

In a recent communication to the cond-mat archives, Suslov [1] severely criticizes a multitude of numerical results obtained by various groups for the critical exponent ν of the localization length at the disorder-induced metal-insulator transition (MIT) in the three-dimensional (3D) Anderson model (AM) of localization as “entirely absurd” and “evident desinformation”. These claims are based on the observation that there still is a large disagreement between analytical, numerical and experimental results for the critical exponent [2]. The author proposes, based on a “simple procedure to deal with corrections to scaling”, that the numerical data support $\nu \approx 1$, whereas recent numerical papers find $\nu = 1.58 \pm 0.06$ [3–5].

As we show below, these claims are entirely wrong. The proposed scheme does neither yield any improved accuracy when compared to the existing finite-size scaling (FSS) methods, nor does it give $\nu \approx 1$ when applied to high-precision data.

FSS at the Anderson MIT has a noteworthy history, reaching a first peak with the seminal papers of Pichard/Sarma [6,7] and MacKinnon/Kramer [8,9]. Especially in Ref. [9], the groundwork for a reliable, numerical FSS procedure was laid and scaling curves could be constructed that proved the existence of an MIT in 3D. In these and later studies based on the same analysis technique [2], the critical exponent ν , as estimated from the divergence of the infinite-size localization and correlation lengths $\xi(W)$ at the transition W_c , i.e., $\xi \propto |1 - W/W_c|^{-\nu}$, is systematically *underestimated*, since the divergent nature at the transition can only be poorly captured by FSS of data obtained for small system sizes and large errors ε in these finite-size data. However, as more powerful computers became available in the last decade, one observed a trend towards larger values of $\nu \approx 1.35$ [10–13] for $\varepsilon \leq 1\%$.

In 1994, high-precision data ($\varepsilon \leq 0.2\%$) showed a hitherto neglected systematic shift of the transition point W_c with increasing system size. Taking this into account phenomenologically, $\nu = 1.54 \pm 0.08$ was found [14]. A subsequent approach by Slevin/Ohtsuki [3–5] incorporated these shifts as irrelevant scaling variables and further allowed for corrections to scaling due to nonlinearities. With higher-precision data ($\varepsilon \approx 0.1\%$), they found $\nu = 1.57 \pm 0.04$. Further results for, e.g., the AM with anisotropic hopping [15–17], the off-diagonal AM [18,19], the AM in a magnetic field [20,21], confirmed this value of ν within the error bars (see Fig. 1). Also, ν is identical for the MIT as a function of disorder or energy [18,19]. We emphasize that a properly performed Slevin/Ohtsuki scaling (SOS) procedure needs to assume various fit functions and that the final estimates are to be suitably extracted from many such functional forms [16–18]; bootstrap [3–5] or Monte Carlo methods [16–18] then need to be employed for a precise estimate of error bars.

We have tested the method proposed by Suslov [1] first with the transfer-matrix (TM) data of Refs. [15,18,19]

with $\varepsilon \leq 0.1\%$; we find $\nu_{\text{Suslov}} = 1.75 \pm 0.17$ for the anisotropic and 1.55 ± 0.04 for the random-hopping AM. The SOS gives $\nu = 1.61 \pm 0.07$ [15] and $\nu = 1.54 \pm 0.03$ [18,19], respectively. Using for a second test energy-level-statistics (ELS) data [16] with $\varepsilon \approx 1\%$, we find $\nu_{\text{Suslov}} = 1.51 \pm 0.25$, whereas SOS gives 1.45 ± 0.2 [16]. Last, for artificially generated data with precisely known $W_c = 16.5$ and varying $\nu \in [0.5, 2.0]$ the results of the Suslov method are comparable to the results of the MacKinnon/Kramer FSS and slightly less reliable than the SOS.

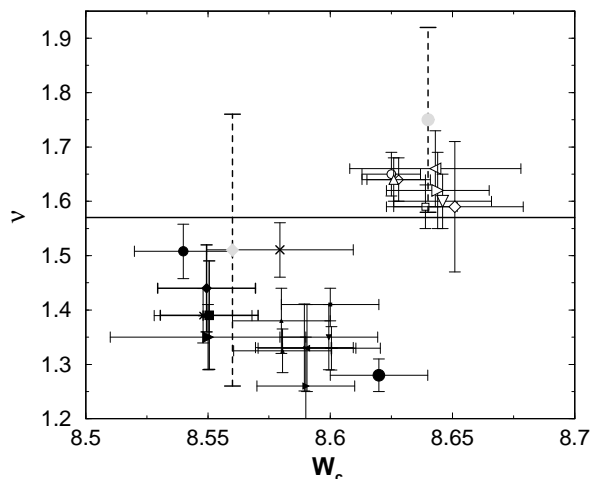


FIG. 1. Results for W_c and ν , for the *anisotropic* AM [15,16] using SOS of TM data (open symbols) and ELS data (filled symbols) for various fit functions. The error bars show the 95% confidence intervals. The accuracy of TM localization lengths data is an order of magnitude higher than that of the ELS data and the system sizes of TM data are larger than for ELS data, giving systematically larger ν values for the former. The goodness of a fit is reflected in the size of the symbol. The 2 thick error bars mark high quality ELS fits for large system sizes. The gray \circ and \square and the corresponding error bars (dashed lines) represent ν_{Suslov} of TM data and ELS data for the anisotropic AM, respectively. The solid line marks the result of [3].

We conclude that the method proposed by Suslov also yields $\nu \approx 1.58$ and not $\nu \approx 1$ for the MIT of the AM.

In principle, the Suslov method does not need to assume any functional form of the FSS curves just as the MacKinnon/Kramer method. As a numerical tool, the Suslov method is not unreasonable, but certainly not better than the established methods: it does not take into account the systematic shift due to irrelevant scaling variables, it relies on an a-priori knowledge of W_c and inherently produces rather large error bars for the critical exponent. We note that Suslov in his numerical test [1] used data for 3 small system sizes 6^3 , 12^3 and 28^3 , while currently sizes $\sim 50^3$ (for ELS) and $18^2 \times 10^8$ (for TM) are standard. It is evident to people with experience in FSS that Suslov’s erroneously small ν is due to his use of too few and too small system sizes.

In conclusion, high-precision numerics with error $\varepsilon \approx 0.1\%$ together with all the above mentioned FSS methods produce a critical exponent $\nu \approx 1.58 > 1$ for 3D. The numerical values of ν for dimensions $2 < d < 3$ [22] and 4 [22,20] remain valid, they are certainly not “entirely absurd” although there is only limited agreement with the field theoretic approach [2]. Similarly precise data are much harder to obtain for our experimental colleagues, but recent advances in this direction show a clear trend towards increasing ν [23,24].

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