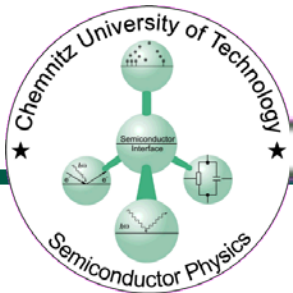


ELECTRON AND HOLE CONCENTRATIONS

What are the **hole concentration** and the **electron concentration**?

What are the carrier concentrations in undoped and doped semiconductors?

These questions will be answered by relating the **Fermi level** and the **carrier concentrations**.



Derivation of n and p from $D(E)$ and $f(E)$

1. Concentration of Electrons in the conduction band

$$n = \int_{E_c}^{\text{Top of conduction band}} \underbrace{f(E)D_c(E)}_{\text{number of energy states between } E \text{ and } E + dE \text{ for each cubic centimeter}} dE$$

number of energy states between E and $E + dE$
for each cubic centimeter

a) set the upper limit of integration at infinity, acceptable because of the rapid fall of $f(E)$ with increasing E

b) replacing/using:

$$D_c(E) = \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3}, \quad E \geq E_c$$

$$D_v(E) = \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3}, \quad E \leq E_v$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c} e^{-(E - E_F)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{-(E_c - E_F)/kT} \int_0^{\infty} \sqrt{E - E_c} e^{-(E - E_c)/kT} d(E - E_c)$$

Introducing a new variable

$$x = (E - E_c)/kT$$

Reveals that the integral is a **gamma function** equal to:

$$\int_0^{\infty} \sqrt{x} e^{-x} dx = \sqrt{\pi}/2$$

$$n = N_c e^{-(E_c - E_F)/kT}$$

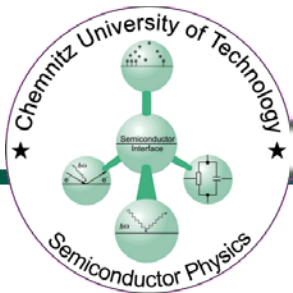
important equation!
should be memorized!

$$N_c \equiv 2 \left[\frac{2\pi m_n kT}{h^2} \right]^{3/2}$$

= effective density of states

Note

- ❖ N_c is called the **effective density of states**, as if all the energy states in the conduction band were *effectively* squeezed into a single energy level, E_c , which can hold N_c electrons (per cubic centimeter).
- ❖ As a result, the electron concentration n is simply the product of N_c and the probability that an energy state at E_c is occupied.



2. Concentration of Holes in the valence band

An expression for the hole concentration can be derived in the same way. The probability of an energy state being occupied by a hole is the probability of it not being occupied by an electron, i.e., $1 - f(E)$:

$$p = \int_{\text{Valence band bottom}}^{E_v} D_v(E)(1 - f(E)) dE$$

Performing the same replacements as before:

