



TECHNISCHE UNIVERSITÄT  
CHEMNITZ

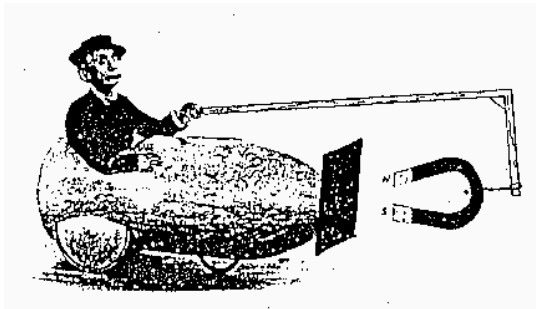
Gastvorlesung „Magnetischen Funktionsmaterialien“  
im Rahmen der Vorlesung

„Komplexe Materialien“ (Prof. Deibel)

Prof. Dr. Olav Hellwig

7.5.2020

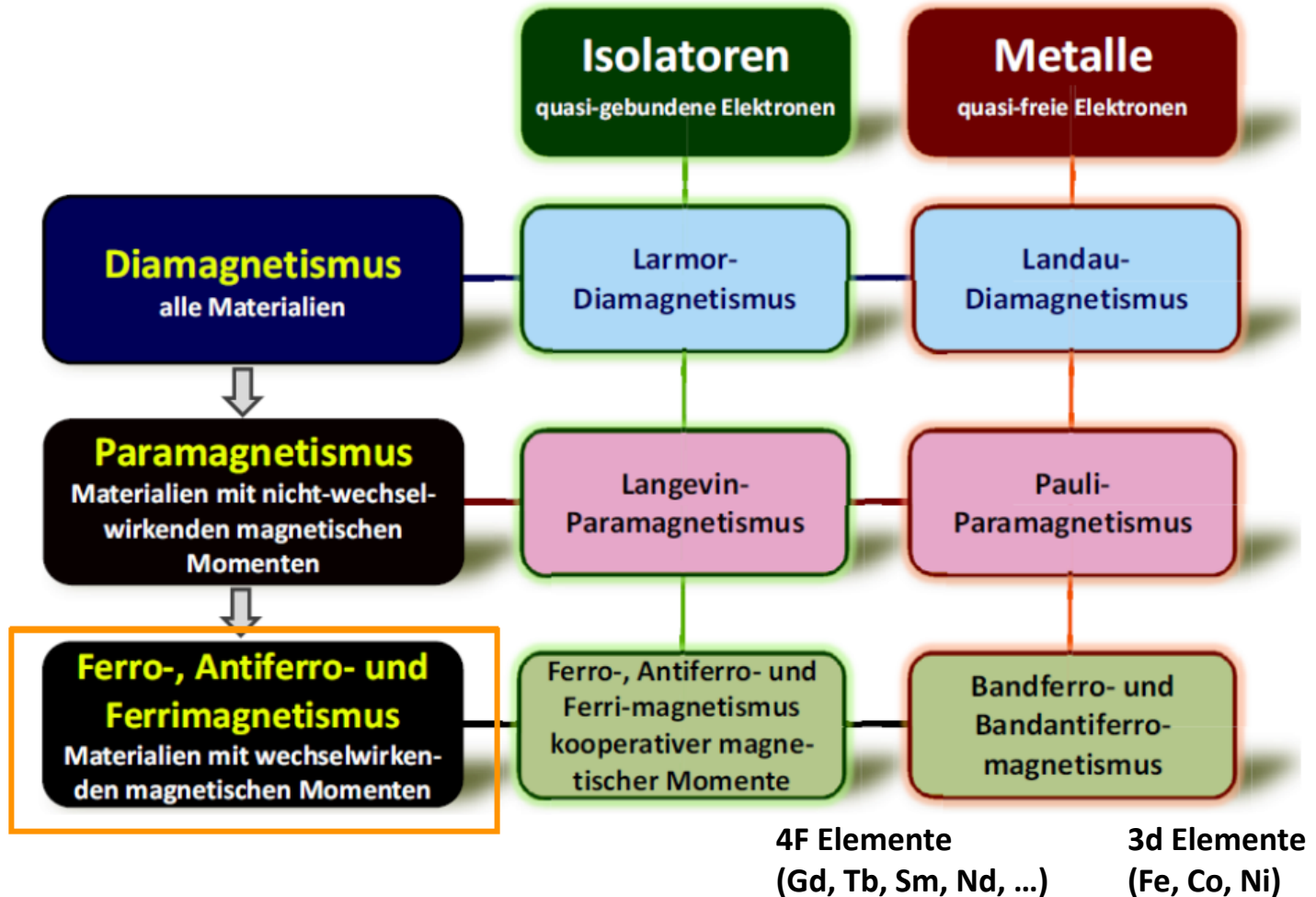
Magnetische Funktionsmaterialien  
SS 2020



## Ferromagnetische (Funktionale) Materialien

- **Einordnung und Einleitung**
- **Energien und Energiedichten einer ferromagnetischen Probe**
  - Austauschwechselwirkung
  - Streufeld- oder Demagnetisierungsenergie, Formanisotropie
  - Anisotropie (außer Formanisotropie = Demagnetisierungsenergiedichte)
  - Zeemann Energie, äußeres Feld
- **Wechselseitige Konkurrenz verschiedener magnetischer Energieterme**
- **Hysterese-Effekte, Stoner-Wohlfarth Modell, Basis für binäre magn. Datenspeicher**
- **Magnetische Funktionsmaterialien zur Datenspeicherung**
  - Entwicklung der Festplatte: Von magnetischen Mikrosystemen zu Nanosystemen
  - GMR (Riesenmagnetwiderstand) und TMR Effekte für empfindlichere Leseköpfe
  - Zukünftige Festplattentechnologien
  - Neue Effekte in der Nanowelt: Spin transfer torque in Nanokontakten
  - Separation von Ladungs und Spinströmen: Spin orbit torque in Dünnschichtsystemen
  - Anwendungen im Magnetic Random Access Memory (MRAM)
  - Die Spinwelle als Informationsträger (HZDR-movie)

# Klassifizierung der mikroskopischen Ursachen unterschiedlicher magnetischer Phänomene

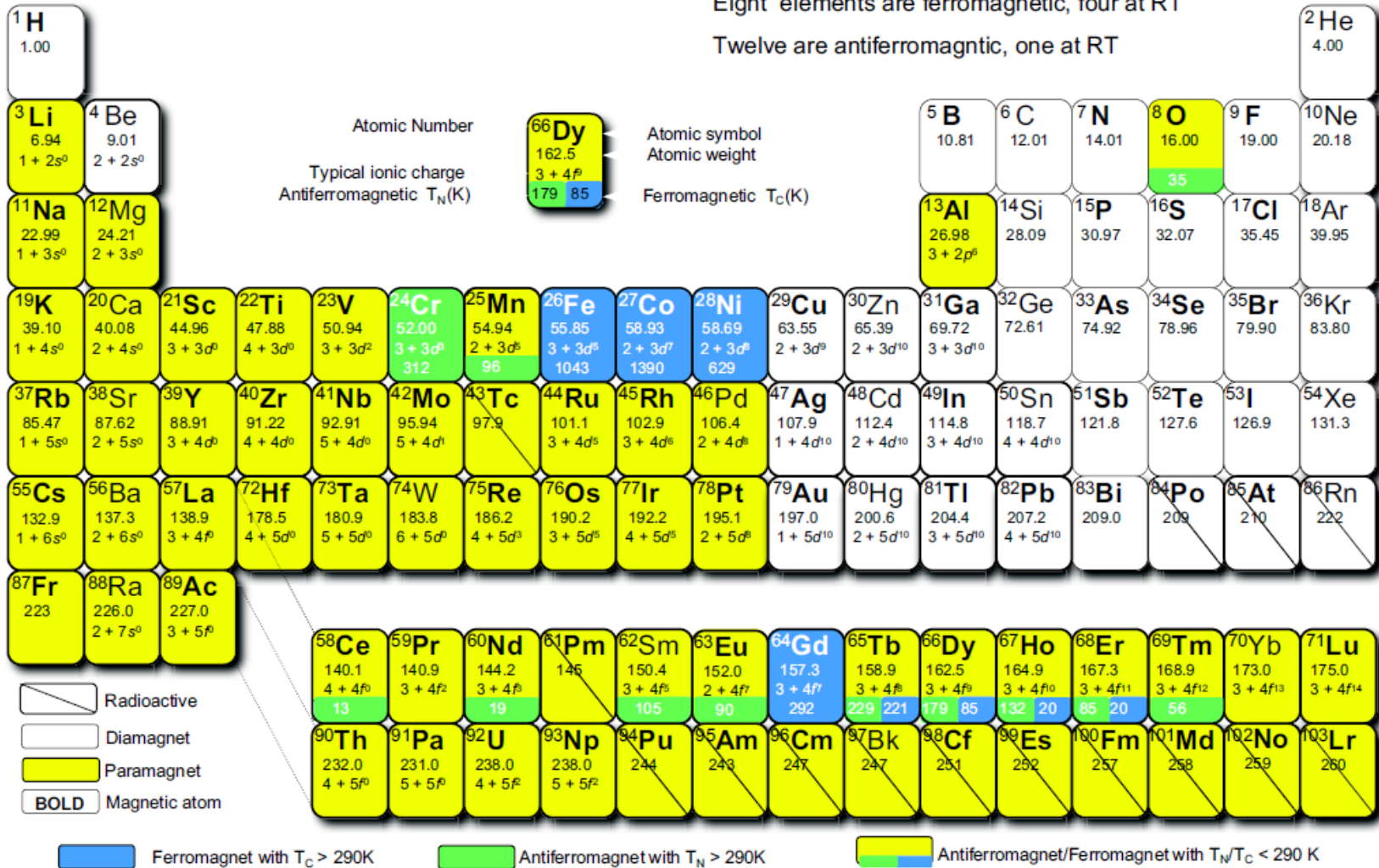


# Magnetic Elements

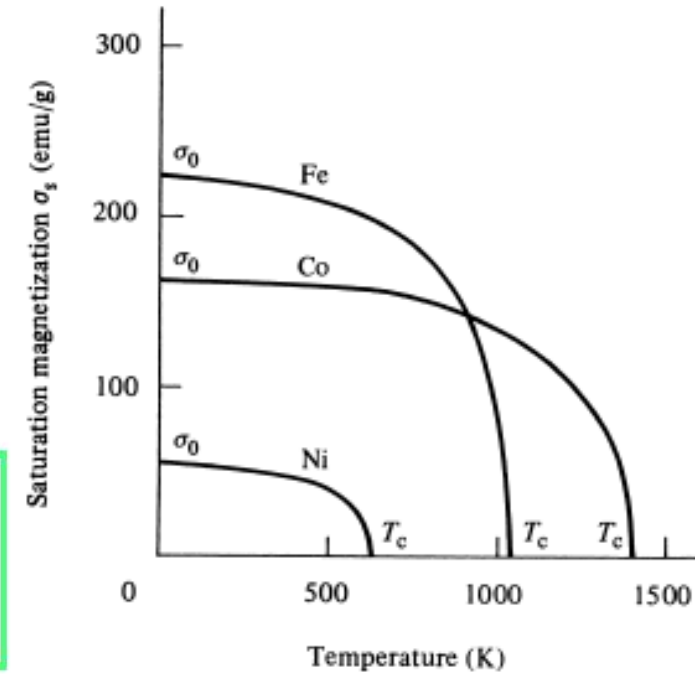
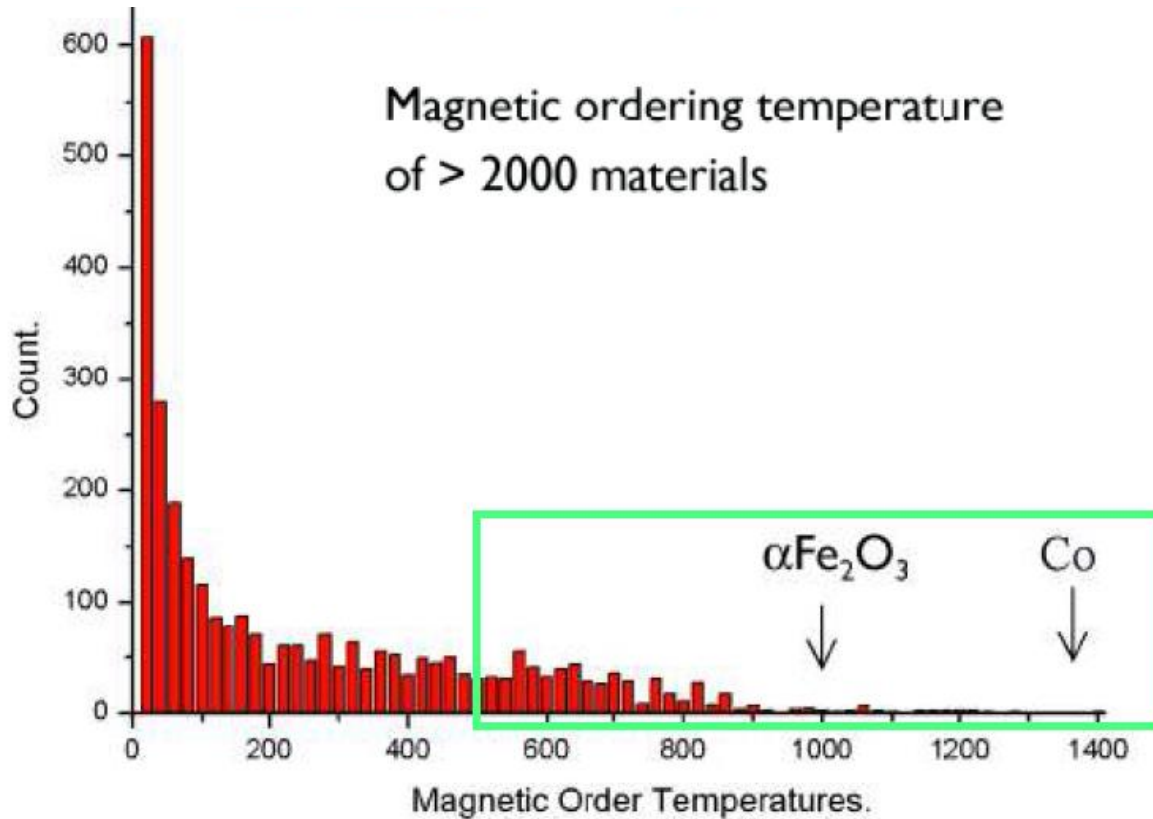
## The Magnetic Periodic Table

Eight elements are ferromagnetic, four at RT

Twelve are antiferromagnetic, one at RT



# $T_C$ of magnetic materials



A useful magnetic material needs to be able to operate from -50 C to 120 C.

