensitivity to a magnetic field perpendicular to the plane of the ring. The characteristic scale of the magnetic field, causing a pronounced effect, corresponds approximately to one flux quantum, Φ_0, threading the interior of a ring.

Excitonic luminescence of nanorings has not been studied yet. However, this study and, in particular, the dependence of positions of emission lines on a perpendicular magnetic field would be of great conceptual interest. Indeed, unlike free carriers (electrons or holes) which are coupled to the vector potential of magnetic field via their charge, an exciton being a *neutral* entity, should not be sensitive to the flux through the ring. This statement is rigorous for a point-like neutral particle. Meanwhile, an exciton has a finite size (the Bohr radius a_B). Here we demonstrate [18] that this finite a_B gives rise to a periodic dependence of the exciton energy, as well as of the luminescence intensity, on the flux Φ threading the ring. The period in Φ is equal to Φ_0. In other words, we show that the excitonic luminescence from a nanoring exhibits the Aharonov-Bohm oscillations [19,20].

II. EXCITONIC MAGNETOLUMINESCENCE: PEAK POSITIONS AND SPECTRAL INTENSITIES

We adopt a simple model for an exciton on a ring. In particular, we assume that the thickness of the ring is much smaller than the Bohr radius. We also assume the attraction potential between electron and hole is short-ranged. Denote with φ_e and φ_h the azimuthal coordinates of the electron and hole, respectively. In the absence of interaction the wave functions of electrons and holes are given by

$$\Psi_N^{(e)}(\varphi_e) = \frac{1}{\sqrt{2\pi}} e^{iN\varphi_e}, \quad \Psi_{N'}^{(h)}(\varphi_h) = \frac{1}{\sqrt{2\pi}} e^{iN'\varphi_h}, \tag{1}$$

where N and N' are integers. The corresponding energies are

$$E_N^{(e)} = \frac{\hbar^2}{2m_e\rho^2}\left(N - \frac{\Phi}{\Phi_0}\right)^2, \quad E_{N'}^{(h)} = \frac{\hbar^2}{2m_h\rho^2}\left(N' + \frac{\Phi}{\Phi_0}\right)^2. \tag{2}$$

Here ρ is the radius of the ring, and m_e, m_h stand for the effective masses of electron and hole, respectively. In the presence of an interaction $V\Big[R(\varphi_e - \varphi_h)\Big]$, where $R(\varphi_e - \varphi_h) = 2\rho\sin(\frac{\varphi_e - \varphi_h}{2})$ is the distance between electron and hole, we search for the wave function of the exciton in the form

$$\Psi(\varphi_e, \varphi_h) = \sum_{N,N'} A_{N,N'}\Psi_N^{(e)}(\varphi_e)\Psi_{N'}^{(h)}(\varphi_h). \tag{3}$$

The coefficients $A_{N,N'}$ are to be found from the equation

$$\sum_{N,N'} A_{N,N'}\Big[E_N^{(e)}+E_{N'}^{(h)}-\Delta\Big]\Psi_N^{(e)}(\varphi_e)\Psi_{N'}^{(h)}(\varphi_h)+V\Big[R(\varphi_e-\varphi_h)\Big]\Psi(\varphi_e,\varphi_h)=0, \tag{4}$$

where Δ is the energy of the exciton. It follows from the definition Eq. (3) that the coefficients $A_{N,N'}$ satisfy the condition

$$\int_0^{2\pi} d\varphi \Psi(\varphi,\varphi)=\sum_N A_{N,-N}. \tag{5}$$

Making use of the assumption that $V(R)$ is short-ranged, we set $\varphi_e=\varphi_h$ in the last term of (4). This allows us to express coefficients $A_{N,-N}$ through the average interaction strength $V_0<0$ defined as $V_0=\frac{1}{2\pi}\int d\varphi V\Big[R(\varphi)\Big]$. Then the substitution of $A_{N,-N}$ into (5) yields a closed equation for the exciton energy Δ

$$1+V_0\sum_N \frac{1}{E_N^{(e)}+E_{-N}^{(h)}-\Delta}=0. \tag{6}$$

We emphasize that other types of interactions will modify the explicit form of V_0, but not the functional form of Eq. (6).