$$p = N_v e^{-(E_F - E_v)/kT}$$

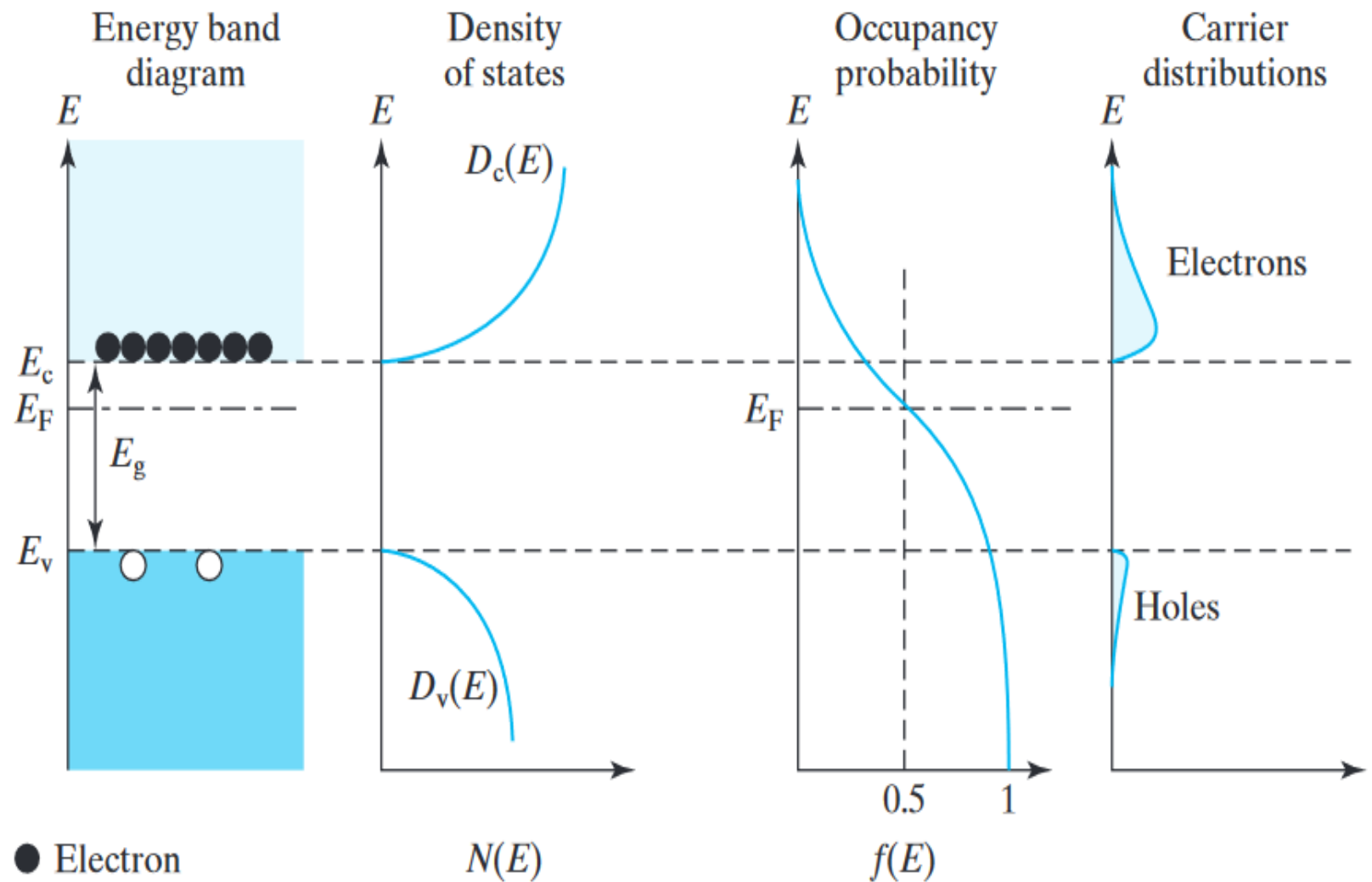
$$N_v \equiv 2 \left[\frac{2\pi m_p kT}{h^2} \right]^{3/2}$$

Note

- ❖ N_v is the **effective density of states of the valence band**
- ❖ The values of N_c and N_v , differ only because m_n and m_p are different.
- ❖ N_c and N_v vary from one semiconductor to another because of the variation in the effective masses, too.

Values of N_c and N_v for Ge, Si, and GaAs at 300 K.

	Ge	Si	GaAs
$N_c \text{ (cm}^{-3}\text{)}$	1.04×10^{19}	2.8×10^{19}	4.7×10^{17}
$N_v \text{ (cm}^{-3}\text{)}$	6.0×10^{18}	1.04×10^{19}	7.0×10^{18}



Schematic band diagram, density of states, Fermi–Dirac distribution, and carrier distributions versus energy.

The np product!

is a **constant** for a **given semiconductor** and **T** and **independent of the dopant concentrations**.

One may explain why the np product is a constant:

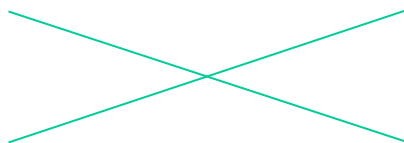
- The electron–hole **recombination rate** is **proportional** to the np product
- When $np = n_i^2$, the **recombination rate** equals the rate of **thermal generation** of electron–hole pairs.



The **np product** is an important relationship usually expressed in the following form:

$$n = N_c e^{-(E_c - E_F)/kT}$$

$$p = N_v e^{-(E_F - E_v)/kT}$$

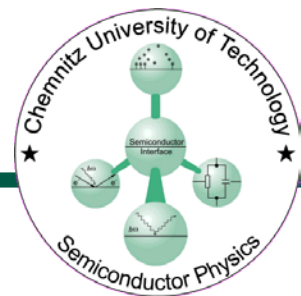


$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$



$$np = n_i^2$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$



The previous equations suggest that there are always some electrons and holes present—whether dopants are present or not.

