Harmonic Oscillator Interacting with Random Ising Spins:
A Hybrid Model for Dynamical Systems with Complex Hysteresis

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Abstract. We aim at an understanding of the dynamical properties of a periodically driven damped harmonic oscillator coupled to a system showing complex hysteresis, such as a random field Ising model at zero temperature. In this work we investigate simplified models, the harmonic oscillator coupled to independent or pairwise coupled spins in quenched random local fields. Because of the appearance of discrete and smooth degrees of freedom, the model can be characterized as a hybrid system. By applying established methods of dynamical systems and piecewise-smooth system theory we show, how chaos emerges. Furthermore we investigate the dynamical behavior of the system for an increasing number of spins, especially as the thermodynamic limit is approached.

Introduction

Hysteresis phenomena can be found in very different research fields, such as magnetic materials, porous materials, shape memory alloys or superconducting systems. The two most prominent models of hysteresis are the purely phenomenological Preisach model [1] and the more physical zero-temperature Random Field Ising Model (RFIM), initially established to study the effects of disorder on phase transitions [2]. While the hysteretic behavior of Preisach-type models has been investigated and applied to a variety of topics, not much is known about scenarios which arise if a hysteretic system is coupled dynamically to its environment. Some work has been done on ordinary differential equations with Preisach nonlinearity, e. g. in the context of ferroresonance phenomena [3], or on the dynamics of an iron pendulum in a magnetic field [4]. In this work we replace in the latter system the phenomenological Preisach model by a more physical model – an ensemble of Ising spins interacting with an external magnetic field. To understand the behavior of this hybrid system, we first treat independent spins and investigate the dynamics of the system especially as the number of discrete elements (spins) goes to infinity (thermodynamic limit), and secondly we introduce hysteretic behavior by a pairwise coupling of the spins, which results in an ensemble of 2-spin 'molecules'.

Results

Model

A schematic illustration of the model can be seen in Figure 1. An iron mass is coupled to a periodically driven damped harmonic oscillator in an external magnetic field, which changes linearly in dependence on the position of the iron mass. By modeling the ferromagnetic iron mass as an ensemble of Ising spins in quenched random local fields, there exists a feedback mechanism between the oscillator and the RFIM, which is illustrated in Figure 2. Because of the smooth degrees of freedom of the oscillator and the discrete number of two-state

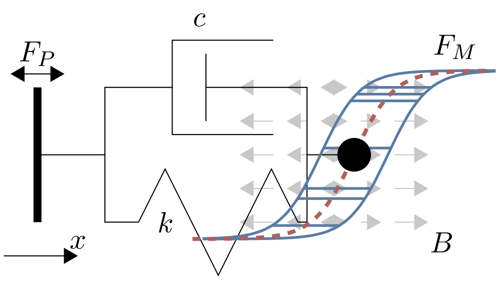

Figure 1: Schematic illustration of a periodically driven damped harmonic oscillator with an iron mass. Hysteresis appears in the interaction between the magnetized iron mass and the surrounding external magnetic field.

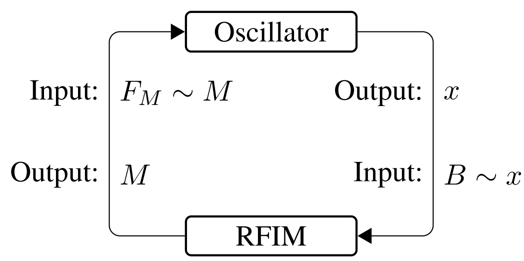

Figure 2: The feedback mechanism of the RFIM coupled to a harmonic oscillator: The actual position x of the oscillator acts as an external field B (input) of the RFIM. Therefore the magnetization M (output) changes with x. The magnetization then acts again on the oscillator as an additional external force.

Ising spins in the RFIM the system can be classified as a hybrid system. By investigating the dynamics of this system we use established methods of piecewise-smooth system theory, such as the *discontinuity mapping*, which describes transversal and grazing intersections with phase space boundaries, especially when calculating Lyapunov exponents and observing bifurcation scenarios [5].

Thermodynamic limit

In the thermodynamic limit we are able to formulate the exact equations of motion which contains an effective nonlinearity. For independent spins the nonlinearity is simply an error function (see dashed line in Figure 1), and for a special case of pairwise coupled spins the nonlinearity is described by a hysteretic play operator (see solid line in Figure 1). This means, the hybrid character of the piecewise-smooth system should vanish in the thermodynamic limit.

