Scaling the localisation lengths for two interacting particles in one-dimensional random potentials

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Abstract

Using a numerical decimation method, we compute the localisation length λ_2 for two onsite interacting particles (TIP) in a one-dimensional random potential. We show that an interaction $U > 0$ does lead to $\lambda_2(U) > \lambda_2(0)$ for not too large U and test the validity of various proposed fit functions for $\lambda_2(U)$. Finite-size scaling allows us to obtain infinite sample size estimates $\xi_2(U)$ and we find that $\xi_2(U) \sim \xi_2(0)^{\alpha(U)}$ with $\alpha(U)$ varying between $\alpha(0) \approx 1$ and $\alpha(1) \approx 1.5$. We observe that all $\xi_2(U)$ data can be made to coalesce onto a single scaling curve. We also present results for the problem of TIP in two different random potentials corresponding to interacting electron-hole pairs.

In two recent articles [1,2], we studied as a simple and tractable approach to the problem of interacting electrons in disordered materials the case of only two interacting particles (TIP) in 1D random potentials. Previous considerations [3] had led to the idea that attractive as well as repulsive interactions between TIP give rise to the formation of particle pairs whose localisation length λ_2 is much larger than the single-particle (SP) localisation length $\lambda_1 \approx 105/W^2$,

$$\lambda_2 \sim U^2 \lambda_1^2 \qquad (1)$$

at two-particle energy $E = 0$, with U the Hubbard interaction strength. Although many papers have numerically investigated the TIP effect [3–9], an unambiguous reproduction of Eq. (1) is still lacking. However, it appears well established that some TIP delocalisation such as $\lambda_2 > \lambda_1$ does indeed exist due to the interaction. Recently, a duality in the spectral statistics for U and $\sqrt{24}/U$ has been proposed [11] for small and very large $|U|$.

Preprint submitted to Elsevier Preprint $Revision: 1.3$*; compiled 20 August 1998*

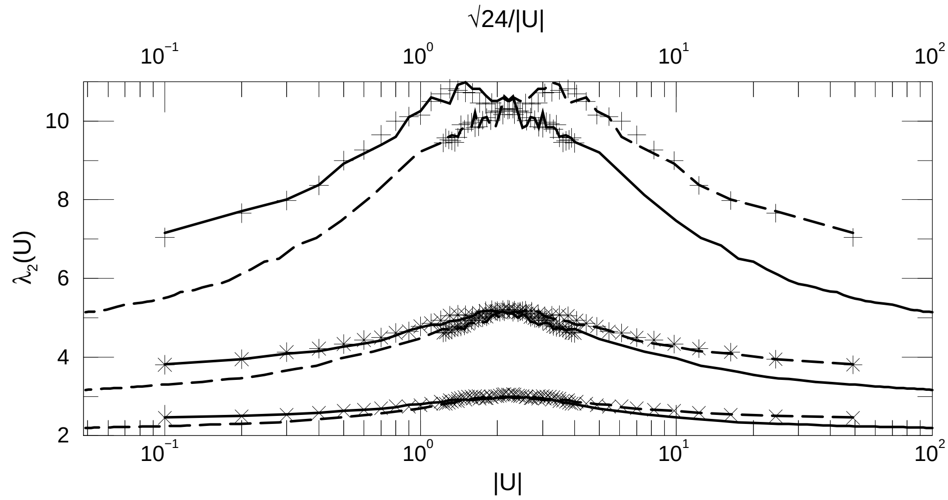

Fig. 1. $\lambda_2(U)$ for TIP as a function of $|U|$ (solid lines) and $\sqrt{24}/|U|$ (dashed lines) at $E = 0$ for disorders $W = 3$ (+), 4 (∗), and 5 (×) and $M = 201$. The data are averaged over 100 samples. The lines (symbols) indicate data for $U > 0$ ($U < 0$).

In Refs. [1,2], we have employed a numerical decimation method [10], i.e., we replaced the full Hamiltonian by an effective Hamiltonian for the doubly-occupied sites only. In [1], we considered the case of TIP with n, m corresponding to the positions of each particle on a chain of length M and random potentials $\epsilon^1_n = \epsilon^2_n \in [-W/2, W/2]$. In [2], we studied the case where ϵ^1_n and ϵ^2_n are chosen independently from the interval $[-W/2, W/2]$, which may be viewed as corresponding to an electron and a hole on the same chain (IEH). Via a simple inversion, we then obtained the Green function matrix elements $\langle 1, 1|G_2|M, M\rangle$ between doubly-occupied sites $(1, 1)$ and (M, M) and focused on the localisation length λ_2 obtained from the decay of the transmission probability from one end of the system to the other, i.e.,

$$\frac{1}{\lambda_2} = -\frac{1}{|M-1|} \ln |\langle 1, 1|G_2|M, M\rangle|. \tag{2}$$