With energies $E_N^{(e)}$, $E_{-N}^{(h)}$ given by (2), the summation over N can be performed explicitly. Then Eq. (6) takes the concise form

$$\left(\frac{\Delta}{\varepsilon_0}\right)^{1/2}=-\left(\frac{\pi V_0}{\varepsilon_0}\right)\frac{\sin\Big(2\pi(\Delta/\varepsilon_0)^{1/2}\Big)}{\cos\Big(2\pi(\Delta/\varepsilon_0)^{1/2}\Big)-\cos\Big(2\pi(\Phi/\Phi_0)\Big)}, \tag{7}$$

where $\varepsilon_0=\hbar^2/2\mu\rho^2$ is the size quantization energy of an exciton; $\mu=m_em_h/(m_e+m_h)$ denotes the reduced mass of electron and hole. It is seen from Eq. (7) that the structure of the excitonic spectrum is determined by a dimensionless ratio $|V_0|/\varepsilon_0$. From the definition of V_0 it follows that it falls off as $1/\rho$ with increasing ρ. Thus, $|V_0|/\varepsilon_0$ is proportional to the radius of the ring. In the limit of large ρ, when $|V_0|\gg\varepsilon_0$, Eq. (7) can be solved analytically. For the ground state energy we get

$$\Delta_0=-E_B\left[1+4\cos\Big(\frac{2\pi\Phi}{\Phi_0}\Big)\exp\Big(-\frac{2\pi\rho}{a_B}\Big)\right], \tag{8}$$

where $E_B=\pi^2V_0^2/\varepsilon_0$ and $a_B=\pi|V_0|\Big(2\mu/\hbar^2\varepsilon_0\Big)^{-1/2}$ are, correspondingly, the binding energy and the Bohr radius of the one-dimensional exciton in the limit $\rho\to\infty$.

The second term in the brackets of Eq. (8) describes the sought Aharonov-Bohm oscillations in the position of the exciton luminescence line. These oscillations also show up in the intensity, $F_0(\Phi)$, of the line. The easiest way to demonstrate it, is to note that F_0 and Δ_0 are connected through the relation $F_0(\Phi) \propto \partial\Delta_0/\partial V_0$, which is straightforward to check.

III. NUMERICAL RESULTS

In Fig. 1 we show the numerical solution of Eq. (7) together with the asymptotic solution (8), valid in the limit of large ρ, for the variation of the exciton energy with Φ. As expected, the Aharonov-Bohm oscillations are close to sinusoidal for large values of $2\pi\rho/a_B$, whereas for $2\pi\rho/a_B = 1$, unharmonicity is already quite pronounced. The increase of the exciton energy as the flux is switched on has a simple physical interpretation. The flux tends to move electron and hole in opposite directions, thus, suppressing their binding. Fig. 1 illustrates how the amplitudes of the Aharonov-Bohm oscillations decrease with increasing the ring perimeter as described by Eq. (8). The oscillations in the line intensity with Φ are plotted in Fig. 2. As expected, the shift is most pronounced for $\Phi = \Phi_0/2$, and the relative magnitude is nearly 80% for the smallest value $2\pi\rho/a_B = 1$. For larger values of $2\pi\rho/a_B$, the oscillations in $F_0(\Phi)$ become increasingly sinusoidal. Excited states of the exciton according to Eq. (7) do also exhibit oscillations as shown in Fig. 3. Note that in this case the oscillations do not decay exponentially with $2\pi\rho/a_B$ as shown in Ref. [18].

IV. DISCUSSION

Equation (8) illuminates the physical origin of the Aharonov-Bohm effect for an exciton. Indeed, the factor $\exp(-2\pi\rho/a_B)$ which determines the magnitude of the effect represents the amplitude for bound electron and hole to tunnel in the opposite directions and meet each other "on the opposite side of the ring" (opposite with respect to the point where they were created by a photon). The fact that electron and hole tunnel in the *opposite* directions in combination with the fact that coupling constants to magnetic flux for electron and hole have the *opposite* signs, results in a certain *compensation* effect. That is, regardless of the point on the ring where the meeting takes place, the *net* phase acquired by the wave function of the exciton at the moment of the meeting is always the same, and is equal to $2\pi\Phi/\Phi_0$. This mechanism of "catching the tails" is illustrated in Fig. 4.

