

Mechanisms of Carrier Collisions and Scattering



phonon scattering and ionized impurity scattering.

τ_{mn} and τ_{mp} can vary significantly with **temperature** and the **doping concentration**.

Crystal vibrations distort the periodic crystal structure and thus scatter the electron waves. Instead of electron waves and vibration waves, it is more intuitive to think of **electron particles scattering off phonon particles**.

The mobility due to phonon scattering alone:

$$\mu_{\text{phonon}} = q\tau_{\text{ph}}/m$$

τ_{ph} is the mean free time of phonon scattering.

But, what determines the phonon scattering mean free time?

The **mean free time of phonon scattering** is

inversely proportional to the phonon density and the electron speed, which is basically the thermal velocity.

the **phonon density** is known to be **proportional to the absolute temperature, T** .

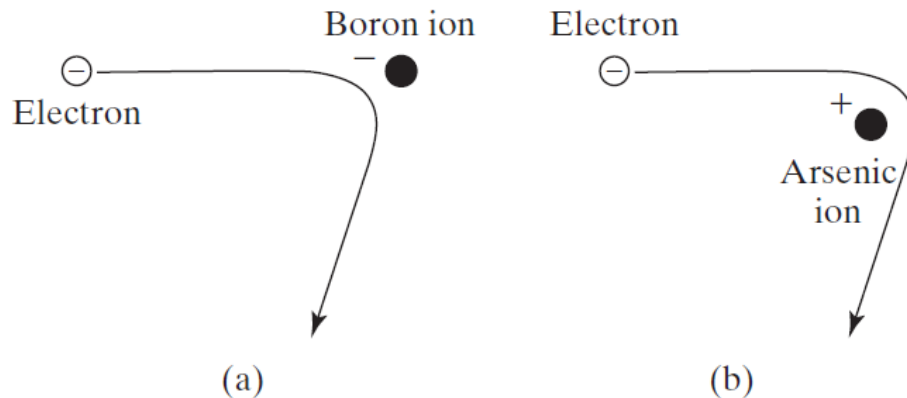
$$\mu_{\text{phonon}} \propto \tau_{\text{ph}}$$

$$\propto \frac{1}{\text{phonon density} \times \text{carrier thermal velocity}} \propto \frac{1}{T \cdot T^{1/2}} \propto T^{-3/2}$$

What about the **impurity scattering mobility**?

The dopant ions are fixed charges in the semiconductor crystal.

They can make electrons and holes change the direction of motion through the coulombic force.



An electron can be scattered by an acceptor ion (a) and a donor ion (b) in a similar manner, even though the ions carry opposite types of charge. The same is true for a hole.

The **mobility due to impurity scattering** is

inversely proportional to the sum of the donor and acceptor ion concentrations.

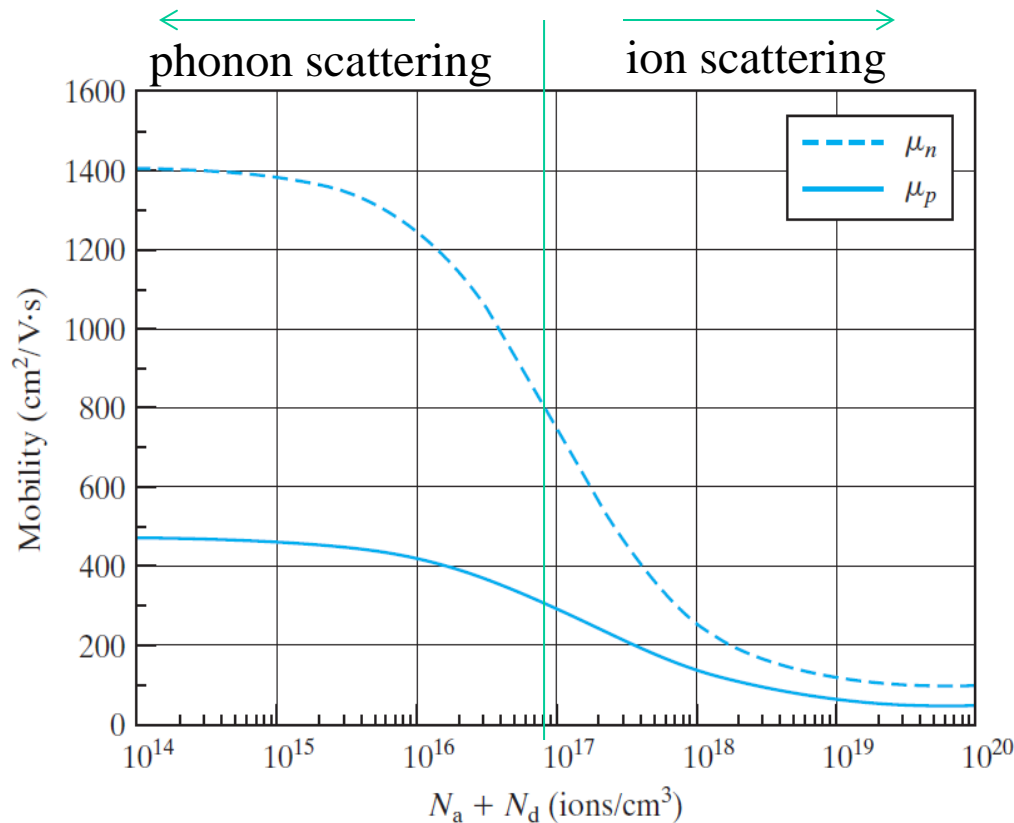
proportional to $T^{3/2}$.