The Curie temperature needs to be > 500 K

Co has the highest  $T_C$  of all magnetic materials



# Three ways to approach magnetism

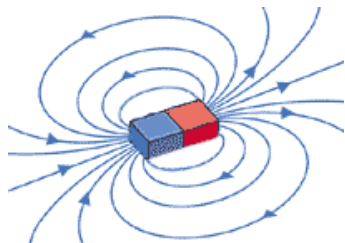
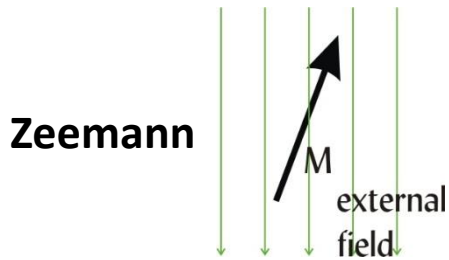
**Maxwell's Equations:**  
Phenomenological macroscopic equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

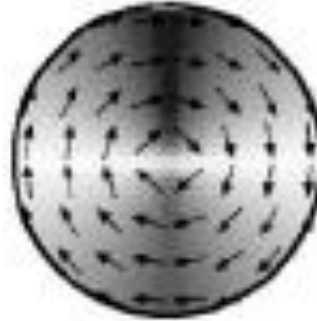
$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$



**Demagnetization  
Stray field  
Shape anisotropy**

**Nanomagnetism:**



**Basics:**

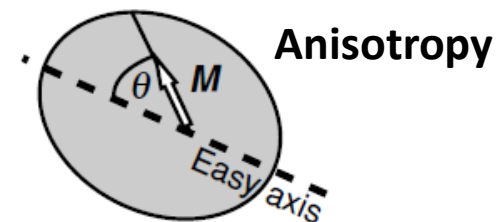
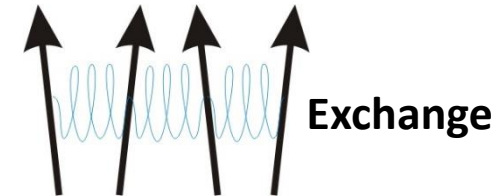
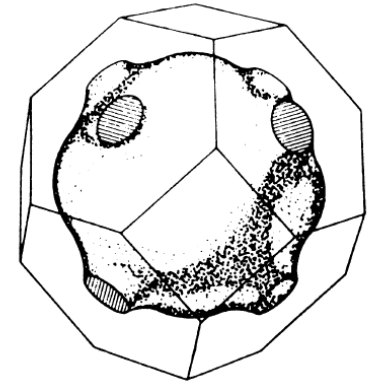
- Micromagnetic ground state
- Characteristic length scales
- Exchange length, domain wall width
- magnetic hardness parameter
- Stoner Wohlfarth theory
- Magnetic field reversal, hysteresis

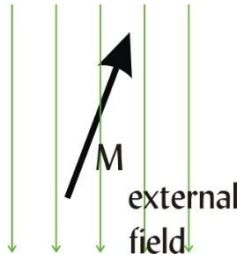
**Applications:**

- **Hard Disk Drives**
- **Magnetoresistance**
- **Spintronics, spin torques**
- **Magnonics**

**Quantum Mechanics:**

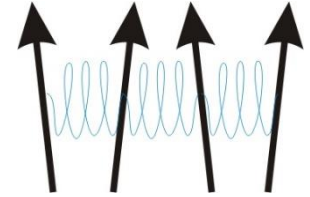
$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$





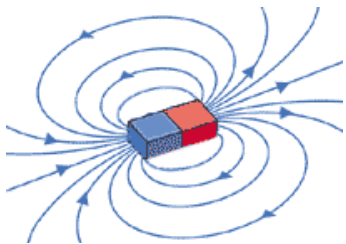
$$E_{external} = -\vec{H} \cdot \vec{M}$$

$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



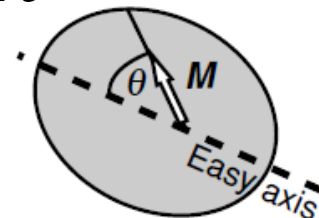
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + surface + others$$

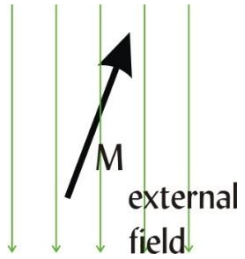
$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



$$E_{anisotropy} = g(\theta)$$

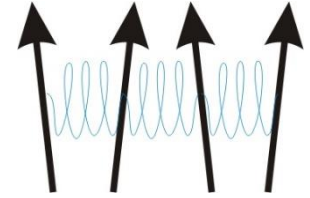
$$= K_U \sin^2 \theta$$





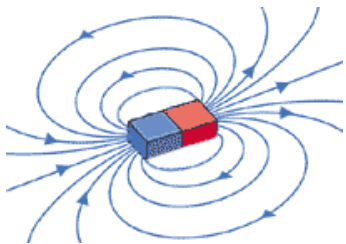
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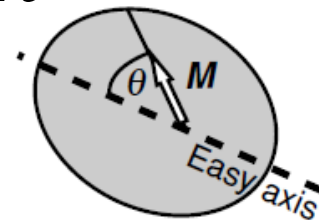
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + surface + others$$

$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$

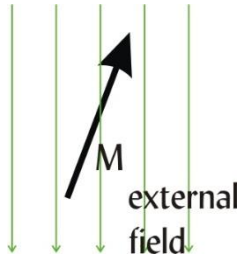


$$E_{anisotropy} = g(\theta)$$

$$= K_U \sin^2 \theta$$

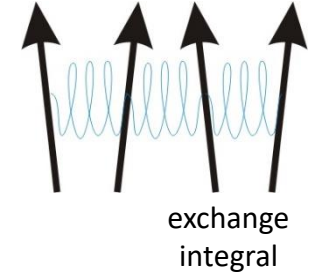






$$E_{external} = -\vec{H} \cdot \vec{M}$$

H earth  $\approx 50\mu\text{T} = 0.00005\text{ T}$ , H electromagnet  $\approx 2.5\text{ T}$   
H superconducting  $\approx 15\text{ T}$ , H pulsed  $\approx 100\text{ T}$  (factor  $10^7$ )



$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$

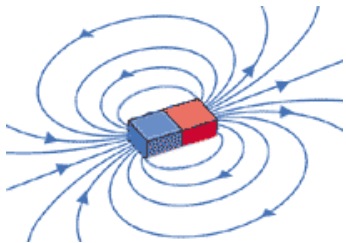
exchange  
integral

A varies by a factor of less than 10

$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + \text{surface} + \text{others}$$

M varies by a factor of  $\sim 10$  (for useful, large M materials)

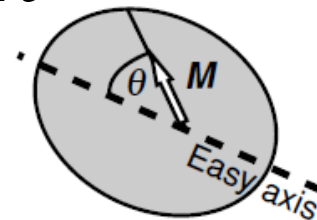
$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



$K_u$  varies by a factor of 100 000 or more ...

$$E_{anisotropy} = g(\theta)$$

$$= K_U \sin^2 \theta$$



# Micromagnetic Energies determining the magnetic state of a sample

external field

$E_{external} = -\vec{H} \cdot \vec{M}$

$= -H M \cos\theta$

H earth  $\approx 50\mu\text{T} = 0.00005\text{ T}$ , H electromagnet  $\approx 2.5\text{ T}$   
 H superconducting  $\approx 15\text{ T}$ , H pulsed  $\approx 100\text{ T}$  (factor  $10^7$ )

exchange integral

$\mathcal{H} = -2 \sum_{i>j} J_{ij} S_i \cdot S_j$

$E_{exchange} = A \left( \frac{\partial\theta}{\partial x} \right)^2$

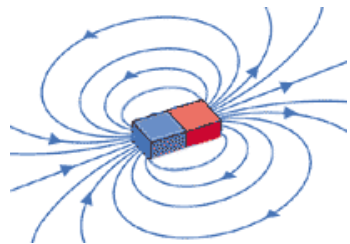
very short range,  
nearest neighbor interactions only

$A$  varies by a factor of less than 10

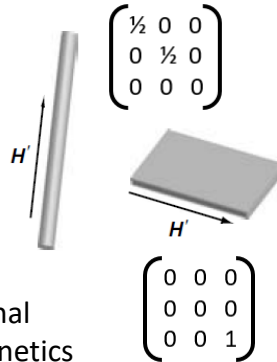
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + \text{surface} + \text{others}$$

$M$  varies by a factor of  $\sim 10$  (for useful, large  $M$  materials)

$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



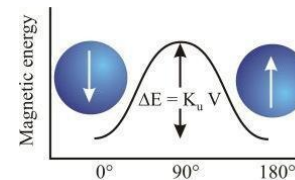
long range,  
everything interacts,  
takes most computational  
resources in micro-magnetics



$$E_{magnetostatics} = -\frac{1}{2} \int_{sample} \vec{H}_d \cdot \vec{M} dV = -\frac{1}{2} \int_{sample} N \vec{M}^2 dV = -\frac{1}{2} N \vec{M}^2 V$$

$K_U$  varies by a factor of 100 000 or more ...

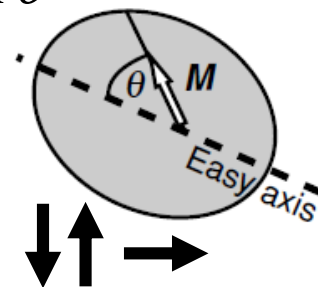
$$E_{anisotropy} = g(\theta)$$

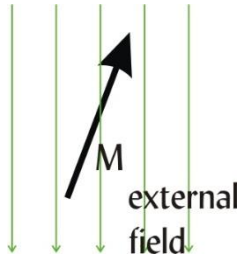


$$= K_U \sin^2\theta$$

directionally varying energy,  
source is often the crystal structure

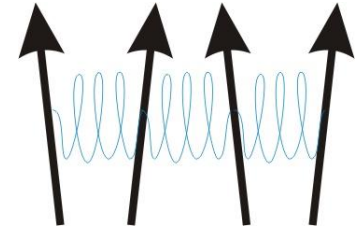
simplest case = uni-axial





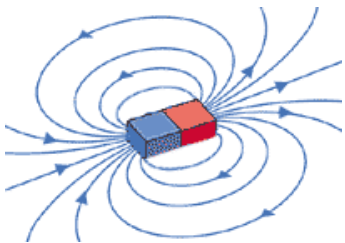
$$E_{external} = -\vec{H} \cdot \vec{M}$$

$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



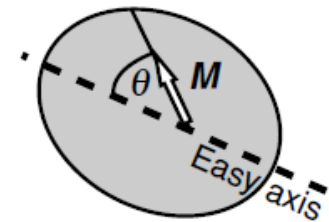
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + \text{surface} + \text{others}$$

$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



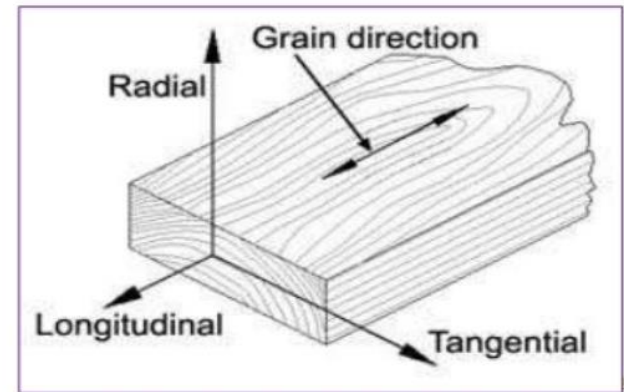
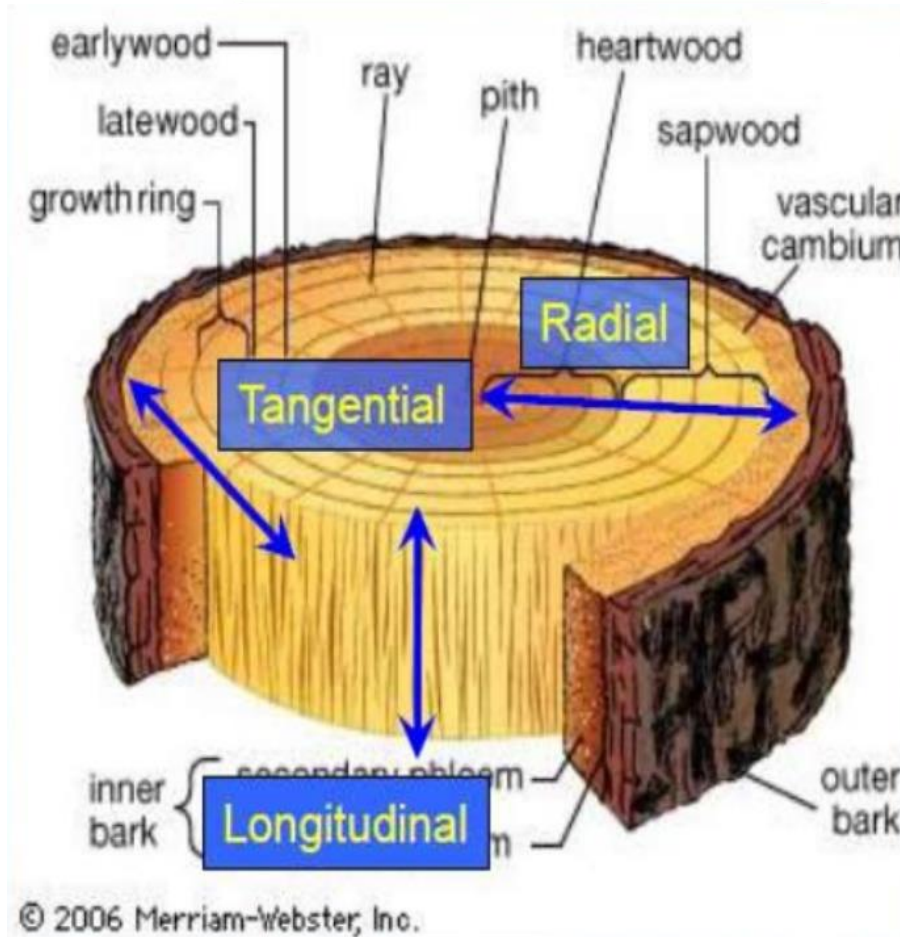
$$E_{anisotropy} = g(\theta)$$

$$= K_U \sin^2 \theta$$



# Anisotropy of wood

The dimensional changes in wood are unequal along the three structural directions (longitudinal, tangential and radial).