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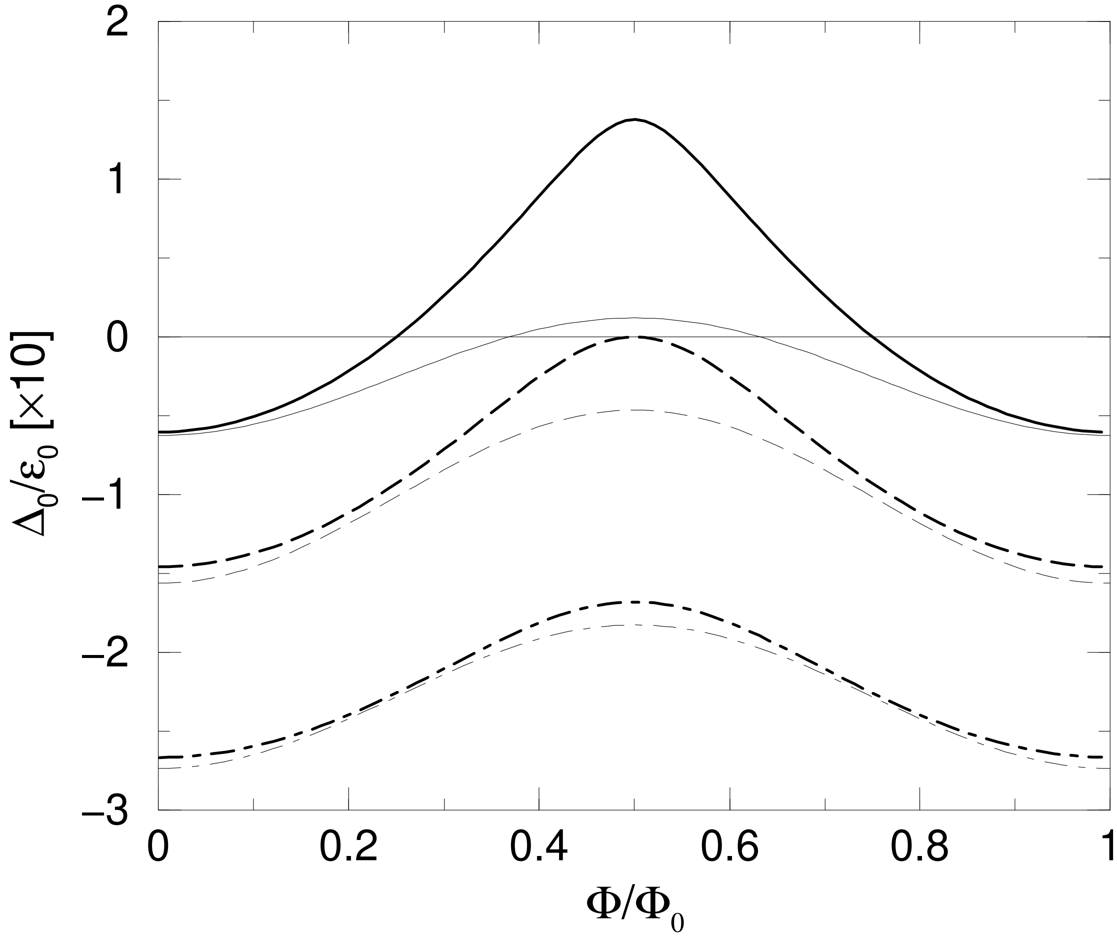

FIG. 1. The Aharonov-Bohm oscillations of the exciton energy is shown for three values of the dimensionless ring perimeter $2\pi\rho/a_B = 1$ (solid lines), 2 (dashed lines) and 3 (dot-dashed lines). The thick and thin lines are drawn from Eq. (7) and Eq. (8), respectively.