Why is the mobility higher, i.e., the scattering weaker, at a higher T ?

At a higher temperature, the electron has a higher thermal velocity and flies by the ion in a shorter time, and its direction of motion is thus less affected by the ion.

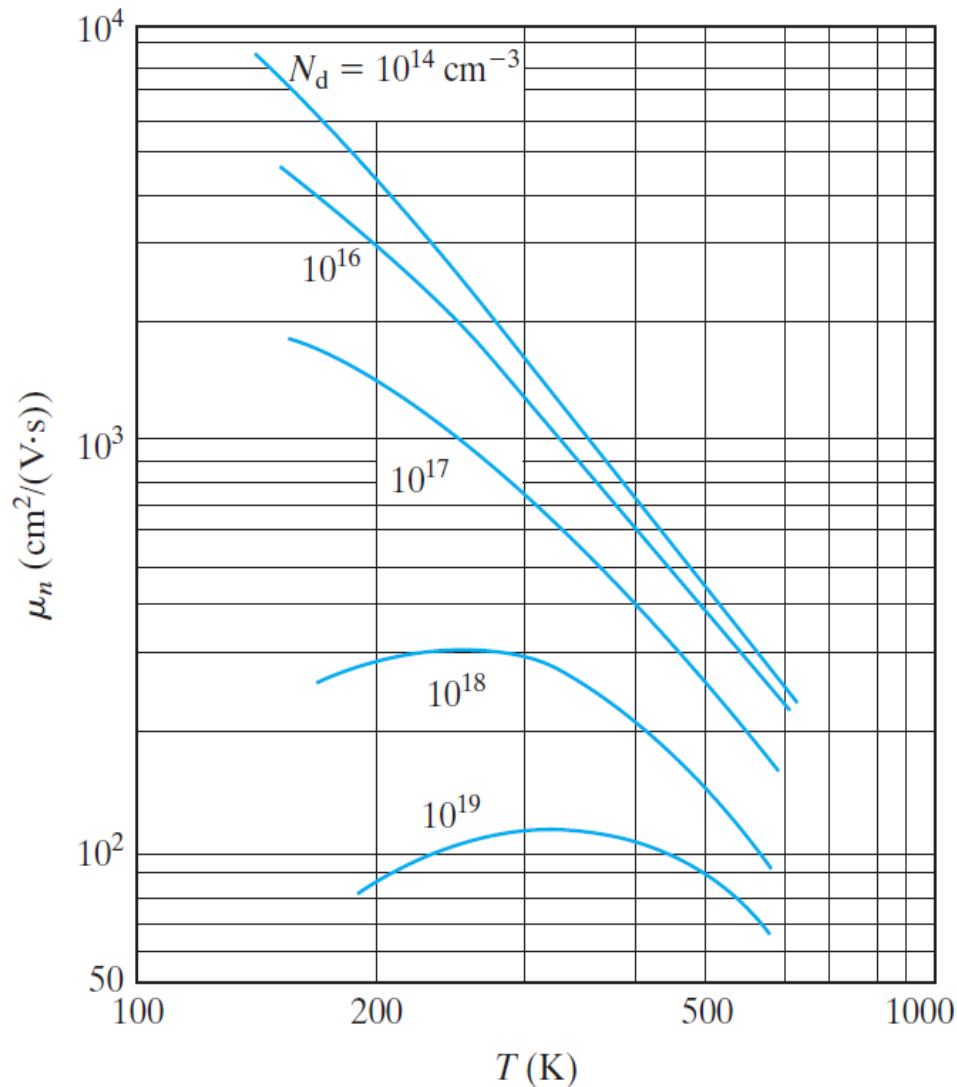
$$\mu_{\text{impurity}} \propto \frac{T^{3/2}}{N_a + N_d}$$

The graph shows the silicon electron and hole mobilities at 300 K.



- ✓ **Hole mobility is about one-third of the electron mobility.** Part of this difference in mobility can be explained by the **difference in the effective mass**. The rest is attributable to the **difference in the scattering mean free time**.
- ✓ At low dopant concentration, the electron mobility is dominated by phonon scattering;
- ✓ At high dopant concentration, it is dominated by impurity ion scattering.