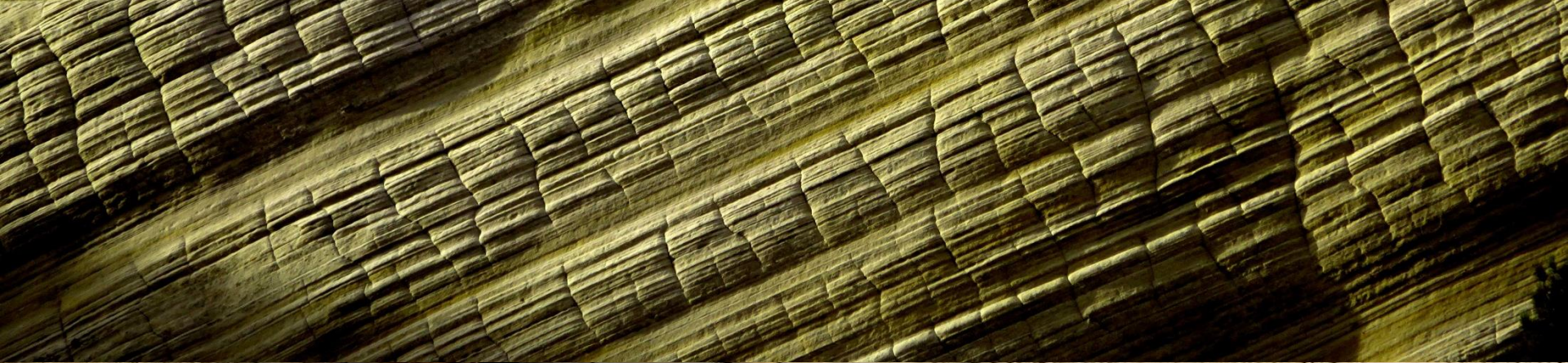


Other examples:

- snow flakes
- icicles
- thin films
- nanotubes
- graphene
- Crystals in general

➔ Magnetic anisotropy ...

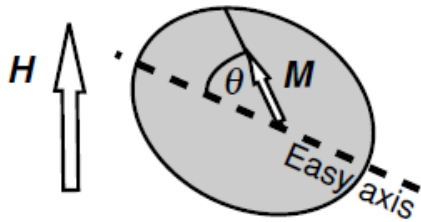






The direction of magnetization  $\mathbf{M}(\mathbf{r})$  in a macroscopic ferromagnetic domain lies along one or several easy axes.

$$E_a = K_1 \sin^2 \theta \quad (\text{uniaxial anisotropy, leading term, simplest case})$$



### Symmetry:

- Uniaxial (magnetic recording)
- Cubic (Fe, Ni)
- Hexagonal (Co)
- Teragonal (FePt)
- more complex symmetries

Magnetization is not necessarily parallel to applied field, unless  $H$  is applied in an easy direction.

will see this later in the Stoner-Wohlfarth model ...

### Types of magnetic anisotropy:

- Magneto-crystalline
- Shape
- Surface, interface ( $\rightarrow$  MLs)
- Magneto-elastic, strain
- other ...

Anisotropy limits the coercivity available in hard magnets  
Anisotropy leads to unwanted coercivity in soft magnets

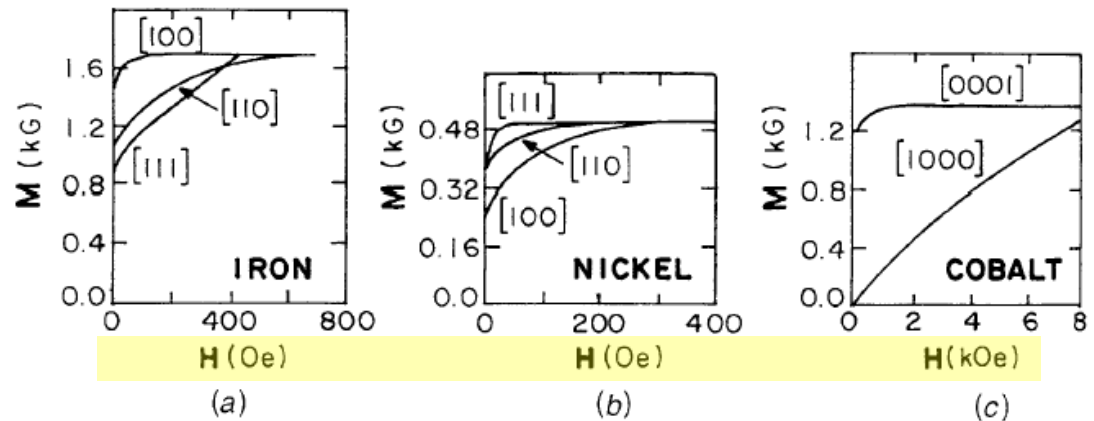
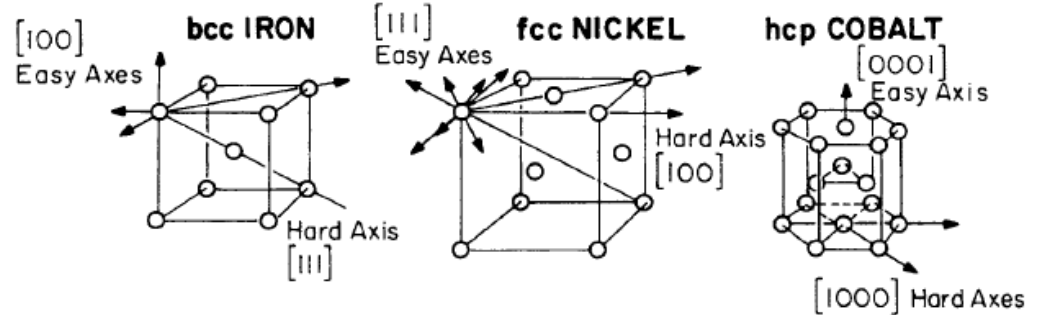


# Magneto-crystalline anisotropy energy

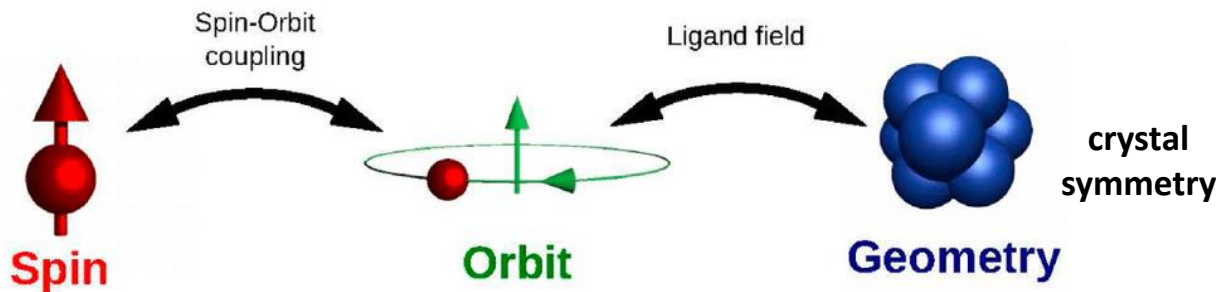
Experimental observation:  
single-crystalline magnetic  
materials have magnetic “easy  
and “hard” axes

Physical origin:

- spin-orbit coupling and
- low symmetry crystal field,  
resulting in asymmetric charge  
distribution



**Figure 6.1** Crystal structure showing easy and hard magnetization directions for Fe(a), Ni(b), and Co(c), above. Respective magnetization curves, below.



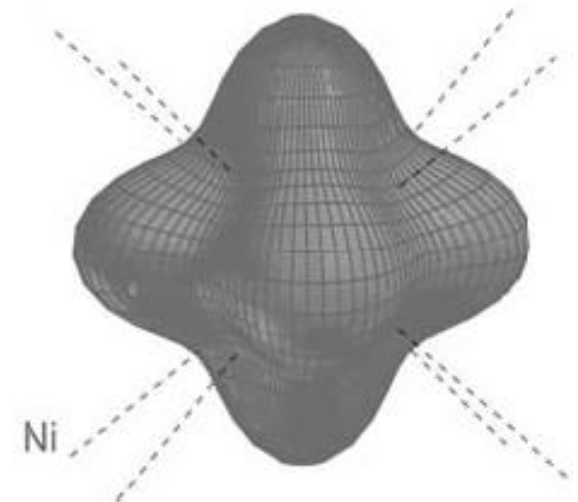
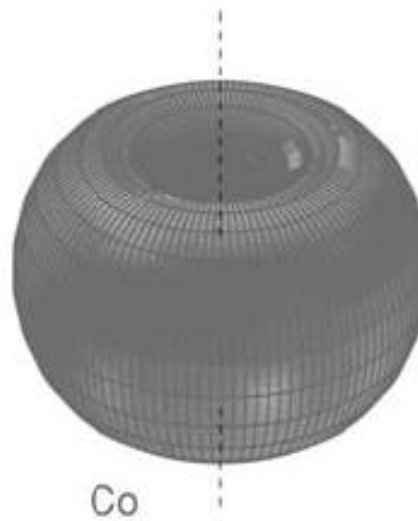
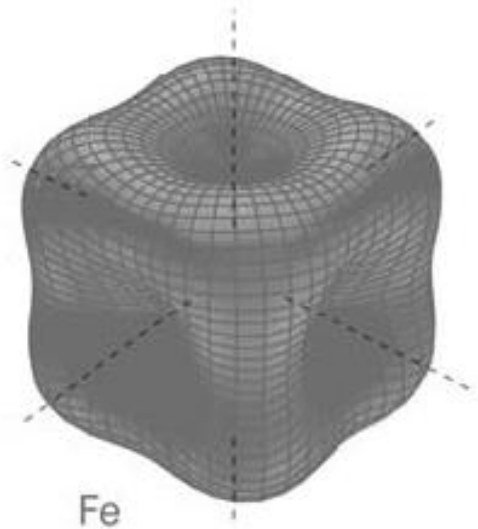
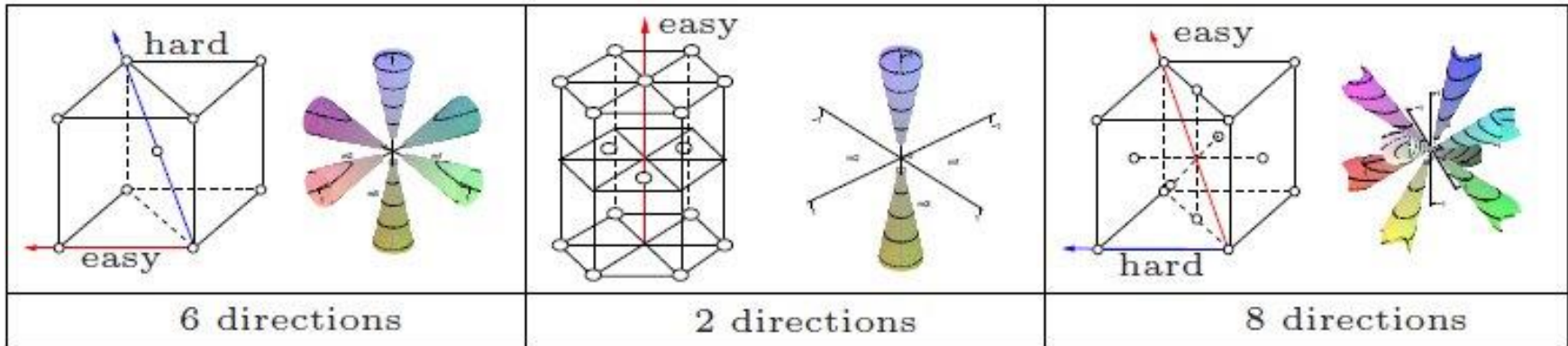
Anisotropy reflects  
the crystal symmetry

# Anisotropy

## Fe bulk

## Co bulk

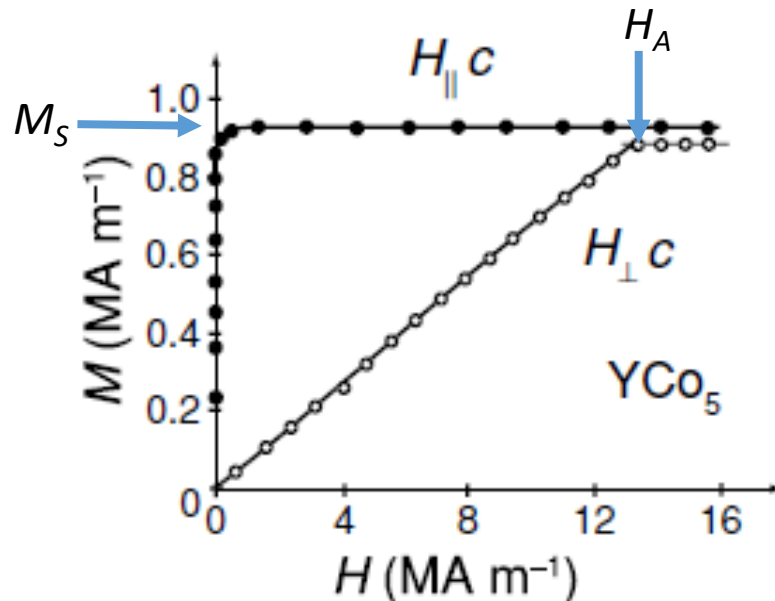
## Ni bulk



To move  $M$  from one direction into another direction an external magnetic field  $B_0$  has to do work against the magneto-crystalline anisotropy:

$$E = \int \vec{B}_0 d\vec{M}$$

The magnetic anisotropy energy is defined as the difference of the free energy between different directions of  $M$  at constant temperature:



$$K_U = \frac{1}{2} H_A M_S \quad (\text{cgs})$$

for uniform magnetization  
(single domain objects)

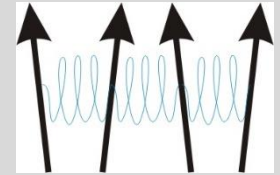
$$K_U = \frac{\mu_0}{2} H_A M_S \quad (\text{SI})$$

The magnetic easy axis is usually used as a reference direction.