Simulations

We simulate the ensemble of independent and pairwise interacting spins for different realization of the local disorder. For a large number of spins the resulting magnetization curves match the analytically calculated error function and the hysteretic play operator, respectively. Furthermore, for an increasing number of independent spins we numerically calculate for each disorder realization the box counting and the Kaplan-Yorke dimension of a typical chaotic attractor of the system. To compare this with the system in its thermodynamic limit, we also determine the fractal dimensions for the smooth system numerically. For independent spins we show in Figures 3 and 4 how for an increasing number of spins the relative variance of the fractal dimensions obtained from different disorder realizations vanishes and how their mean values converge to the values of the fractal dimensions in the thermodynamic limit. This means that both fractal dimensions are self-averaging quantities

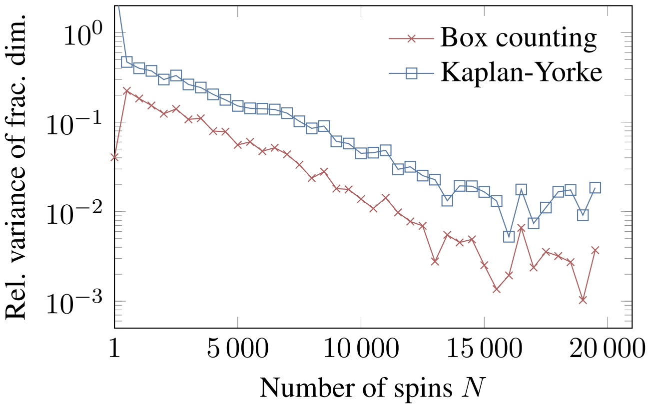

Figure 3: The relative variance of the box counting (—×—) and Kaplan-Yorke (—□—) dimension in dependency on the number of spins obtained from 512 realizations of the random fields. The system shows self-averaging with respect to these dynamical properties.

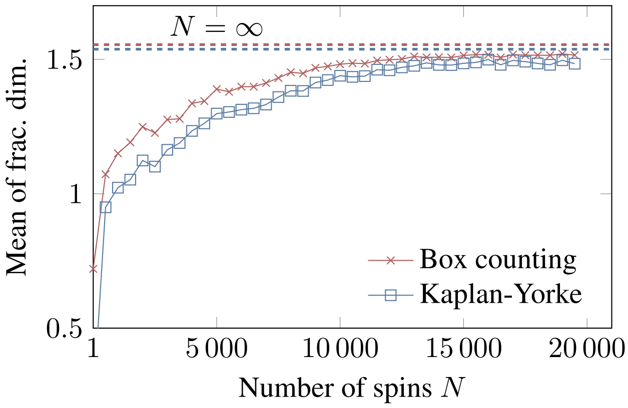

Figure 4: The box counting (—×—) and Kaplan-Yorke (—□—) dimensions of the hybrid system converge to their values in the thermodynamic limit (- - - -,- - - -).

and therefore can be calculated from one typical disorder realization of a large system. Similar results were obtained for pairwise coupled spins, although convergence to the thermodynamic limit slows down considerably from exponential to power law behaviour. Besides fractal dimensions we compare the bifurcation scenarios of the smooth system in its thermodynamic limit with those for a small and a very large number of spins. For the system with few spins chaos emerges by the occurrence of grazing bifurcations, which are typical for piecewise-smooth systems, whereas in the system with many spins these bifurcations vanish and the bifurcation diagrams become similar to those of the smooth system. This indicates the disappearance of the discontinuities in the case of the number of spins approaching infinity. To understand the interplay between an increasing number of spins with an simultaneously increasing smoothness of the system, we also analytically showed, that the largest short-time Lyapunov exponent for the piecewise-smooth system with infinitely many spins can be greater than zero for a short segment on the attractor. This explains, why the system can still produce chaos, even though the discontinuities of the system disappear in the thermodynamic limit and the hybrid character vanishes.

Conclusions

In this work we showed, that a periodically driven damped harmonic oscillator coupled to independent and pairwise coupled Ising spins can be treated using techniques of piecewise-smooth system theory. We found, that for independent and pairwise interacting spins the approach to the thermodynamic limit results in an asymptotic nonlinearity in form of a error function or as a hysteretic play operator. For an increasing number of spins we also found, that the fractal dimensions of the chaotic attractors of the piecewise-smooth system approach those of the system in the thermodynamic limit.

References

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