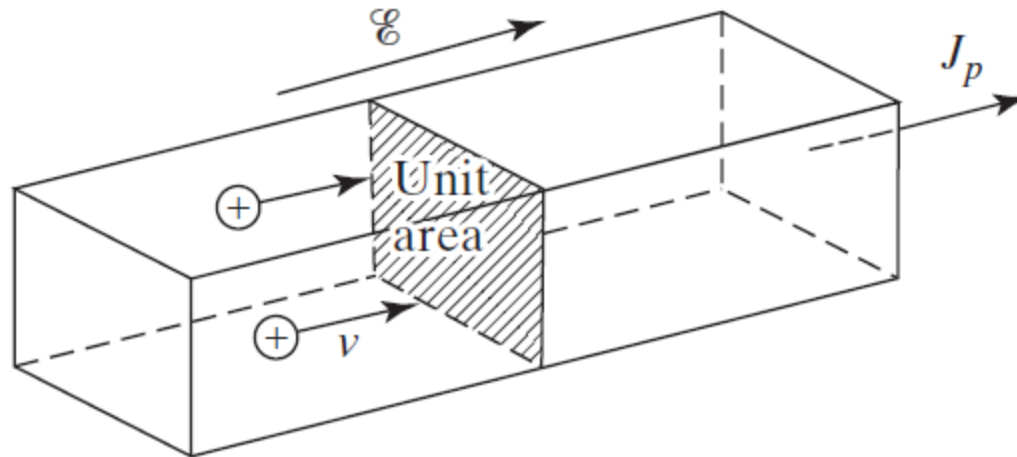
The graph shows the temperature dependence of the electron mobility in Si.



- ✓ At small dopant concentrations, μ decreases with increasing T , indicative of the dominance of phonon scattering.
- ✓ At very high dopant concentration and low temperature, μ increases with increasing T : impurity scattering dominates.

Drift Current and Conductivity

Let us turn our attention to the current that flows in a semiconductor as a result of carrier drift. The **current density**, J , is the **charge per second crossing a unit area plane normal to the direction of current flow**.

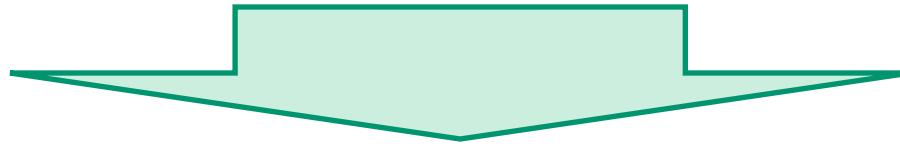


P-type semiconductor

$$J_{p,\text{drift}} = qp v = qp \mu_p \mathcal{E}$$

N-type semiconductor

$$J_{n,\text{drift}} = -qn v = qn \mu_n \mathcal{E}$$



$$J_{\text{drift}} = J_{n,\text{drift}} + J_{p,\text{drift}} = (qn \mu_n + qp \mu_p) \mathcal{E}$$