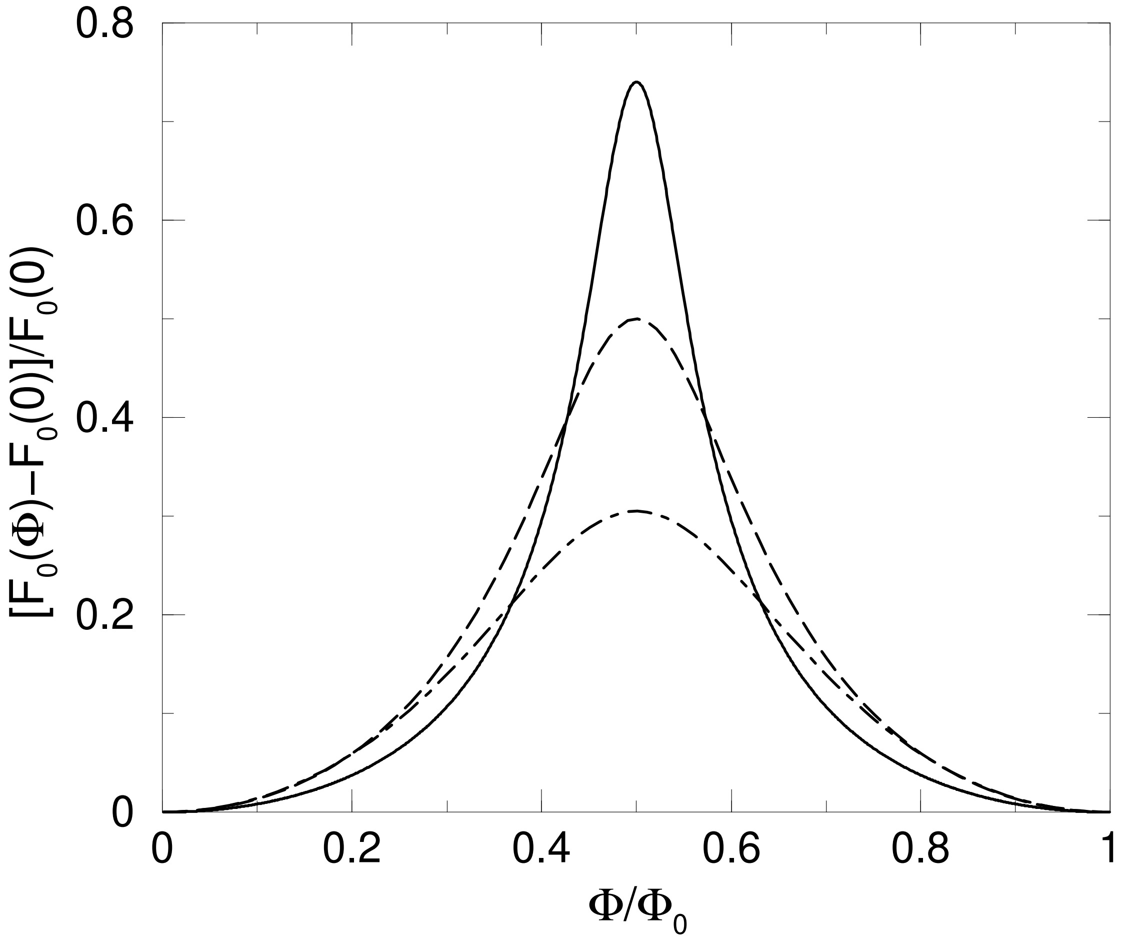

FIG. 2. The Aharonov-Bohm oscillations of the exciton line intensity for the three values of the dimensionless ring perimeter $2\pi\rho/a_B = 1$ (solid line), 2 (dashed line) and 3 (dot-dashed line).

