

Computational Science 1

<http://www.tu-chemnitz.de/physik/THUS/de/lehre/CSM-WS1516.php>

Seminar Exercises

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Exercise 6 (10.12.2015):

Statistical test of randomness

from *An Introduction to Computer Simulation Methods*,
Chapter 7, Problem 7.35

A rather simple but very fast random number generator can be constructed with the linear congruential method $x_n = (ax_{n-1} + c) \bmod m$, where a , c , m , and x_n are integers. The random numbers take values between 0 to m and should be divided by m to fit into the interval $[0, 1)$.

- a) **Uniformity.** A random number sequence should contain numbers distributed in the unit interval with equal probability. The simplest test of uniformity is to divide this interval into M equal size bins. For example, consider the first $N = 10^4$ numbers generated with $a = 106$, $c = 1283$, and $m = 6075$. Place each number into one of $M = 100$ bins. Is the number n_i of entries in each bin approximately equal? What happens if you increase N ? Plot the distribution of the n_i 's for different N and M .
- b) **Filling sites.** Although a random number sequence might be distributed in the unit interval with equal probability, the consecutive numbers might be correlated in some way. One test of this correlation is to fill a square lattice of L^2 sites at random. Consider an array $n(x, y)$ that is initially empty, where $1 \leq x_i, y_i \leq L$. A site is selected randomly by choosing its two coordinates x_i and y_i from two consecutive numbers in the sequence. If the site is empty, it is filled and $n(x_i, y_i) = 1$; otherwise it is not changed. This procedure is repeated t times, where t is the number of Monte Carlo steps per site. That is, the time is increased by $1/L^2$ each time a pair of random numbers is generated. Because this process is analogous to the decay of radioactive nuclei, we expect that the fraction of empty lattice sites should decay as e^{-t} . Determine the fraction of unfilled sites using the random number generator that you have been using for $L = 10, 15$, and 20 . Are your results consistent with the expected fraction? Repeat the same test with $a = 65539$, $c = 0$, and $m = 2^{31}$.
- c) **Hidden correlations.** Another way of checking for correlations is to plot x_{i+k} versus x_i . If there are any obvious patterns in the plot, then there is something wrong with the generator. Use $a = 16807$, $c = 0$, and $m = 2^{31} - 1$. Can you detect any structure in the plotted points for $k = 1$ to $k = 5$?
- d) **Short-term correlations.** Another measure of short term correlations is the autocorrelation function

$$C(k) = \frac{\langle x_{i+k}x_i \rangle - \langle x_i \rangle^2}{\langle x_i^2 \rangle - \langle x_i \rangle^2}, \quad (1)$$

where x_i is the i th term in the sequence. If x_{i+k} and x_i are not correlated, then $\langle x_{i+k}x_i \rangle = \langle x_{i+k} \rangle \langle x_i \rangle$ and $C(k) = 0$. Is $C(k)$ identically zero for any finite sequence? Compute $C(k)$ for $a = 106$, $c = 1283$, and $m = 6075$.