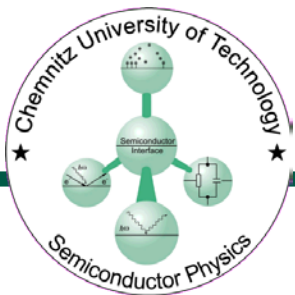
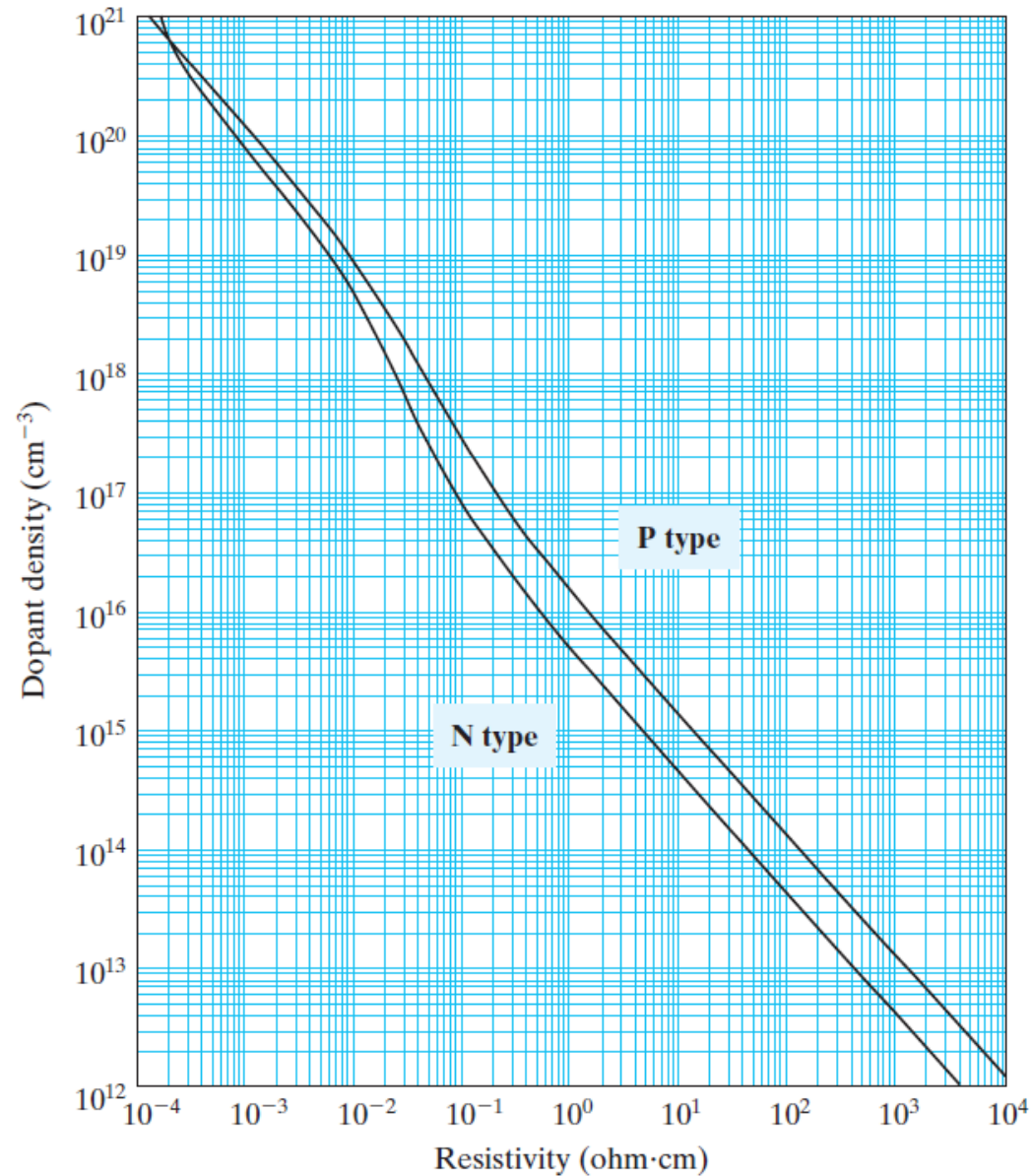
The quantity in the parentheses is the
conductivity, σ , of the semiconductor

$$\sigma = qn\mu_n + qp\mu_p$$

Notes

- Usually **only one of the components is significant** because of the large ratio between the majority and minority carrier densities.
- The **resistivity, ρ** , is the **reciprocal** of the **conductivity**.
- The **standard units** of σ and ρ are A/V·cm (or S/cm, S being siemens) and $\Omega\cdot\text{cm}$, respectively.

In the graph ρ is shown as a function of the dopant density for silicon at room temperature





Summary

10th lecture

Charge carriers move due to:

Temperature: random, no current just noise

In the presence of an **electric field**, charge carriers gain a drift velocity and produce a drift current density proportional to the applied Electric field E :

$$v_p = \mu_p \mathcal{E}$$

$$v_n = -\mu_n \mathcal{E}$$

$$J_{p,\text{drift}} = qp\mu_p \mathcal{E}$$

$$J_{n,\text{drift}} = qn\mu_n \mathcal{E}$$

μ_p and μ_n are called the hole and electron mobility.

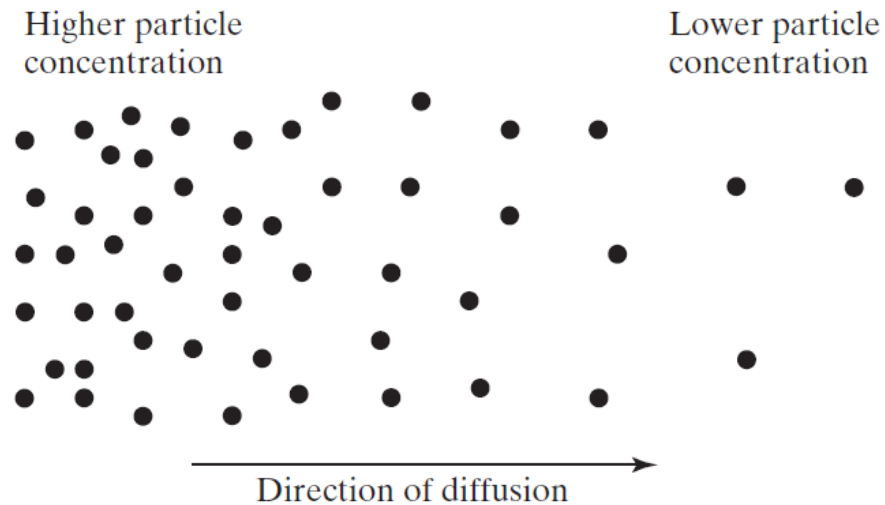
They are determined by how frequently the carriers collide with phonons or dopant ions and lose their drift momentum.