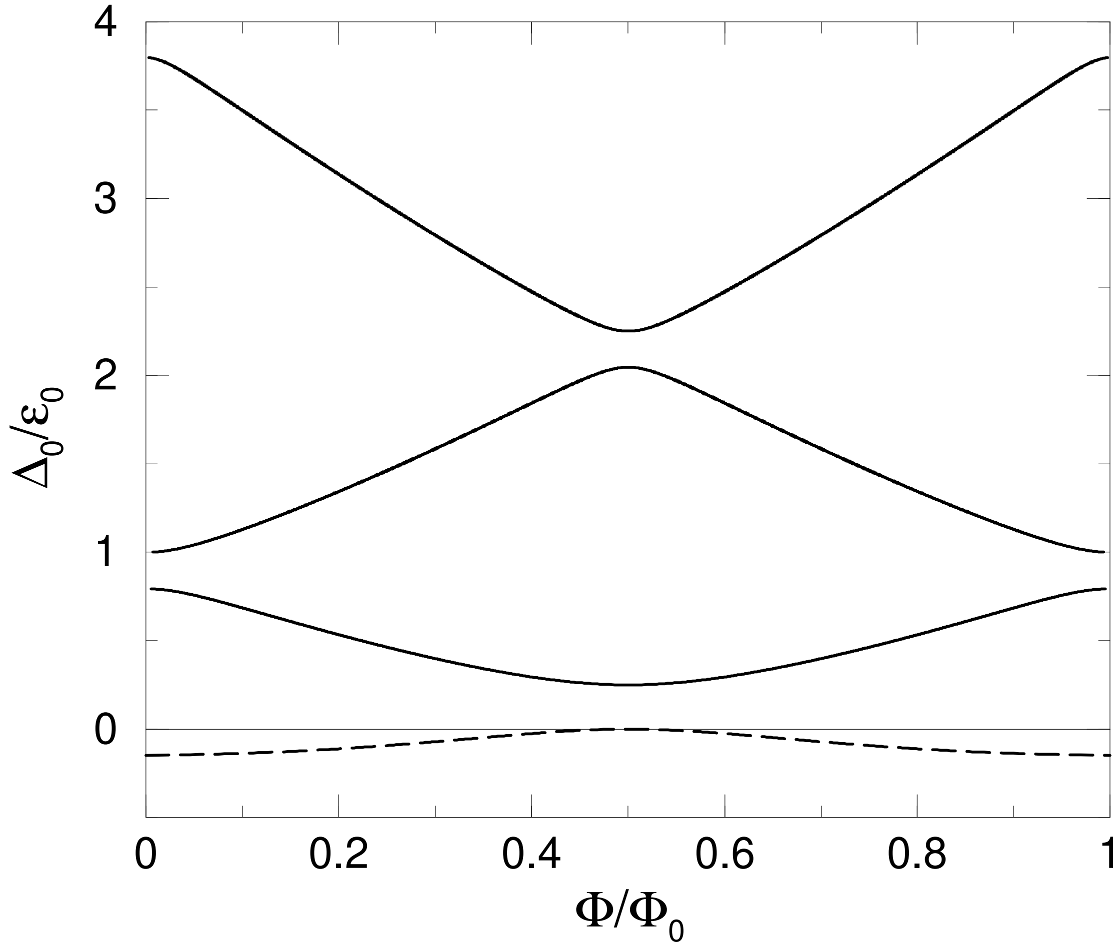

FIG. 3. The Aharonov-Bohm oscillations of the first 3 excited states of an exciton for dimensionless ring perimeter $2\pi\rho/a_B = 2$. Also shown (dashed line) is the corresponding ground state as in Fig. 1.

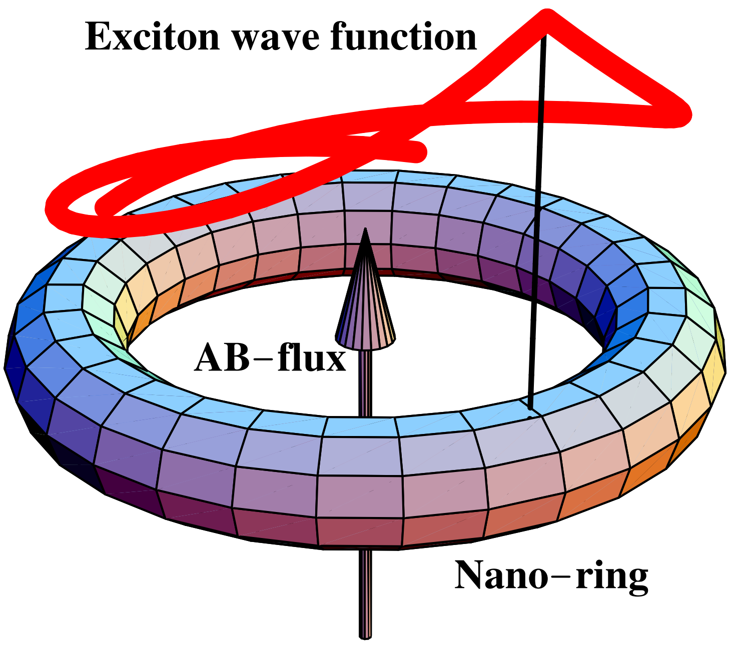

FIG. 4. Schematic illustration of the origin of the Aharonov-Bohm effect for an exciton. The thin line marks the point where the exciton is created, the thick gray line atop the ring presents the decay of the envelope of the excitonic wave function.