In Fig. 1 we present data for $\lambda_2(U)$ obtained for three different disorders for system sizes $M = 201$ at $E = 0$. In agreement with the previous arguments and calculations [6,7,11], we find that the enhancement is symmetric in U and decreases for large $|U|$. In [11] is has been argued that at least for $\lambda_1 \approx M$, there exists a critical $U_c = 24^{1/4} \approx 2.21$, which should be independent of W, at which the enhancement is maximal. We find that in the present case with $\lambda_1 < M$ the maximum of $\lambda_2(U)$ depends somewhat on the specific value of disorder used. The data in Fig. 1 may be compatible with the duality of Ref. [11], but only for the large disorder $W = 5$. For the smaller disorders and for the range of interactions shown, we do not observe the duality. We emphasize that the duality observed in [11] is for spectral statistics and need not apply to quantities such as the localisation length λ_2.

In order to reduce the possible influence of the finiteness of the chain length, we constructed finite-size-scaling (FSS) curves for 11 interaction values $U = 0, 0.1, \ldots, 1$ from the λ_2 data for 26 disorder values W between 0.5 and 9, for 24 system sizes M between 51 and 251, averaging over 1000 samples in each case. In Fig. 2 we show the infinite-size localisation lengths (scaling parameters) ξ_2 obtained from these 11 FSS curves. A simple power-law fit $\xi_2 \propto W^{-2\alpha}$ in the

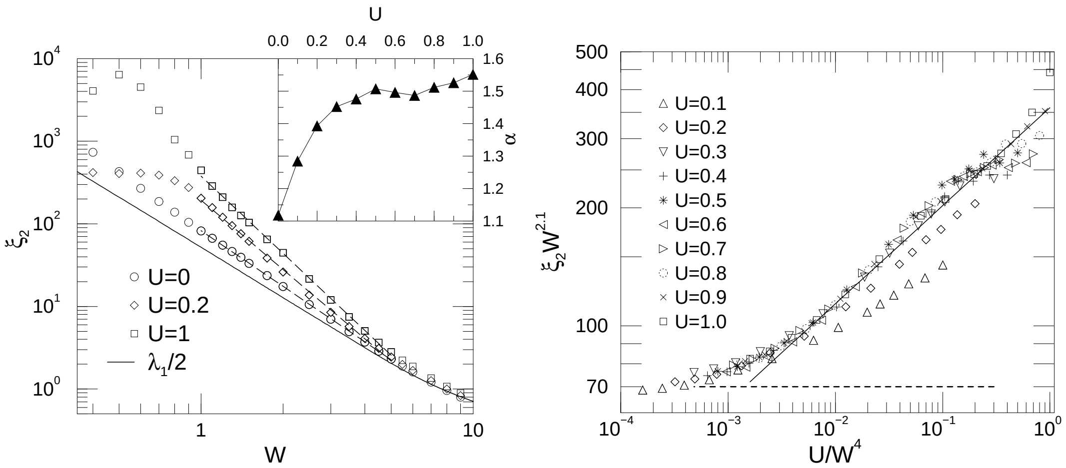

Fig. 2. Left panel: TIP localisation lengths ξ_2 after FSS. The dashed lines represent power-law fits. Inset: Exponent α obtained by the power-law fits. Right panel: Scaling plot according to [5] with TIP localisation lengths $\xi_2(U)$ for $W \in [1, 5]$. The solid line indicates a slope of 1/4, the dashed line the value of $\xi_2 W^{2.1}$ in the limit $U = 0$.

disorder range $W \in [1, 5]$ yields an exponent α which increases with increasing U as shown in the inset of Fig. 2, e.g., $\alpha = 1.55$ for $U = 1$ and $\alpha = 1.1$ for $U = 0$. Because of the latter, in the following we will compare $\xi_2(U \neq 0)$ with $\xi_2(0)$ when trying to identify an enhancement of the localisation lengths due to interaction.

Song and Kim [5] suggested that the TIP localisation data may be described by a scaling form $\xi_2 = W^{-\alpha_0} g(|U|/W^{\Delta})$ with g a scaling function. They obtain $\Delta = 4$ by fitting the data. Our data can be best described when α_0 is related to the disorder dependence of ξ_2 as $(\alpha - \alpha_0)/\Delta \approx 1/4$. As shown in Fig. 2, the scaling is only good for $W \in [1, 5]$ and $U \geq 0.3$. We note that assuming an interaction dependent exponent $\alpha(U)$, we still do not obtain a good fit to the scaling function with the data for all U.