Mobilities are functions of **temperature** and the **total dopant concentration**.

Measured mobility data are routinely presented in graphs.

DIFFUSION CURRENT

In addition to the drift current, there is a second component of current called the **diffusion current**.



It is the familiar process by which particles move from a point of higher particle density toward a point of lower density.

The low conductivity and the ease of creating nonuniform carrier densities make diffusion an important process in semiconductors. (not an important consideration in metals because of their high conductivities)

If the charge carrier concentration is not uniform, there will be an **electron diffusion current**, which is proportional to the gradient of the electron concentration.

The rate of particle movement by diffusion is proportional to the concentration gradient.

electrons

$$J_{n,\text{diffusion}} \propto \frac{dn}{dx}$$

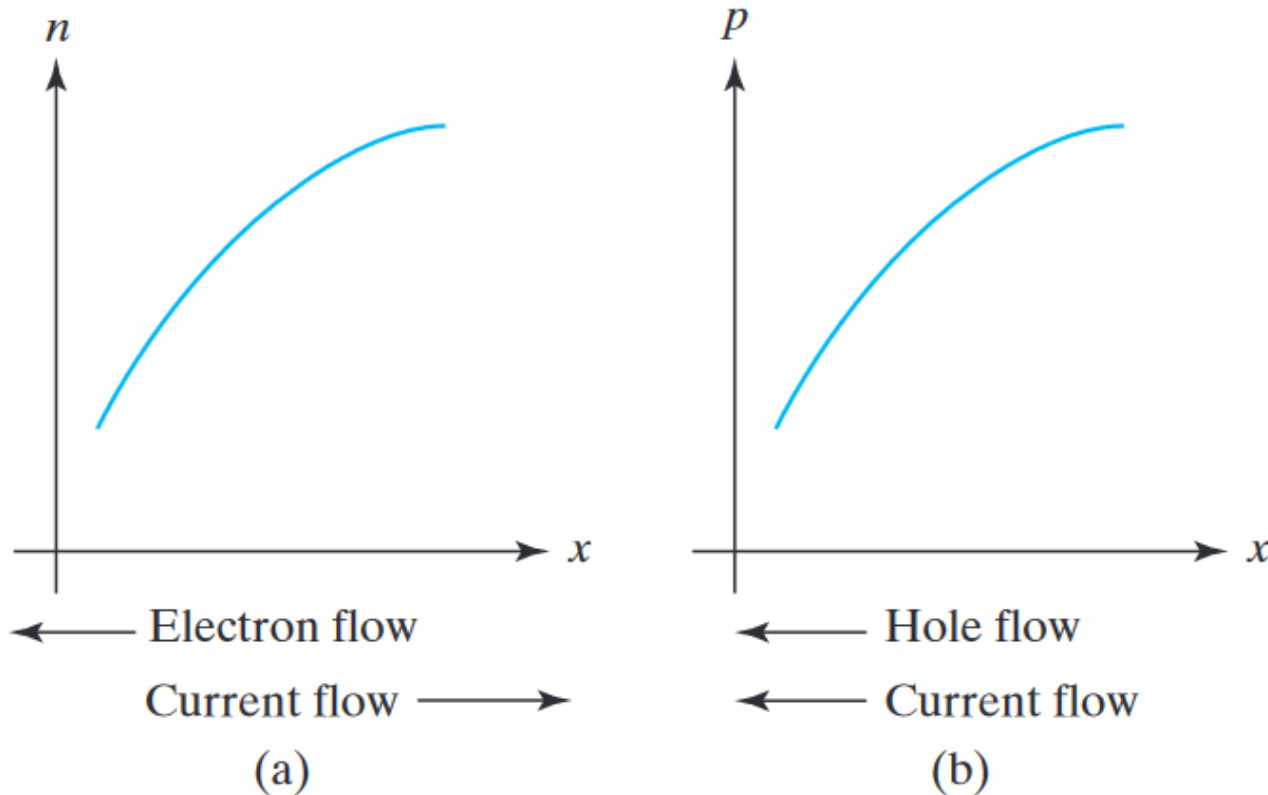
$$J_{n,\text{diffusion}} = qD_n \frac{dn}{dx}$$

holes

$$J_{p,\text{diffusion}} = -qD_p \frac{dp}{dx}$$

q is the elementary charge ($+1.6 \times 10^{-19}$ C), and $D_{n,p}$ is called the charge carrier **diffusion constant**.

The larger the $D_{n,p}$ the faster the charge carriers diffuse.

Note

- ✓ A positive slope of carrier concentration produces a positive electron diffusion current (a), but a negative hole diffusion current (b).
- ✓ In (a), electrons diffuse to the left (toward the lower concentration point). Because electrons carry negative charge, the diffusion current flows to the right.
- ✓ In (b), holes diffuse to the left, too. Because holes are positively charged, the hole current flows to the left, i.e., the current is negative.

