Computational Science 2

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Seminar
Exercises

Exercise 1 (11.04.2013):

Continuum percolation

from ***An Introduction to Computer Simulation Methods***,
Chapter 12, Problem 12.4

a) Suppose that disks of unit diameter ($r = 0.5$) are placed at random on the sites of a square lattice with unit lattice spacing. Define ϕ as the area fraction covered by the disks. Convince yourself that $\phi_c = \pi p_c/4$.

b) Modify `PercolationApp` to simulate continuum percolation. Instead of placing the disks on regular lattice sites, place their centers at random in a square box of area L^2. The relevant parameter is the density ρ, the number of disks per unit area, instead of the probability p. We can no longer use the `LatticeFrame` class. Instead arrays are needed to store the x and y locations of the disks. When the mouse is clicked on a disk, your program will need to determine which disk is at the location of the mouse, and then check all the other disks to see if they overlap or touch the disk you have chosen. This check is recursively continued for all overlapping disks. It also is useful to have an array that keeps track of the `clusterNumber` for each disk. Only disks that have not been assigned a cluster number need to be checked for overlaps.

c) Estimate the value of the density ρ_c at which a spanning cluster first appears. Given this value of ρ_c, use a Monte Carlo method to estimate the corresponding area fraction ϕ_c. Choose points at random in the box and compute the fraction of points that lie within any disk. Explain why ϕ_c is larger for continuum percolation than it is for site percolation. Compare your direct Monte Carlo estimate of ϕ_c with the indirect value of ϕ_c obtained from $\phi = 1 - \exp(-\rho\pi r^2)$ using the value of ρ_c. Explain any discrepancy.

d) A variation of the cookie problem is to place disks with unit diameter at random in a box with the constraint that the disks do not overlap. Continue to add disks until the fraction of successful attempts becomes less than 1%, that is, when one hundred successive attempts at adding a disk are not successful. Does a spanning cluster exist? If not, increase the diameters of all the disks at a constant rate (in analogy to the baking of the cookies) until a spanning cluster is attained. How does ϕ_c for this model compare with the value of ϕ_c found in part (c)?