Übungen zur Thermodynamik

Computational Science Bachelor

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Teil III: Motoren, Kreisprozesse

(**HA** Punkte in Klammern, ohne Punkte: im Seminar)

1. Der Einzylindermotor eines Motorrads hat ein Kompressionsverhältnis von 6:1. Der Kolben verursacht eine maximale Volumenänderung im Zylinder von 200 cm^3. Wie groß ist die Leistung durch die Expansion des Gases, wenn der Motor eine Drehzahl von 3000 Upm (50Hz) hat ? Es wird adiabatische Expansion eines idealen Gases mit P_1=20atm vorausgesetzt. (**HA** 2)

2. Man berechne den Wirkungsgrad eines Benzinmotors (Otto) und einer Carnot-Maschine. Wie groß ist der Wirkungsgrad bei einem Kompressionsverhältnis von 8:1 ? (**HA** 3)

3. Berechnen Sie die Entropie für ein ideales Gas, das aufgeheizt wird unter a) isobaren und b) isochoren Bedingungen. (**HA** 2)

4. Ein Liter ideales Gas mit einer Temperatur von 300 K und einem Druck von 1bar wird adiabatisch kompremiert auf 3bar. Wie groß ist dann das Volumen und die Temperatur ? Welche Arbeit wurde verrichtet ? (**HA** 2)

5. Betrachten Sie folgenden Keisprozeß eines idealen einatomigen Gases (V_0=100l, P_0=1atm, R=0.082 l atm (mol K)$^{-1}$: (a-b) Isobare Expansion bei P=P_0,(b-c) Isotherme Expansion bei T=600 K, (c-d) Isochore Abkühlung bei V=2 V_0, (d-a) Isotherme Kompression bei T=400K.

   a) Berechnen Sie die Arbeiten W_{ab}, W_{bc}, W_{cd}. (**HA** 2)

   b) Bestimmen Sie $Q_{ab}, Q_{bc}, Q_{cd}, Q_{da}$ als positive Größen und geben Sie für jedes Q an, ob es sich dabei um vom System aufgenommene oder abgegebene Wärme handelt. (Werte für $W_{da} = -200/3\ln 3$ l atm) (**HA** 3)

   c) Leiten Sie die Formel für den Wirkungsgrad als Funktion von $Q_{ab}, Q_{bc}, Q_{cd}, Q_{da}$ her und berechnen Sie den numerischen Wert. (**HA** 2)

6. The fundamental thermodynamic relation for a paramagnetic material takes the form $dU = TdS - MdH$ wher H is the external magnetic field and U, S, and $M$are the energy, entropy, and magnetic moment per unit volume. Assume that the magnetization can be expressed in the form $M[T,H] = H\chi[T]$.

   a) Show that the constant-H heat capacity can then be expressed as $c_H[T,H] = c_H[T,0] + H^2 f[T]$ and determine the relationship betweem $f[T]$ and $\chi[T]$.(**HA** 2)

   b) $c_H[T,0]$ describes nonmagnetic contributions to the heat capacity which tend to be small at very low temperature. Evaluate the adiabatic demagnetization coefficient $\left(\frac{\partial T}{\partial H}\right)_S$ assuming that $c_H[T,0]$ is negligible and that the magnetization for $T > T_c$ is described by the Curie-Weiss law

$$\chi[T] \approx \frac{\chi_0}{T - T_c}$$

where χ_0 and T_c are constants.(**HA** 2)

c) Evaluate the temperature change obtained by demagnetization $H_i \to H_f$. Is your result plausible? If not, why not? (**HA** 2)

7. A magnetic refrigerator employs a paramagnetic salt as the working material in a heat engine. Here we develop the basic principle without dwelling on technical details of implementation. An ideal heat engine performs the following cycle:

   1) The working material is in thermal contact at temperature T_1 with the sample to be refrigerated. The magnetizing field is reduced from H_a to H_b as the paramagnetic salt draws heat from the sample at constant temperature.

   2) The sample is isolated from the working material and the magnetizing field is increased adiabatically from H_b to H_c.

   3) The working material is placed in thermal contact with the reservoir at constant temperature T_2; the reservoir is usually liquid helium boiling under reduced pressure so that T_2 ~1K. The magnetizing field is increased fromH_c to H_d as the entropy of the working material is reduced and heat is expelled into the reservoir and removed by a "high"-temperature refrigerator.

   4) The working material is isolated again and the magnetizing field is decreased from H_d back to H_a, returning the working material to its original state in preparation for the next cycle.

   The net result of this cycle is to transfer heat Q_1 from the cold sample to the warmer reservoir by means of the magnetic work W performed upon the paramagnetic salt during a reversible cycle. In this problem we employ simple models to estimate the heat and work on each leg of the cycle. In previous problems we proved that

   $$\frac{T_f - T_c}{T_i - T_c} = \frac{H_f}{H_i}$$

   for adiabatic transformations for a paramagnet satisfying $M = H\chi[T]$ and $C_H >> C_M$. Assume for the present purposes that $\chi[T] = \chi_0/(T - T_c)$ where χ_0 and T_c are constant.

   a) Sketch the cycle in the TS plane and relate the fields H_a and H_c to the extreme values H_b and H_d. (**HA** 1)

   b) Determine the heat and work for each step of the cycle. (**HA** 3)

   c) The performance coefficient for a refrigerator is defined at the ratio between the heat absorbed from the sample and the work performed upon the working material per cycle. Compute the performance coefficient for this refrigerator. (**HA** 2)

   d) Compute the work per cycle performed on 1 cm^3 of gadolinium sulfate and the heat removed from the sample assuming that the refrigerator operates between temperatures $T_1 = 0.1$K and $T_2 = 1.0$K using fields $H_b = 10$ Gauss and $H_d = 1000$ Gauss. Assume that Curie's law is valid and neglect T_c. The molar susceptibility of gadolinium sulfate measured at 285.5K is 5.4210^{-2}cm^3/mole. The density is 4.14g/cm^3 and the molecular weight of $Gd_2(SO_4)_3$ is 603 g/mole. (**HA** 2)

Max 30 Punkte