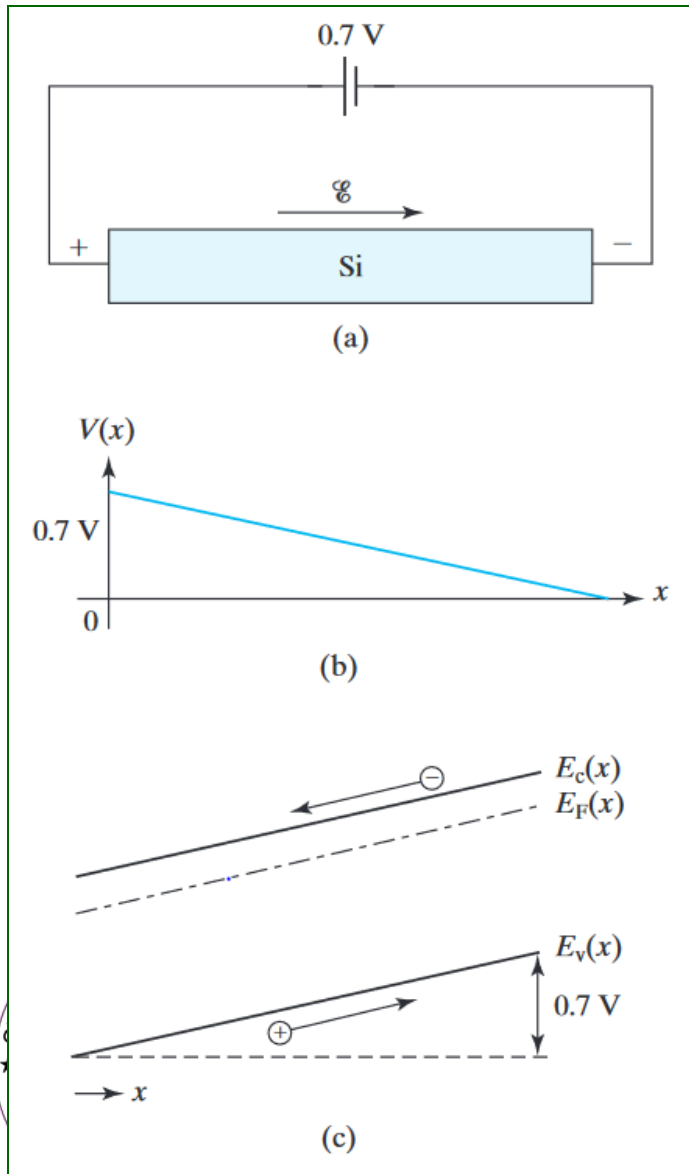
Both **drift** and **diffusion** of **electrons** and **holes** contribute to the current and add up:

$$J_n = J_{n,\text{drift}} + J_{n,\text{diffusion}} = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$J_p = J_{p,\text{drift}} + J_{p,\text{diffusion}} = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}$$

$$J = J_n + J_p$$

RELATION BETWEEN THE ENERGY DIAGRAM AND V, E



(a) When a voltage is applied across a piece of semiconductor as shown in the figure, it alters the band diagram.

By definition, a **positive voltage raises the potential energy of a positive charge and lowers the energy of a negative charge**. It therefore lowers the energy diagrams since the energy diagram plots the energy of an electron (a negative charge).

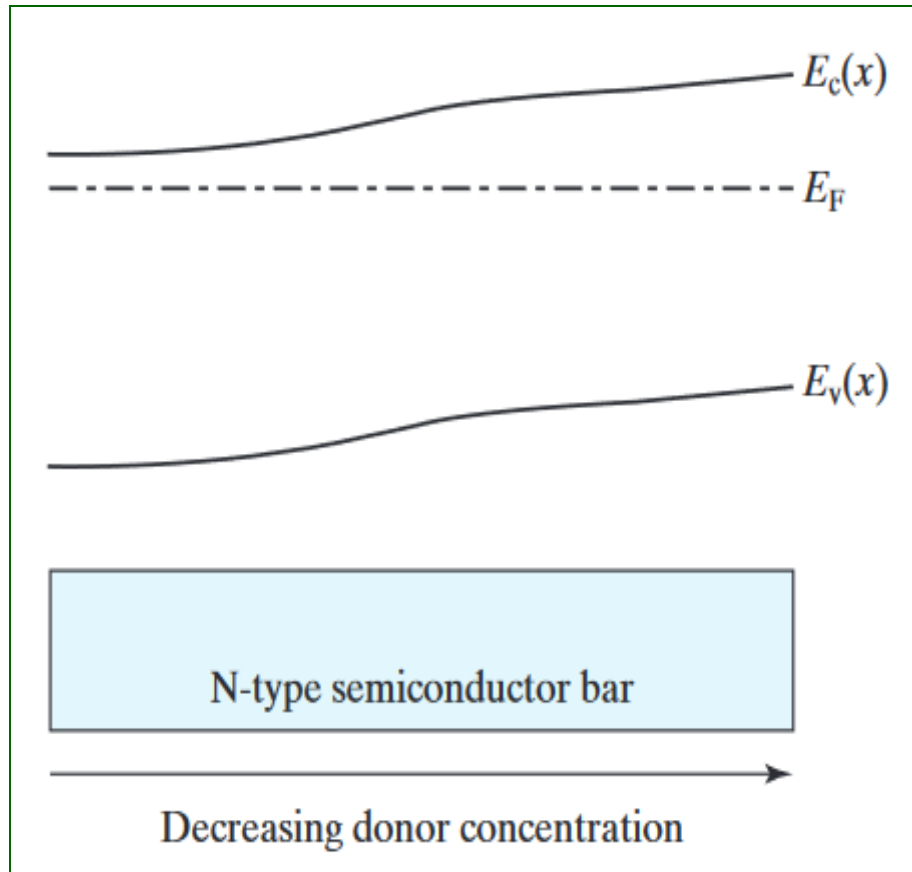
$$\mathcal{E}(x) \equiv -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx}$$