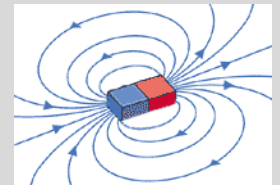
**Table 7.1.** Domain wall parameters for some ferromagnetic materials

	$M_s$ stray/demag (MA m <sup>-1</sup> )	$A$ exchange (pJ m <sup>-1</sup> )	$K_1$ anisotropy (kJ m <sup>-3</sup> )
Ni <sub>80</sub> Fe <sub>20</sub>	0.84	10	0.15
Fe	1.71	21	48
Co	1.44	31	410
CoPt	0.81	10	4900
Nd <sub>2</sub> Fe <sub>14</sub> B	1.28	8	4900
SmCo <sub>5</sub>	0.86	12	17 200
CrO <sub>2</sub>	0.39	4	25
Fe <sub>3</sub> O <sub>4</sub>	0.48	7	-13
BaFe <sub>12</sub> O <sub>19</sub>	0.38	6	330

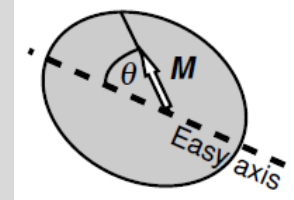
$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



$$E_{anisotropy} = g(\theta) = K_U \sin^2 \theta$$



magnetization

exchange stiffness

anisotropy (energy density)

$$E_{magnetostatics} = -\frac{1}{2} \int_{sample} \vec{H}_d \cdot \vec{M} dV = -\frac{1}{2} \int_{sample} N \vec{M}^2 dV = -\frac{1}{2} N \vec{M}^2 V$$

Variation across materials

less than 5

less than 10

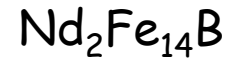
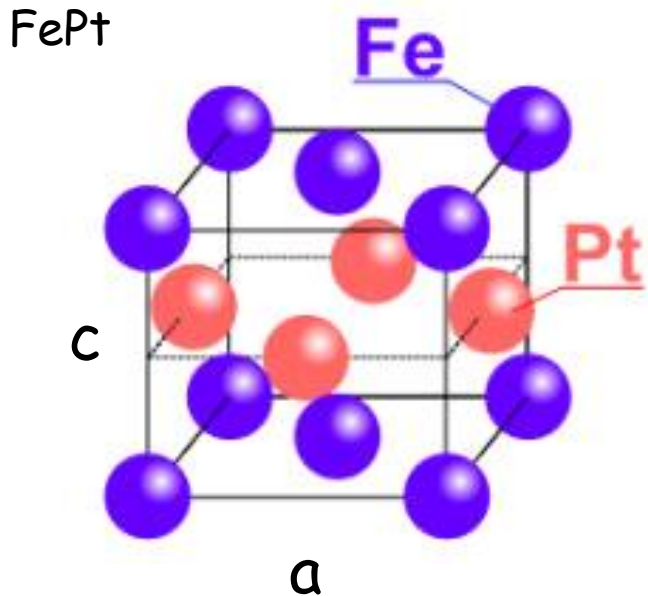
up to 100 000



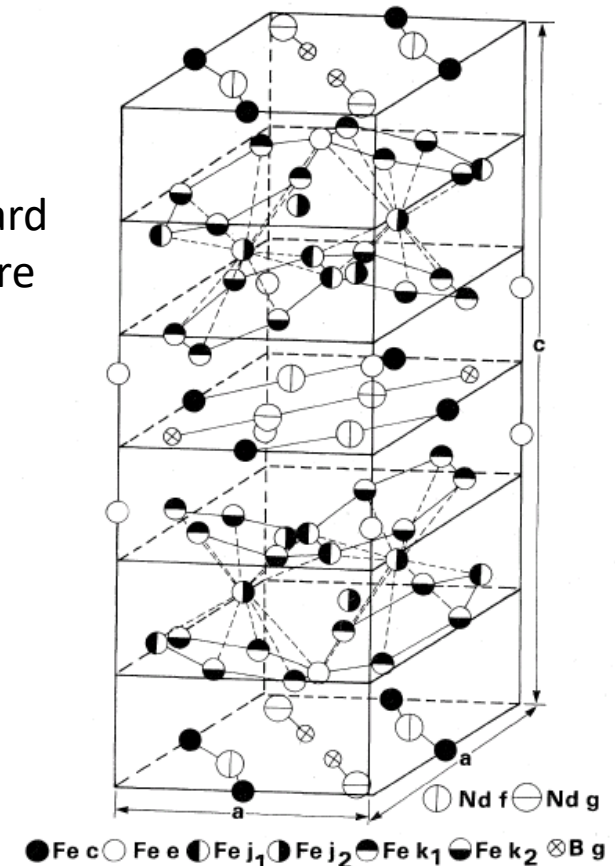
largest variations in anisotropy

# Magneto-crystalline anisotropy energy

Simple hard  
magnetic structure

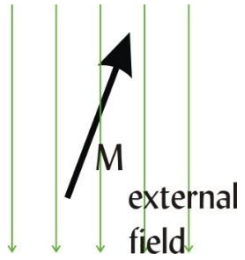


More complex hard  
magnetic structure



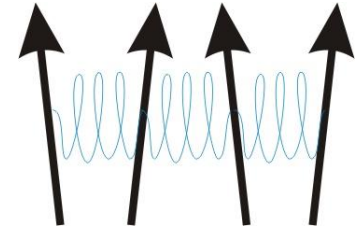
Alloy system	Material	$K_1$ ( $10^7 \text{erg/cm}^3$ )	$M_S$ ( $\text{emu/cm}^3$ )	$H_K$ (kOe)	$T_C$ (K)
Co-alloys	CoPtCr	0.20	298	13.7	—
$L_{10}$ phases	FePt	6.6–10	1140	116	750
Rare-earth	$\text{Fe}_{14}\text{Nd}_2\text{B}$	4.6	1270	73	585
Transition metals	$\text{SmCo}_5$	11–20	910	240–400	1000

# Energies determining the magnetic state of a sample



$$E_{external} = -\vec{H} \cdot \vec{M}$$

$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$

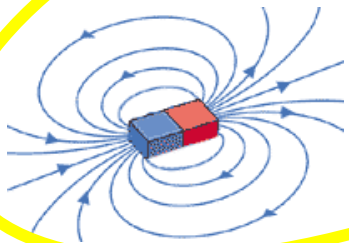


$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + surface + others$$

originates from macroscopic dipole-dipole interaction

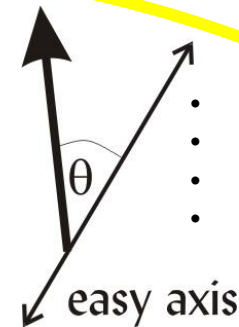
originates from spin-orbit interaction in conjunction with the crystal field, i.e. from microscopic (electronic and atomic) dipole-dipole interactions

$$E_{stray} = -\frac{1}{2} \vec{H}_S \cdot \vec{M}$$



demag energy  
stray field energy  
→ (sample) shape anisotropy

$$E_{anisotropy} = g(\theta) = K_U \sin^2 \theta$$



- Magneto-crystalline
- Magneto-elastic
- Surface (mix: ex+intr)
- etc.

Crystalline and induced anisotropies

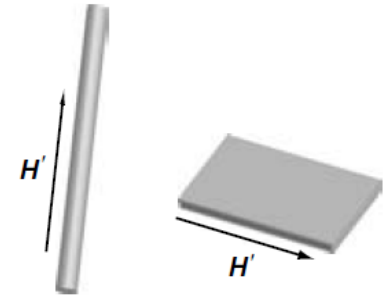
**Intrinsic/mixed anisotropy**

**extrinsic anisotropy**



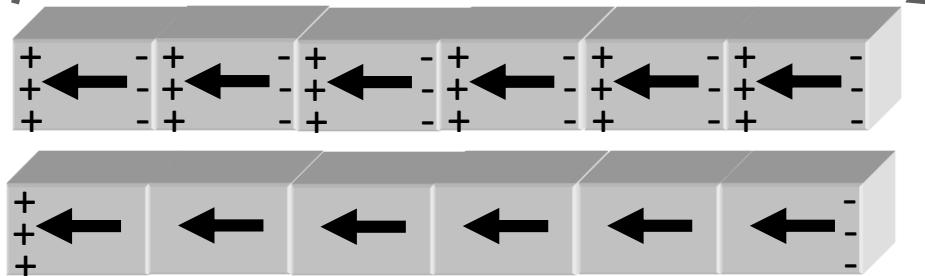
The magnetic hardness parameter is the dimensionless ratio of anisotropy to stray field (or dipole) energy:

$$\kappa = \sqrt{|K_1| / \mu_0 M_s^2}$$

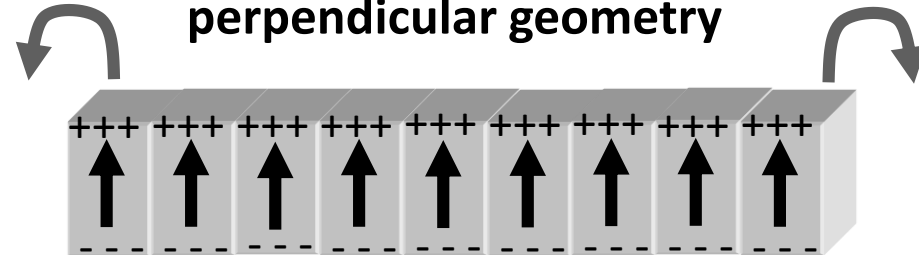


It should be greater than 1 for a permanent magnet and much less than 1 for a good temporary magnet.

**longitudinal geometry**



**perpendicular geometry**



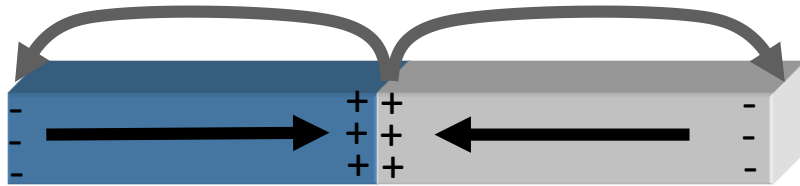
**Alternative definition: magnetic Q-factor**

