

# Computational Science 1

[http://www.tu-chemnitz.de/physik/THUS/de/lehre/CSM\\_WS1718.php](http://www.tu-chemnitz.de/physik/THUS/de/lehre/CSM_WS1718.php)

## Seminar Exercises

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Exercise 5 (23.11.2017):

## Statistical test of randomness

from *An Introduction to Computer Simulation Methods*,  
Chapter 7, Problem 7.35

A rather simple but very fast random number generator can be constructed with the linear congruential method  $x_n = (ax_{n-1} + c) \bmod m$ , where  $a$ ,  $c$ ,  $m$ , and  $x_n$  are integers. The random numbers take values between 0 to  $m$  and should be divided by  $m$  to fit into the interval  $[0, 1)$ .

- Uniformity.** A random number sequence should contain numbers distributed in the unit interval with equal probability. The simplest test of uniformity is to divide this interval into  $M$  equal size bins. For example, consider the first  $N = 10^4$  numbers generated with  $a = 106$ ,  $c = 1283$ , and  $m = 6075$ . Place each number into one of  $M = 100$  bins. Is the number  $n_i$  of entries in each bin approximately equal? What happens if you increase  $N$ ? Plot the distribution of the  $n_i$ 's for different  $N$  and  $M$ .
- Filling sites.** Although a random number sequence might be distributed in the unit interval with equal probability, the consecutive numbers might be correlated in some way. One test of this correlation is to fill a square lattice of  $L^2$  sites at random. Consider an array  $n(x, y)$  that is initially empty, where  $1 \leq x_i, y_i \leq L$ . A site is selected randomly by choosing its two coordinates  $x_i$  and  $y_i$  from two consecutive numbers in the sequence. If the site is empty, it is filled and  $n(x_i, y_i) = 1$ ; otherwise it is not changed. This procedure is repeated  $t$  times, where  $t$  is the number of Monte Carlo steps per site. That is, the time is increased by  $1/L^2$  each time a pair of random numbers is generated. Because this process is analogous to the decay of radioactive nuclei, we expect that the fraction of empty lattice sites should decay as  $e^{-t}$ . Determine the fraction of unfilled sites using the random number generator that you have been using for  $L = 10, 15$ , and  $20$ . Are your results consistent with the expected fraction? Repeat the same test with  $a = 65539$ ,  $c = 0$ , and  $m = 2^{31}$ .
- Hidden correlations.** Another way of checking for correlations is to plot  $x_{i+k}$  versus  $x_i$ . If there are any obvious patterns in the plot, then there is something wrong with the generator. Use  $a = 16807$ ,  $c = 0$ , and  $m = 2^{31} - 1$ . Can you detect any structure in the plotted points for  $k = 1$  to  $k = 5$ ?
- Short-term correlations.** Another measure of short term correlations is the autocorrelation function

$$C(k) = \frac{\langle x_{i+k}x_i \rangle - \langle x_i \rangle^2}{\langle x_i^2 \rangle - \langle x_i \rangle^2}, \quad (1)$$

where  $x_i$  is the  $i$ th term in the sequence. If  $x_{i+k}$  and  $x_i$  are not correlated, then  $\langle x_{i+k}x_i \rangle = \langle x_{i+k} \rangle \langle x_i \rangle$  and  $C(k) = 0$ . Is  $C(k)$  identically zero for any finite sequence? Compute  $C(k)$  for  $a = 106$ ,  $c = 1283$ , and  $m = 6075$ .