(c) shows that the energy diagram is lower (at the left) where the voltage is higher. The band diagram is higher where the voltage is lower.

Notes

- E_c and E_v vary in the opposite direction from the voltage.
- E_c and E_v are higher where the voltage is lower.
- E_c and E_v are always separated by a constant, E_g .

$$E_c(x) = \text{constant} - qV(x)$$

RELATION BETWEEN D and μ 

- N-type semiconductor in which the dopant density decreases towards the right.
- The semiconductor is at equilibrium, and therefore the Fermi level E_F is constant.
- The left side is more heavily doped than the right side, and so E_F is closer to E_c on the left side.

Because E_c is not a constant, there is an electric field equal to

$$\mathcal{E}(x) \equiv -\frac{dV}{dx} = \frac{1}{q} \frac{dE_c}{dx} = \frac{1}{q} \frac{dE_v}{dx}$$

This field is internally created and is as real as a field created by an external voltage. Because the semiconductor is at equilibrium, there cannot be any J_n (or J_p):

$$J_n = 0 = qn\mu_n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$0 = qn\mu_n\mathcal{E} - qn \frac{qD_n}{kT} \mathcal{E}$$

electrons

$$D_n = \frac{kT}{q} \mu_n$$

holes

$$D_p = \frac{kT}{q} \mu_p$$

\therefore

Einstein relationships

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$\begin{aligned} \frac{dn}{dx} &= \frac{-N_c}{kT} e^{-(E_c - E_F)/kT} \frac{dE_c}{dx} \\ &= \frac{-n}{kT} \frac{dE_c}{dx} \\ &= \frac{-n}{kT} q \mathcal{E} \end{aligned}$$

- The electron and hole concentrations introduced some lectures ago when we talked about **the np product**:

$$n = N_d \quad \text{and} \quad p = n_i^2 / N_d \quad \text{for an N-type sample}$$

are the equilibrium carrier concentrations. They will be denoted with **n_0** and **p_0** from now on.

- The electron and hole concentrations can become different from n_0 and p_0 , for example, when light shines on the sample and generates electrons and holes.
- The n' and p' are known as the **excess carrier concentrations** denoted by



$$n \equiv n_0 + n'$$

$$p \equiv p_0 + p'$$

If n' and p' are created by light, are **equal** because the electrons and holes are created in pairs.

If n' and p' are introduced by other means, they will still be equal to each other because of charge neutrality : $n + N_a = p + N_d$

Since **charge neutrality** is satisfied at equilibrium when $n' = p' = 0$, any time a non-zero n' is present, an equal p' must be present to maintain the charge neutrality, and vice versa. Otherwise, the net charge will attract or repel the abundant majority carriers until neutrality is restored:

$$n' \equiv p'$$

If the light is suddenly turned off, $\mathbf{n'}$ and $\mathbf{p'}$ will decay with time until they become zero and \mathbf{n} and \mathbf{p} return to their equilibrium values, $\mathbf{n_0}$ and $\mathbf{p_0}$.

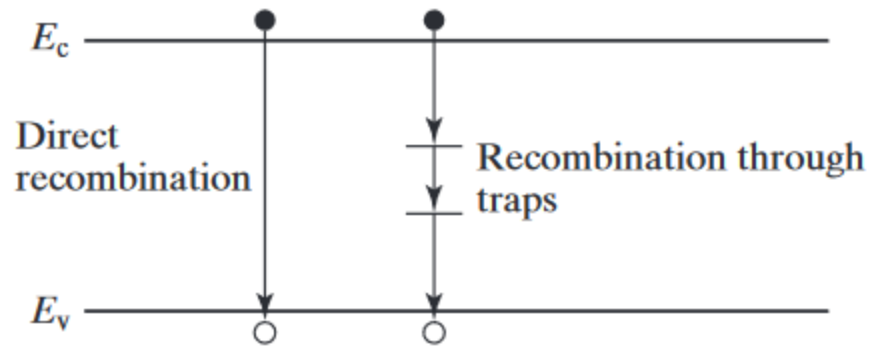
The **process of decay** is **recombination**: an electron and a hole recombine and annihilate each other. The time constant of the decay is called the **recombination time** or **carrier lifetime**, τ .