shape anisotropy of a thin film

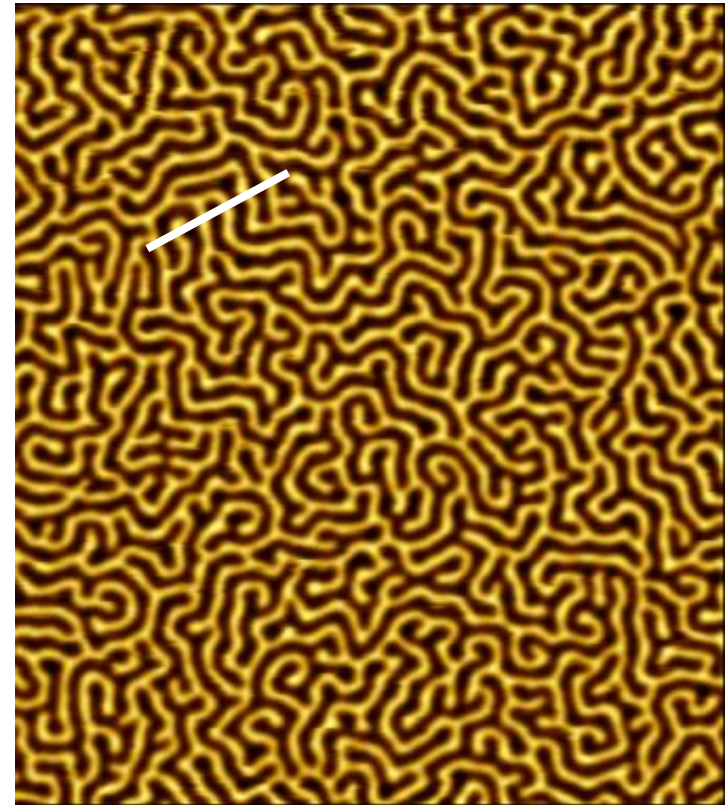
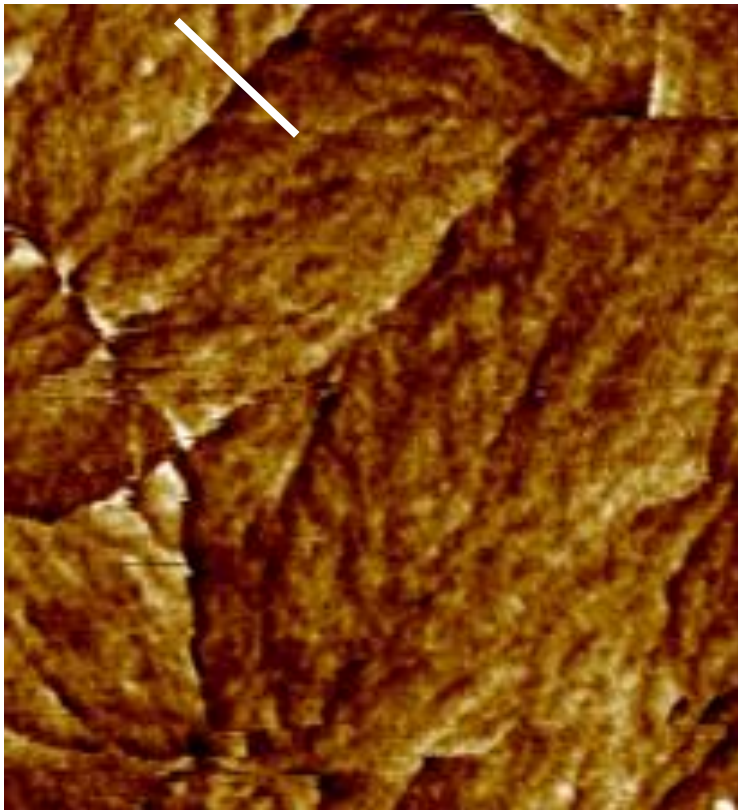
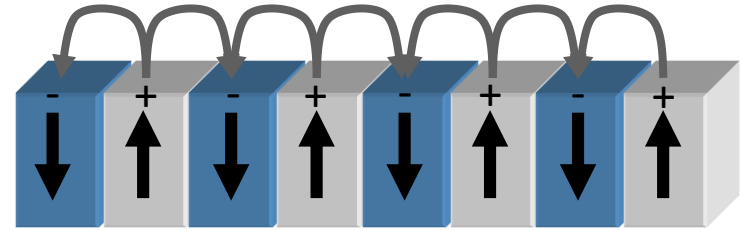
Magnetization of thin films.  $Q = -K_u/K_d$  where  $K_d = (1/2)\mu_0 M_s^2$

If  $Q > 1$  then the anisotropy  $K_1$  is large enough to pull the magnetization out-of-the-plane in a thin film geometry

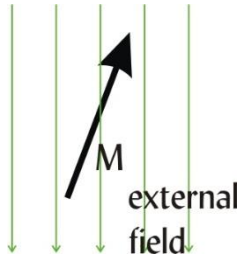
longitudinal geometry



perpendicular geometry

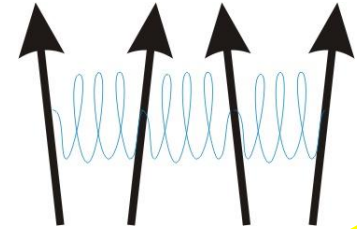


# Energies determining the magnetic state of a sample



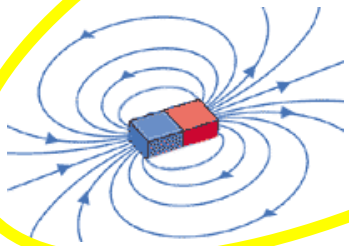
$$E_{\text{external}} = -\vec{H} \cdot \vec{M}$$

$$E_{\text{exchange}} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$

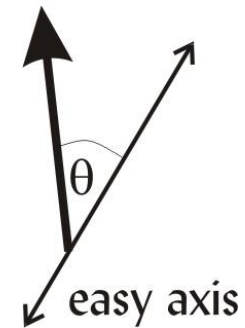


$$E_{\text{total}} = \int_{\Omega} (E_{\text{external}} + E_{\text{stray}} + E_{\text{exchange}} + E_{\text{anisotropy}}) dV + \text{surface} + \text{others}$$

$$E_{\text{stray}} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



$$E_{\text{anisotropy}} = g(\theta) = K_U \sin^2 \theta$$



$$E_{\text{magnetostatics}} = -\frac{1}{2} \int_{\text{sample}} \vec{H}_d \cdot \vec{M} dV = -\frac{1}{2} \int_{\text{sample}} N \vec{M}^2 dV = -\frac{1}{2} N \vec{M}^2 V$$

# Exchange length

Exchange length is the distance over which the magnetization is expected to be able to respond.

Competition between exchange energy and stray field energy is characterized by the exchange length

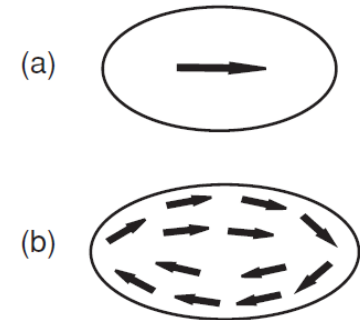
Film shape anisotropy energy  $E_{\text{shape (or stray)}} = 1/2\mu_0 M^2$

$l \sim \frac{\partial x}{\partial \theta}$  length to tilt  $\theta$  against shape anisotropy

$$E_{\text{exchange}} = A \left( \frac{\partial \theta}{\partial x} \right)^2 = \frac{A}{l^2} = E_{\text{stray}} = 1/2\mu_0 M^2$$

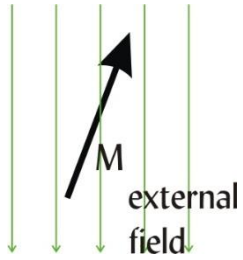
$$l_{\text{ex}} = \sqrt{\frac{A}{\mu_0 M_s^2}}$$

*factor 1/2 is sometimes skipped*



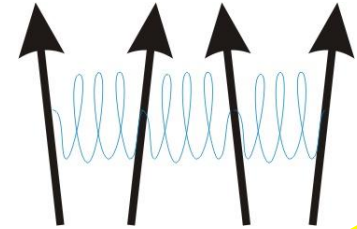
Stable ferromagnetic configurations in a soft spheroidal particle:  
(a) without and (b) with the effect of the demagnetizing field.

	$l_{\text{ex}}$ (nm)
Ni <sub>80</sub> Fe <sub>20</sub>	3.4
Fe	2.4
Co	3.4
CoPt	3.5
Nd <sub>2</sub> Fe <sub>14</sub> B	1.9
SmCo <sub>5</sub>	3.6
CrO <sub>2</sub>	4.4
Fe <sub>3</sub> O <sub>4</sub>	4.9
BaFe <sub>12</sub> O <sub>19</sub>	5.8



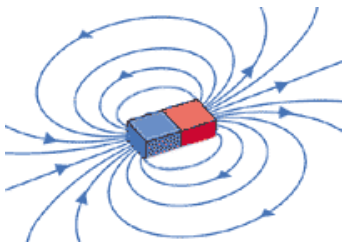
$$E_{external} = -\vec{H} \cdot \vec{M}$$

$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



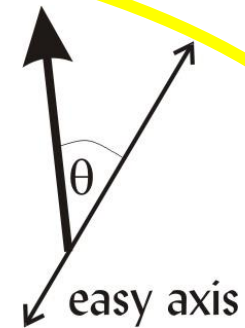
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + surface + others$$

$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



$$E_{anisotropy} = g(\theta)$$

$$= K_U \sin^2 \theta$$



# Domain wall width and energy

walls  $\sigma_w = \text{exchange} + \text{anisotropy}$

$$= \int_{-\infty}^{\infty} A \left( \frac{\partial \theta}{\partial x} \right)^2 + K \sin^2(\theta) dx$$

Minimize the energy (exchange+anisotropy),  
No demag energy included in the domain wall

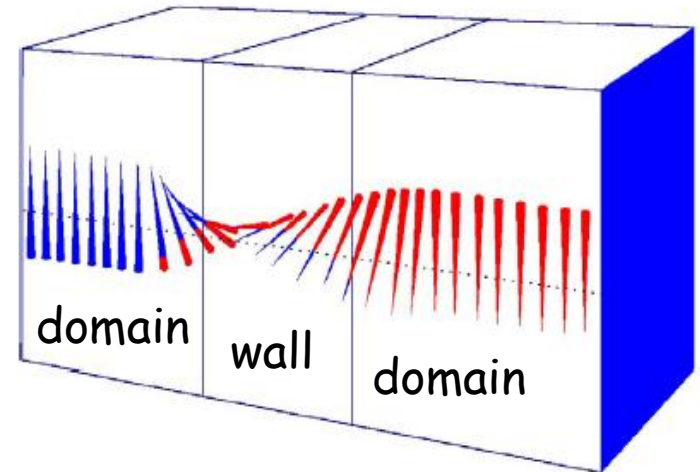
$$\theta(x) = \arctan\left[\sinh\left(\frac{\pi x}{\delta_w}\right)\right] + \pi/2$$

$\sigma_w = 4\sqrt{AK}$   
domain wall energy density

and

$\delta_w = \pi\sqrt{A/K}$   
domain wall width

How do we define where the domain wall ends?



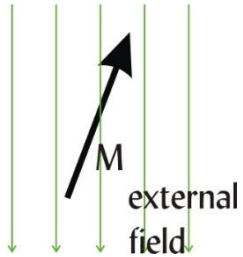
anisotropy K  
(energy density)  
up to 100 000

exchange  
stiffness A  
less than 10

The domain wall does not have a precisely defined width, since the direction of magnetization only approaches the easy axis asymptotically. Anisotropy of some sort is necessary for finite domain wall width.

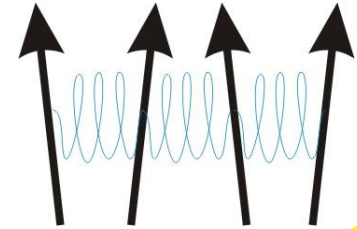
Stray field or demagnetization energy triggers domain formation → domain wall formation  
Exchange wants infinitely thick DW, anisotropy wants infinitely thin DW → compromise





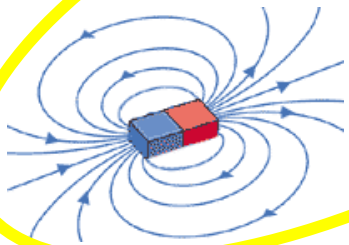
$$E_{\text{external}} = -\vec{H} \cdot \vec{M}$$

$$E_{\text{exchange}} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



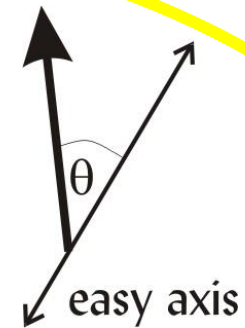
$$E_{\text{total}} = \int_{\Omega} (E_{\text{external}} + E_{\text{stray}} + E_{\text{exchange}} + E_{\text{anisotropy}}) dV + \text{surface} + \text{others}$$

$$E_{\text{stray}} = -\frac{1}{2} \vec{H}_S \cdot \vec{M}$$

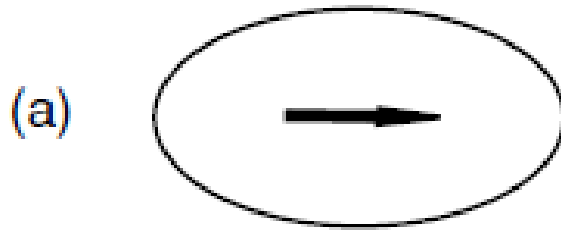


$$E_{\text{anisotropy}} = g(\theta)$$

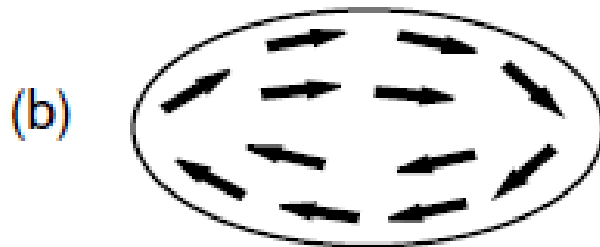
$$= K_U \sin^2 \theta$$



exchange dominates

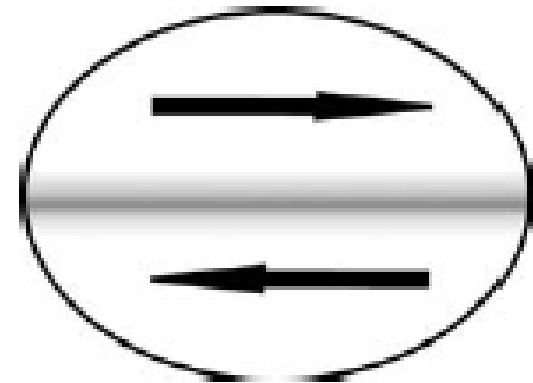


stray fields start to compete



Stable ferromagnetic configurations in a soft spheroidal particle:  
(a) without and (b) with the effect of the demagnetizing field.

stray fields and anisotropy start to compete with exchange



A ferromagnetic domain state resulting from the interplay of uniaxial anisotropy and demagnetizing field. The domain wall is the shaded region.