In Fig. 3, we show that a much better scaling can be obtained when plotting

$$\xi_2(U) - \xi_2(0) = \tilde{g}\left[f(U)\xi_2(0)\right] \tag{3}$$

with $f(U)$ determined by FSS. Now the scaling is valid for *all* U and $W \in [0.6, 9]$. As indicated by the straight lines, we observe a crossover from a slope

2 to a slope 3/2. There are some deviations from scaling, but these occur for large and very small values of $\xi_2(U)$ and are most likely due to numerical inaccuracy [1]. In the inset of Fig. 3, we show the behavior of $f(U)$. For $U \geq 0.3$ a linear behavior $f(U) \propto U$ appears to be valid which translates into a U^2 ($U^{3/2}$) dependence of $\xi_2(U) - \xi_2(0)$ in the regions of Fig. 3 with slope 2 (3/2). For $U \leq 0.5$, we have $f(U) \propto \sqrt{U}$ which yields $\xi_2(U) - \xi_2(0) \propto U$ ($U^{3/4}$).

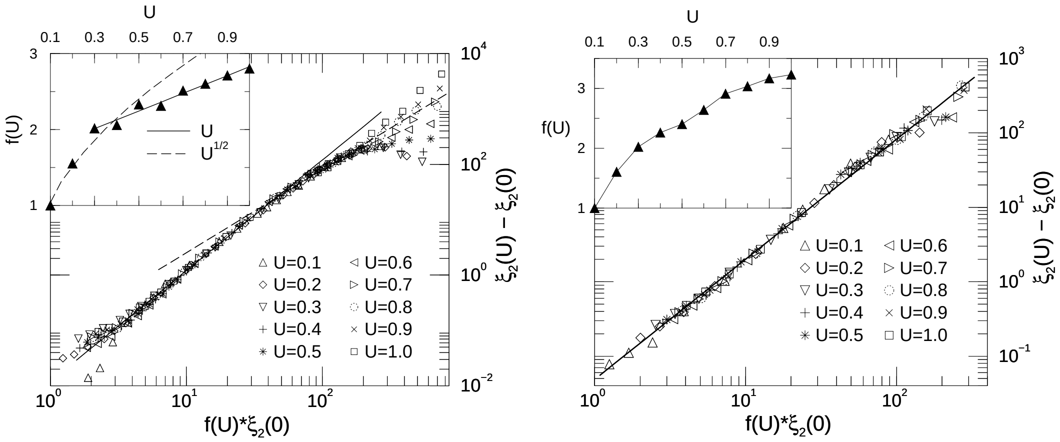

Fig. 3. Left panel: Scaling plot of Eq. (3) for TIP with $W \in [0.6, 9]$. The solid (broken) line indicates the slope 2 (1.5). Inset: The values of $f(U)$ needed to make the data collapse onto the $U = 0.1$ curve. Right panel: IEH localisation length scaled as in the left panel. The solid line indicates slope 1.61 and $W \in [1, 7]$.

Thus in summary it appears that our data cannot be described by a simple power-law behavior with a single exponent as in Eq. (1) neither as function of W, nor as function of $\xi_2(0)$ [1], nor after scaling the data onto a single scaling curve.

As for TIP we computed [2] the IEH localisation lengths by the DM along the diagonal using 100 realizations for each (U, M, W). We find that the data for IEH are very similar to the case of TIP. We again perform FSS and observe that the infinite-size estimates $\xi_2(U)$ are well characterized by an exponent $\alpha(U)$. We can again scale the $\xi(U)$ data for IEH onto a single curve as shown in Fig. 3. However, here the crossover from slope 2 to 3/2 is much less prominent and the data can be described reasonably well by a single slope of 1.61. Also, the crossover behavior in $f(U)$ is suppressed. We remark that these differences may be due to the smaller number of samples used for IEH.

In conclusion, we observe an enhancement of the two-particle localisation length due to onsite interaction both for TIP and IEH. This enhancement persists, unlike for TMM [6,8,9], in the limit of large system size and after constructing infinite-sample-size estimates from the FSS curves. We remark that the IEH case is of relevance for a proposed experimental test of the TIP effect [12].

Acknowledgements

R.A.R. gratefully acknowledges support by the Deutsche Forschungsgemeinschaft (SFB 393).

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