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau}$$

Notes

- The recombination rate (per cubic centimeter per second) is proportional to $\mathbf{n'}$ and $\mathbf{p'}$.
- τ has the dimension of time and is typically around **1 μ s in Si**.

It may range from 1 ns to 1 ms, depending on the **density of trace metal impurities** such as Au and Pt, which form **traps** in the band gap with several energy levels deep in the band gap.



An electron–hole pair recombines when an electron drops from the conduction band into the valence band.

These deep traps **can capture electrons or holes** to facilitate recombination as shown in the figure. They **shorten the recombination time**; they are also called **recombination centers**.

Too short τ is bad for device leakage current, and hence extreme cleanliness is maintained in the semiconductor fabrication plants partly to avoid these metallic contaminants.

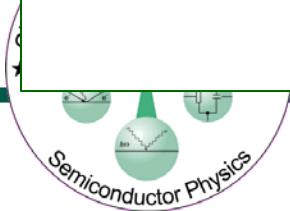
- The **other recombination process** shown in the previous figure - **direct recombination** or **radiative recombination** - is very **inefficient and unimportant in silicon** because the electrons and holes at the edges of the band gap do not have the same wave vectors. These types of semiconductors are called **indirect gap semiconductors**. The lifetime is highly variable and determined by the density of recombination centers.
- The **radiative recombination** process is **very efficient in direct gap semiconductors** such as **GaAs** because the electrons and holes have the same wave vectors and is responsible for light emission in light-emitting diodes and lasers.



Thermal generation is the reverse process of recombination.

- At any nonzero temperature, electron–hole pairs are constantly being generated and lost (by recombination).
- If $n' = p' = 0$, the rate of recombination equals the rate of generation and the net rate of change is zero.
- If is positive, there is a net recombination rate as shown in $\frac{n'}{\tau} = \frac{p'}{\tau}$
- If is negative, i.e., there are fewer electrons than the equilibrium concentration, nature sees to it that there is a net rate of thermal generation rather than recombination. Which is predicted by equating a positive variation in time:

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau}$$



(thermal gen. cont.)

The np product revisited

When $np = ni^2$, the rate of thermal generation is equal to the rate of recombination. Under this condition, n and p are said to be at **thermal equilibrium**.

When $np > ni^2$, there is **net recombination**;

When $np < ni^2$, there is **net thermal generation**.

QUASI-EQUILIBRIUM AND QUASI-FERMI LEVELS

Whenever $np \neq n_i^2$ the semiconductor is not at equilibrium.

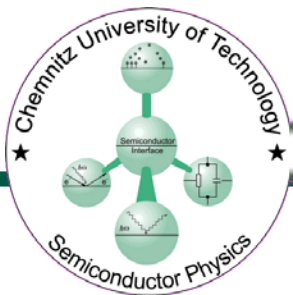


electrons and holes are not at equilibrium with each other.

Nonetheless, we would like to preserve and use, as much as possible, the following equilibrium relationships, which are very useful:

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT}$$



The issue is that the previous equations, when multiplied together, lead to

$$np = ni^2$$

We saw in a previous lecture that the presence of **excess carriers** can easily make

$$np \gg ni^2$$

How do we handle this?

Quasi-Fermi Levels E_{Fn} and E_{Fp}

Previous relationships re-written:

$$n = N_c e^{-(E_c - E_{Fn})/kT}$$

$$p = N_v e^{-(E_{Fp} - E_v)/kT}$$

In which E_{Fn} and E_{Fp} are the electron and hole quasi-Fermi levels.

When electrons and holes are at equilibrium, i.e., when $\mathbf{np} = \mathbf{ni}^2$

E_{Fn} and E_{Fp} coincide and this is known as E_F

Otherwise, $E_{Fn} \neq E_{fp}$.

Notes

- The previous equations indicate that even when electrons and holes, as two groups, are not at equilibrium with each other, each one of them can still be at **equilibrium among themselves**.
- Electrons and holes, as two groups of particles, **can easily be away from equilibrium** because they are only **loosely coupled by the recombination/generation mechanism**, which is a slow process (has a long time constant around $1\ \mu\text{s}$).
- In contrast, the electrons (or holes) are **strongly coupled** among themselves by **exchanging positions and energy through thermal motion at high speed** and by **scattering with very short mean free times** ($\sim 0.1\ \text{ps}$).

Next Lecture

Spectroscopies for characterisation of Semiconductor properties