# Magneto-crystalline anisotropy energy

(across all magnetic materials)

**Table 7.1.** Domain wall parameters for some ferromagnetic materials

	$M_s$ stray/demag (MA m <sup>-1</sup> )	$A$ exchange (pJ m <sup>-1</sup> )	$K_1$ anisotropy (kJ m <sup>-3</sup> )	$\delta_w$ (nm)	$\gamma_w$ (mJ m <sup>-2</sup> )	$\kappa$	$l_{ex}$ (nm)
Ni <sub>80</sub> Fe <sub>20</sub>	0.84	10	0.15	2000	0.01	0.01	3.4
Fe	1.71	21	48	64	4.1	0.12	2.4
Co	1.44	31	410	24	14.3	0.45	3.4
CoPt	0.81	10	4900	4.5	28.0	2.47	3.5
Nd <sub>2</sub> Fe <sub>14</sub> B	1.28	8	4900	3.9	25	1.54	1.9
SmCo <sub>5</sub>	0.86	12	17 200	2.6	57.5	4.30	3.6
CrO <sub>2</sub>	0.39	4	25	44.4	1.1	0.36	4.4
Fe <sub>3</sub> O <sub>4</sub>	0.48	7	-13	72.8	1.2	0.21	4.9
BaFe <sub>12</sub> O <sub>19</sub>	0.38	6	330	13.6	5.6	1.35	5.8

	magnetization	exchange stiffness	anisotropy (energy density)	domain wall width	domain wall energy	hardness parameter	exchange length
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Variation across materials

less than 5	less than 10	up to 100 000	up to 1000	up to 6000	up to 500	~ factor 3
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largest variations in anisotropy ↑

# Magneto-crystalline anisotropy energy

(across all magnetic materials)

**Table 7.1.** Domain wall parameters for some ferromagnetic materials

	$M_s$ stray/demag (MA m <sup>-1</sup> )	$A$ exchange (pJ m <sup>-1</sup> )	$K_1$ anisotropy (kJ m <sup>-3</sup> )	$\delta_w$ exchange vs anisotropy (nm)	$\gamma_w$ exchange vs anisotropy factor (mJ m <sup>-2</sup> )	anisotropy vs stray fields $\kappa$	exchange vs $l_{ex}$ stray (nm)
Ni <sub>80</sub> Fe <sub>20</sub>	0.84	10	0.15	2000	0.01	0.01	3.4
Fe	1.71	21	48	64	4.1	0.12	2.4
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magnetization      exchange stiffness      anisotropy (energy density)      domain wall width      domain wall energy      hardness parameter      exchange length

Variation across materials

less than 5

less than 10

up to 100 000

up to 1000

up to 6000

up to 500

~ factor 3

$$\delta_w = \pi \sqrt{A/K}$$

$$\gamma_w = 4\sqrt{AK}$$

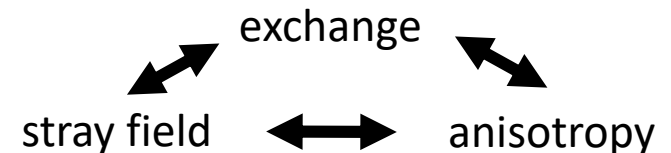
$$\kappa = \sqrt{|K_1| / \mu_0 M_s^2}$$

$$l \approx \sqrt{A/M}$$

largest variations in anisotropy



stray field energy =  
shape anisotropy energy

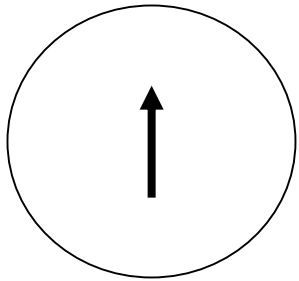


# Single domain particle size

Assume a magnetic sphere of radius  $R$ , moment  $M$ , uniaxial anisotropy  $K$  and exchange parameter  $A$ .

What is the critical radius to form a single domain. For simplicity we will assume the magneto-static energy is zero for the domain state.

## Single domain



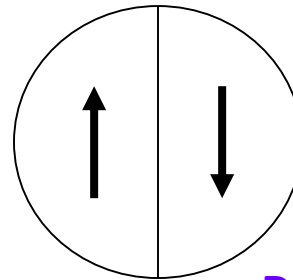
- $H_K$  does not allow for any tilt
- only demag energy

### Demag energy

$$E = \frac{1}{2} H_d M V = \frac{1}{2} \frac{4\pi}{3} M^2 \frac{4\pi}{3} R^3 = \frac{8\pi^2}{9} M^2 R^3$$

we used  $H_d = 4\pi N M = \frac{4\pi}{3} M$ ,  $N_{sphere} = \frac{1}{3}$

## Domain formation



- $H_K$  does not allow for any tilt
- only domain wall energy

### Domain wall energy

$$E \approx 4\sqrt{AK} \pi R^2$$

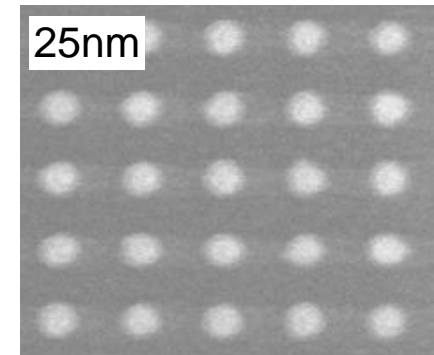
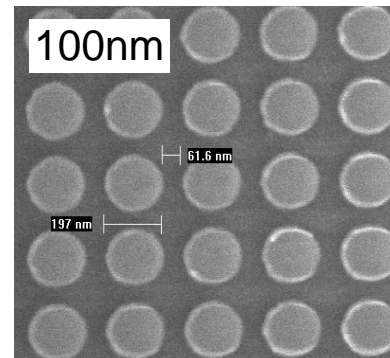
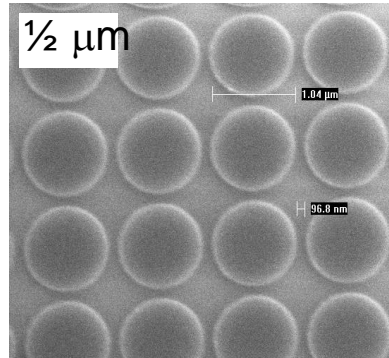
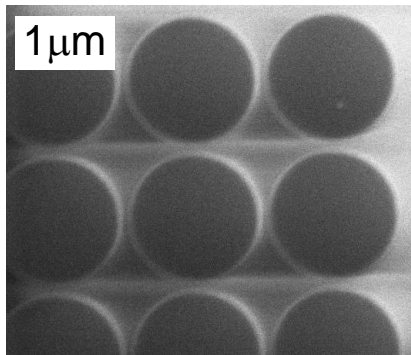
These energies are equal  $\rightarrow R = \frac{9\sqrt{AK}}{2\pi M^2}$

Use typical parameters for magnetic recording media,

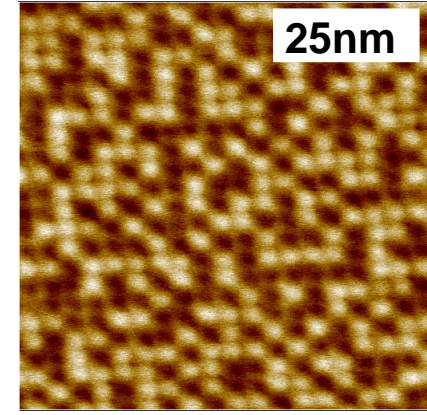
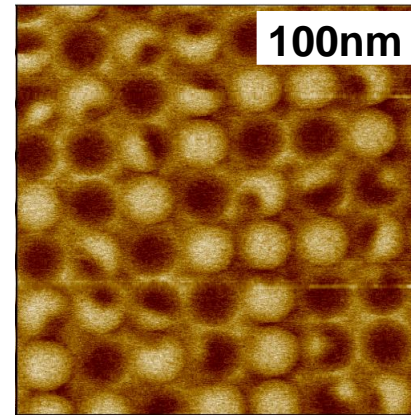
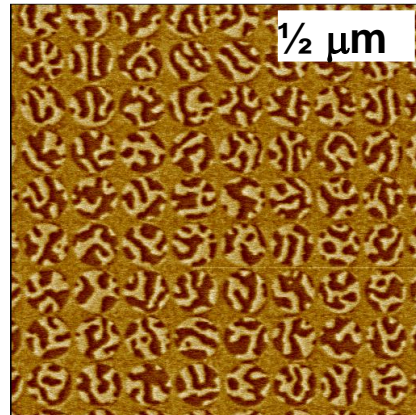
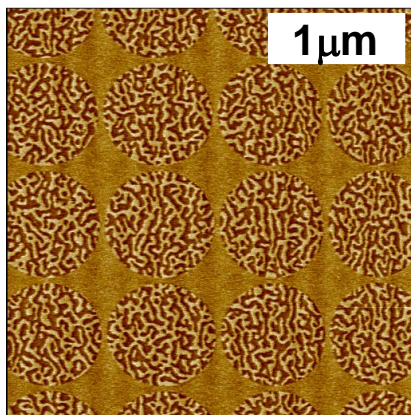
e.g.  $A = 10^{-6}$  erg/cm =  $10^{-11}$  J/m,  $M=1000$  emu/cc,  $K=5 \times 10^6$  ergs/cc gives  $R=32$  nm

# Single domain experiments

- Studied islands with variable sizes, radius ranging from  $5\mu\text{m}$  –  $50\text{ nm}$



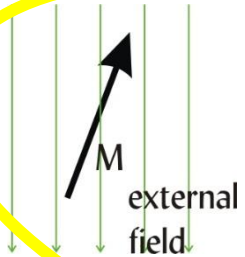
- Single domain behavior is found in islands with a radius  $\sim 50\text{ nm}$ .



MFM images of islands in demagnetized states

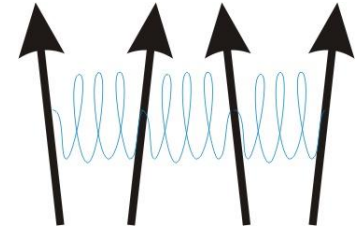


# Energies determining the magnetic state of a sample



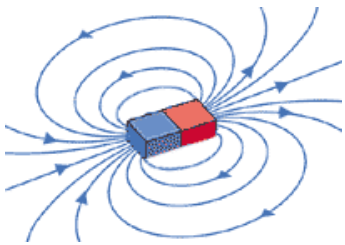
$$E_{external} = -\vec{H} \cdot \vec{M}$$

$$E_{exchange} = A \left( \frac{\partial \theta}{\partial x} \right)^2$$



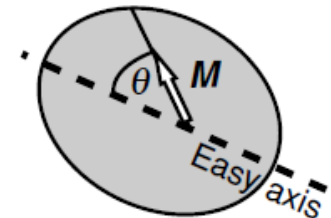
$$E_{total} = \int_{\Omega} (E_{external} + E_{stray} + E_{exchange} + E_{anisotropy}) dV + surface + others$$

$$E_{stray} = -\frac{1}{2} \vec{H}_s \cdot \vec{M}$$



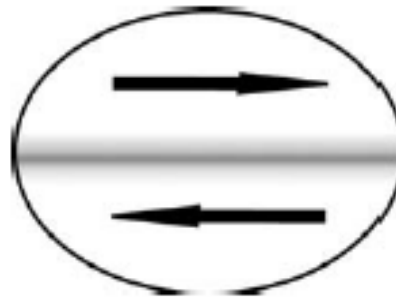
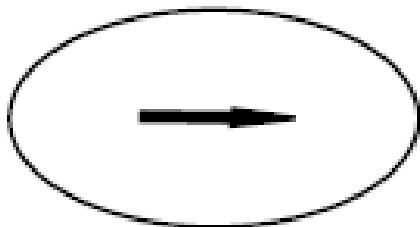
$$E_{anisotropy} = g(\theta)$$

$$= K_U \sin^2 \theta$$



# Reversal mechanisms

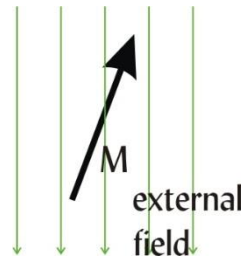
- Assume a small particle, such that the magnetism stays uniform, i.e. the particle is too small to support a domain wall, i.e. particle is smaller or of similar length scale than the typical domain wall width ...
- Treat the magnetization as a single spin known as a macro-spin, this type of model was used in a previous lecture, when we talked about micro-magnetic modeling using computer models ...



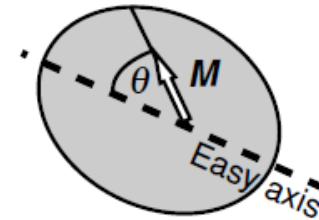
	$\delta_w$ (nm)
Ni <sub>80</sub> Fe <sub>20</sub>	2000
Fe	64
Co	24
CoPt	4.5
Nd <sub>2</sub> Fe <sub>14</sub> B	3.9
SmCo <sub>5</sub>	2.6
CrO <sub>2</sub>	44.4
Fe <sub>3</sub> O <sub>4</sub>	72.8
BaFe <sub>12</sub> O <sub>19</sub>	13.6

# Stoner-Wohlfarth-Model

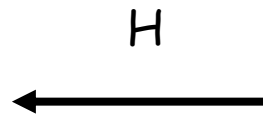
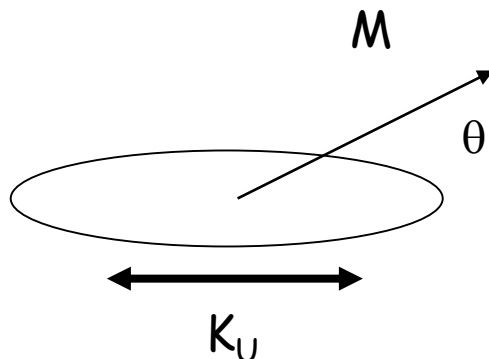
**Simplest possible reversal: Consider only Zeeman and anisotropy energy**  
**Simplest analytical model that exhibits hysteresis, Stoner-Wohlfarth-Model**



$$E_{external} = -\vec{H} \cdot \vec{M}$$



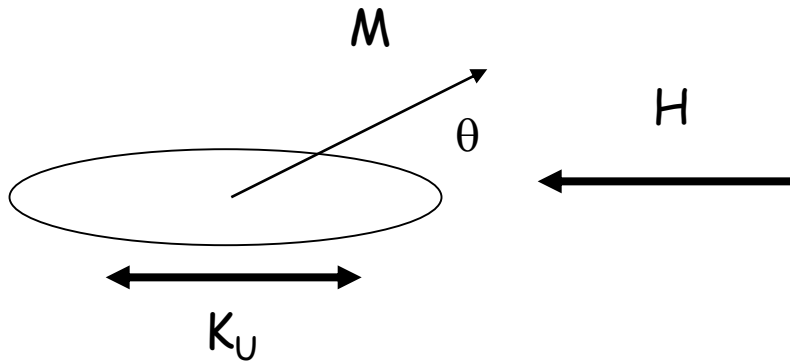
$$E_{anisotropy} = g(\theta) \\ = K_U \sin^2 \theta$$



$$E = -MH \cos \theta + K \sin^2 \theta$$

H term is uni-directional, K term is uniaxial

# Stoner-Wohlfarth-Model

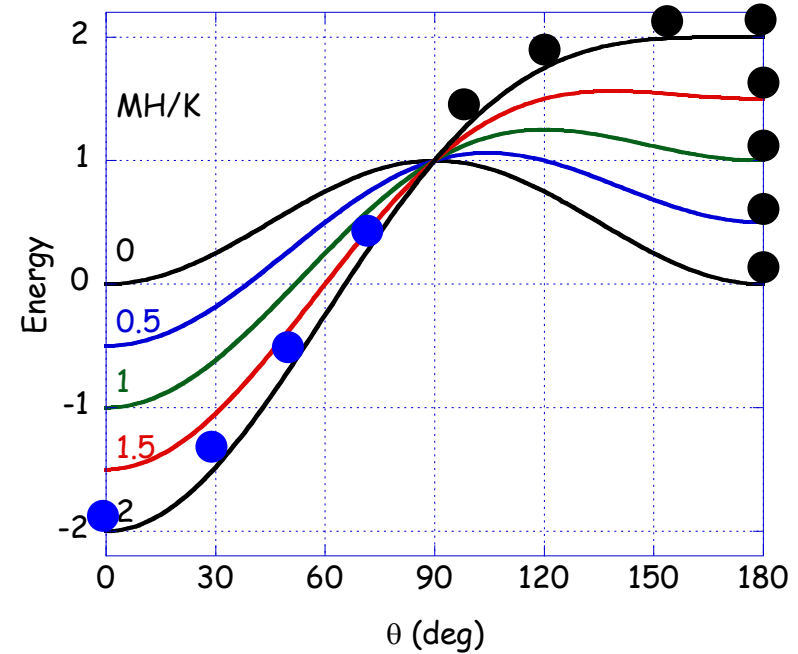


$$E = -MH \cos \theta + K \sin^2 \theta$$

$$\frac{\partial E}{\partial \theta} = \sin \theta (MH + 2K \cos \theta) = 0$$

$$\frac{\partial^2 E}{\partial \theta^2} = MH \cos \theta + 2K \cos(2\theta) > 0$$

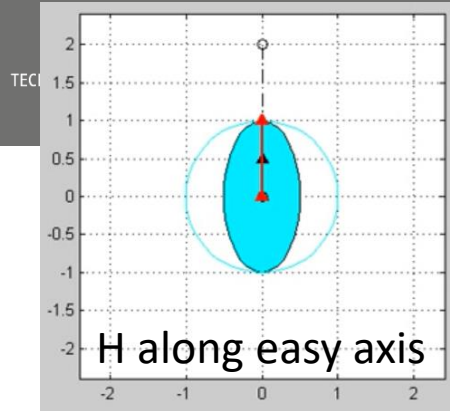
$$H_C = 2K / M$$



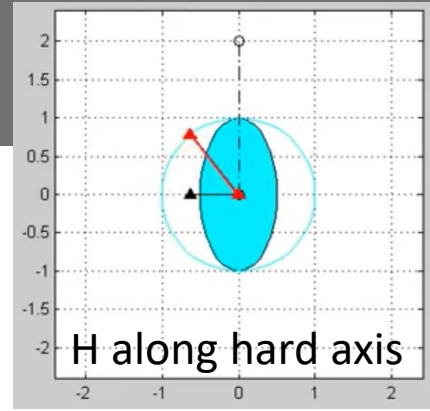
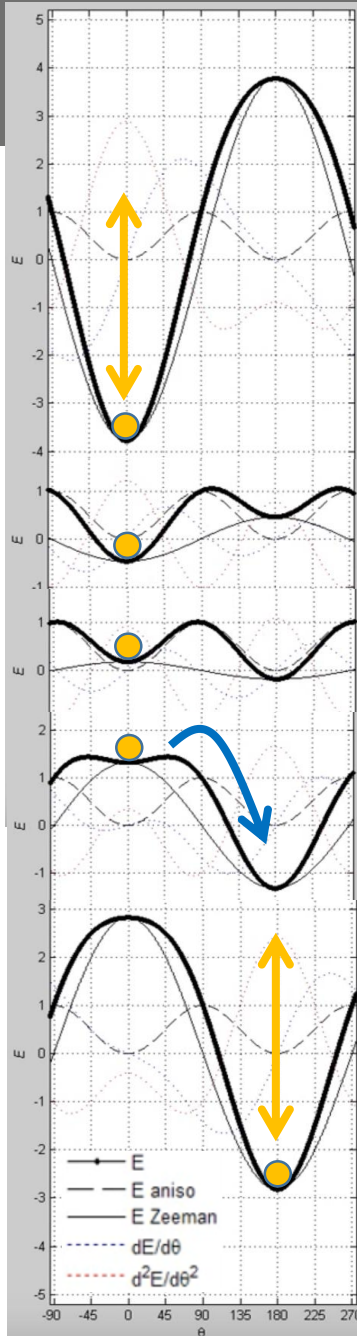
minima stay for all fields  
at 0 and 180 degrees  
Particle switches at  $2K_U/M$

[show easy axis Stoner-Wohlfarth movie](#)

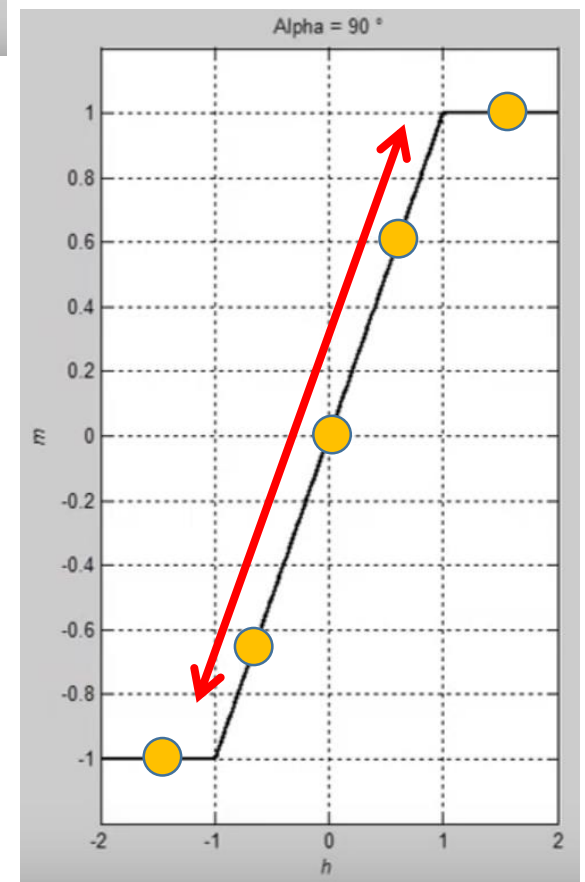
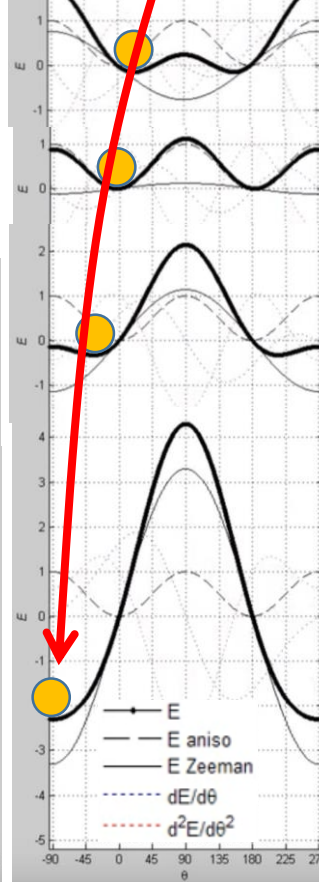
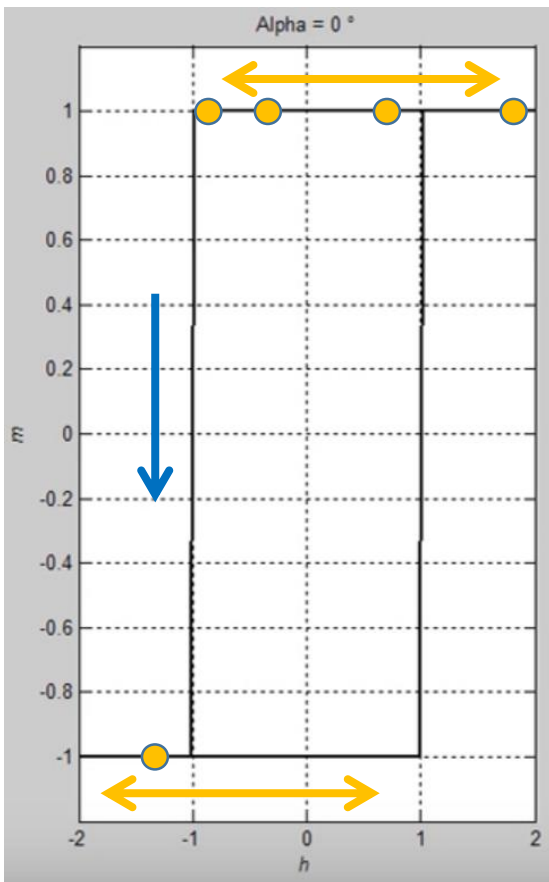
In zero field the magnetic anisotropy term is minimized when the magnetization is aligned with the easy axis. In a large field, the magnetization is pointed towards the field.



<https://www.youtube.com/watch?v=7HNGCJJ5i5A>



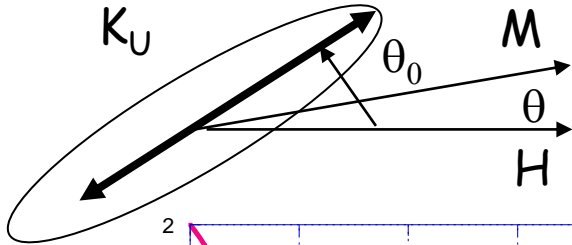
<https://www.youtube.com/watch?v=oN-12IsK6Ik>





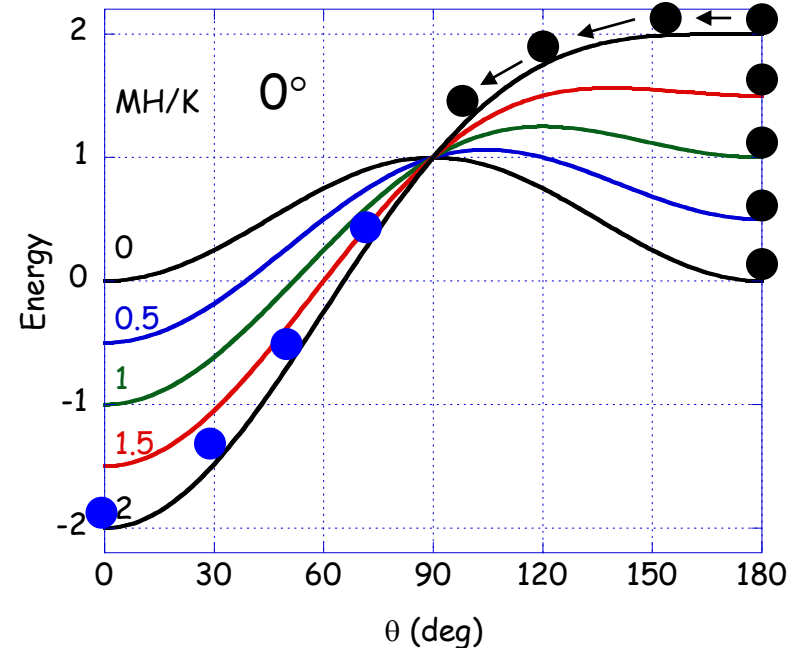
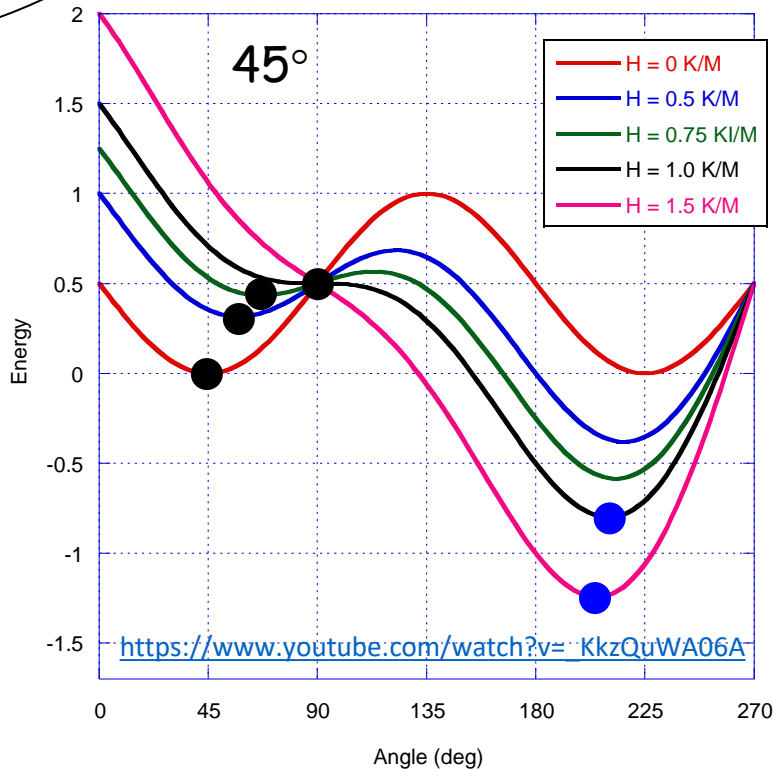
# Stoner-Wohlfarth-Model

Up to know: H along easy or hard direction, now arbitrary directions for Stoner-Wohlfarth-Model



$$E = -MH \cos \theta + K \sin^2 (\theta - \theta_0)$$

Field at an angle with  
the anisotropy axis  $\theta_0$



minima moves from  $45^\circ$  half way towards hard axis ( $90^\circ$ )

Particle switches at  $K_U/M$ , not at  $2K_U/M$

Easiest way to switch a SW-particle is at  $45^\circ$

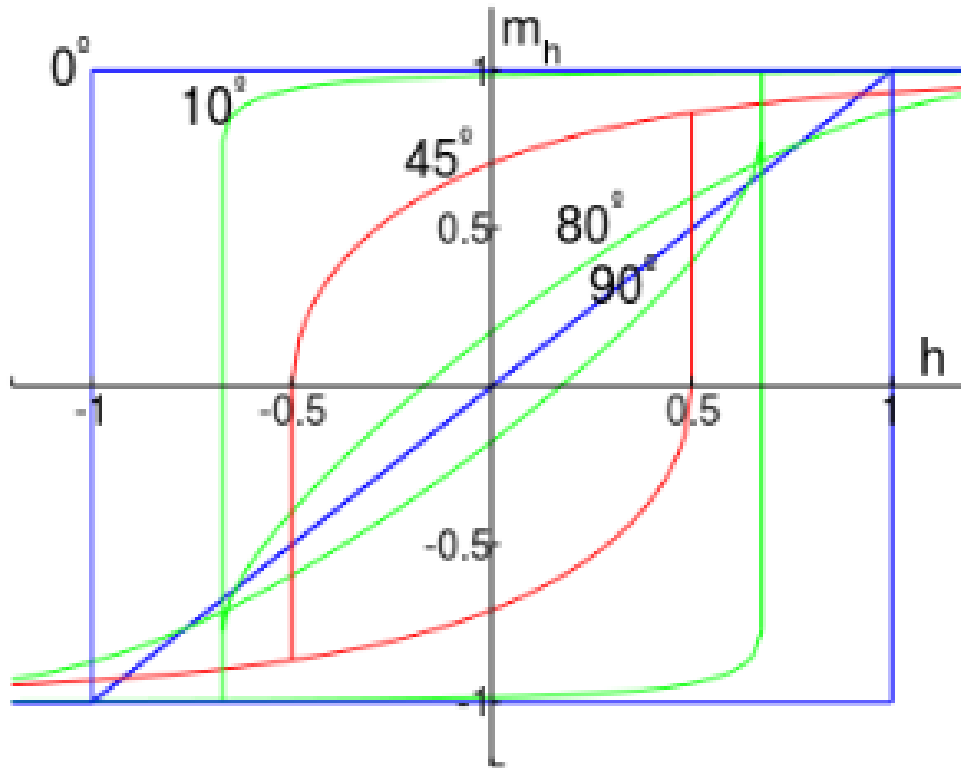


Figure 3. Some hysteresis loops predicted by the Stoner-Wohlfarth model for different angles between the field and easy axis.

<https://www.youtube.com/watch?v=KkzQuWA06A>

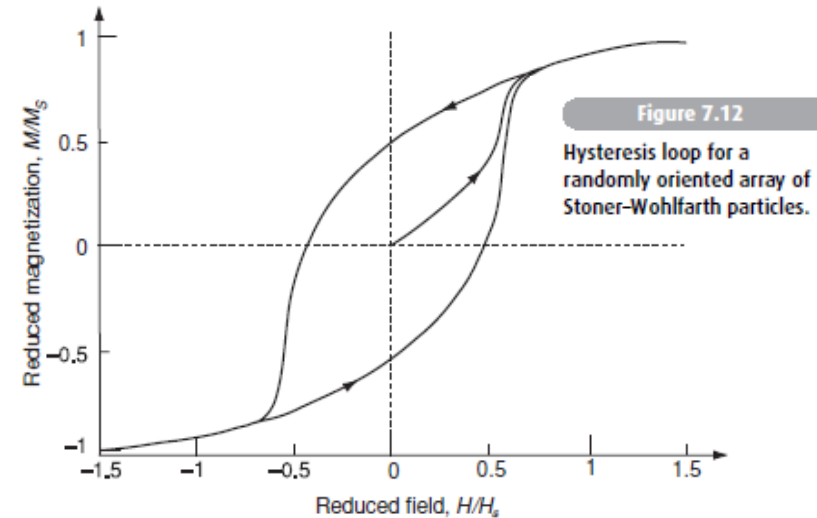
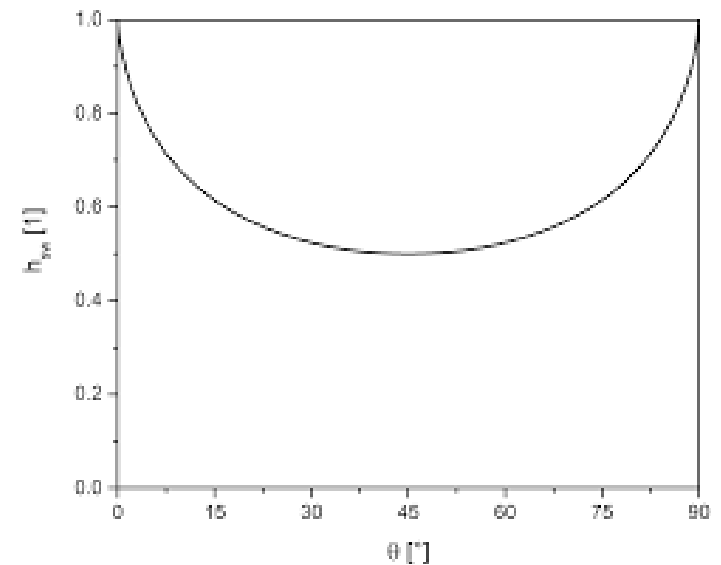
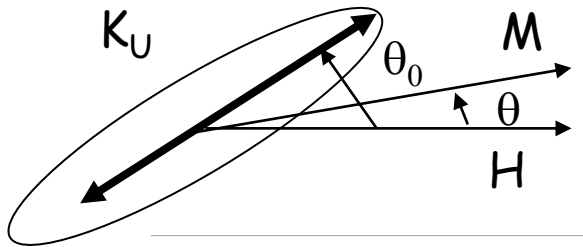


Figure 7.12  
Hysteresis loop for a randomly oriented array of Stoner-Wohlfarth particles.



Reversal movies for 0,15,30,45,60,75, 85 and 90 degrees

# Stoner-Wohlfarth reversal movies for various angles plot single particle illustration, hysteresis loop, energy landscape



An energy landscape with metastable minima gives rise to remanence and coercivity.

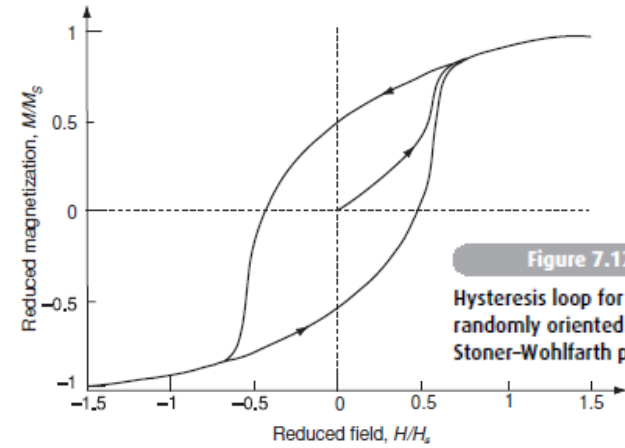
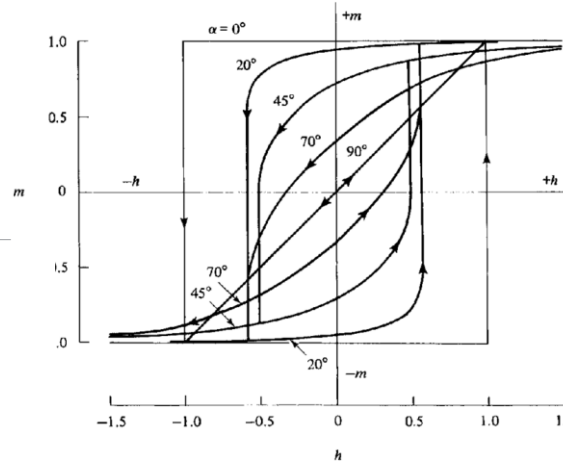
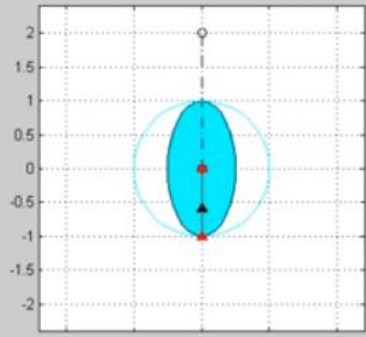
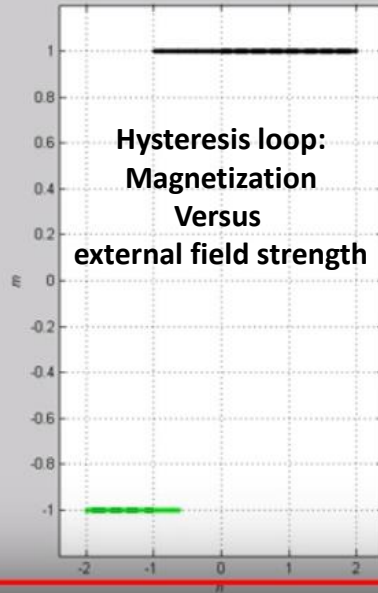


Figure 7.12  
Hysteresis loop for a randomly oriented array of Stoner-Wohlfarth particles.

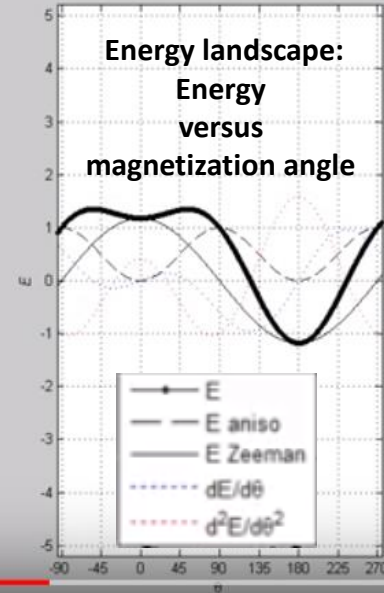
Single particle illustration  
M and H vector illustrations



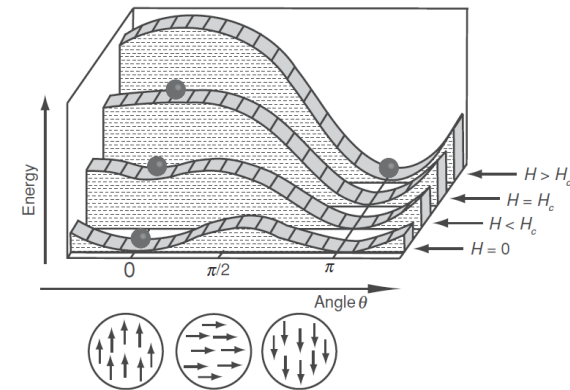
Fixed external field angle  
Sweep external field strength



Hysteresis loop:  
Magnetization  
Versus  
external field strength



Energy landscape:  
Energy  
versus  
magnetization angle



## Ferromagnetische (Funktionale) Materialien

- Einordnung und Einleitung
- Energien und Energiedichten einer ferromagnetischen Probe
  - Austauschwechselwirkung
  - Streufeld- oder Demagnetisierungsenergie, Formanisotropie
  - Anisotropie (außer Formanisotropie = Demagnetisierungsenergiedichte)
  - Zeemann Energie, äußeres Feld
- Wechselseitige Konkurrenz verschiedener magnetischer Energieterme
- Hysterese-Effekte, Stoner-Wohlfarth Modell, Basis für binäre magn. Datenspeicher
- **Magnetische Funktionsmaterialien zur Datenspeicherung**
  - **Entwicklung der Festplatte: Von magnetischen Mikrosystemen zu Nanosystemen**
  - **GMR (Riesenmagnetwiderstand) und TMR Effekte für empfindlichere Leseköpfe**
  - **Zukünftige Festplattentechnologien**
  - **Neue Effekte in der Nanowelt: Spin transfer torque in Nanokontakten**
  - **Separation von Ladungs und Spinströmen: Spin orbit torque in Dünnschichtsystemen**
  - **Anwendungen im Magnetic Random Access Memory (MRAM)**
  - **Die Spinwelle als Informationsträger (HZDR-movie)**