In an **intrinsic semiconductor**, the nonzero n and p are the **results of thermal excitation**, which moves some electrons from the valence band into the conduction band.

Since such movements create electrons and holes in pairs, $n = p$ in intrinsic semiconductors.

$$n = p = n_i$$

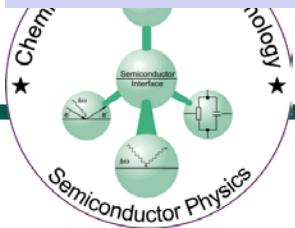
- ✓ n_i is called the **intrinsic carrier concentration**;
- ✓ n_i is a strong function of E_g and T but is independent of the dopant concentration.

Some values

n_i at room temperature is roughly 10^{10}cm^{-3} for Si and 10^7cm^{-3} for GaAs, which has a larger band gap than Si.



For Si, the np product is therefore 10^{20}cm^{-6} regardless of the conductivity type (P type or N type) and the dopant concentrations.



There are 4 types of charged species in a semiconductor: electrons, holes, positive donor ions, and negative acceptor ions. Their densities are represented by the symbols n , p , N_d , and N_a .

In general, all samples are free of net charge. **Charge Neutrality** requires that the densities of the negative particles and positive particles are equal:

$$n + N_a = p + N_d$$



$$np = n_i^2$$



$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2} \right)^2 + n_i^2 \right]^{1/2}$$



The previous relationships can be further simplified:

1. $N_d - N_a \gg n_i$ (i.e., N type),

$$\begin{aligned} n &= N_d - N_a \\ p &= n_i^2 / n \end{aligned}$$

If, furthermore, $N_d \gg N_a$, then

$$n = N_d \quad \text{and} \quad p = n_i^2 / N_d$$

2. $N_a - N_d \gg n_i$ (i.e., P type),

$$\begin{aligned} p &= N_a - N_d \\ n &= n_i^2 / p \end{aligned}$$

If, furthermore, $N_a \gg N_d$, then

$$p = N_a \quad \text{and} \quad n = n_i^2 / N_a$$



Conclusions and Consequences of previous relationships!

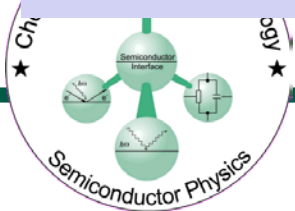
An acceptor/ a donor can effectively negate the effect of a donor/an acceptor:

Dopant Compensation.



We can start with P-type Si and convert a portion of it into N-type simply by adding enough donors.

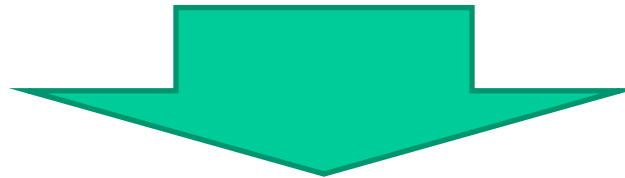
This is one of the techniques employed to make complex devices.



Carrier concentrations at extremely high and low temperatures

- Very high temperatures:

$$n_i \text{ is large} \implies n_i \gg |N_d - N_a| \implies n = p = n_i$$



The semiconductor becomes “intrinsic” !

- Very low temperatures:

E_F may rise above E_d , and most of the donor (or acceptor, in the case of P-type material) atoms can remain nonionized. This phenomenon is called **freeze-out**. In this case, if the doping is not heavy enough to form an impurity band the dopants are not totally ionized.

The carrier concentration may be significantly less than the dopant concentration. The exact analysis is complicated, but the result is,

$$n = \left[\frac{N_c N_d}{2} \right]^{1/2} e^{-(E_c - E_d)/2kT}$$

Application of the Freeze - Out

Case studies: Often it is desirable to detect or image the **black-body radiation** emitted by warm objects, e.g., to detect **tumors** (which restrict blood flow and produce cold spots), to identify inadequately **insulated building windows**, to **detect people and vehicles at night**, etc.

Requirements: This requires a photodetector that responds to photon energies around 0.1 eV. For this purpose, one can use a **semiconductor photoconductor with $E_g < 0.1$ eV**.

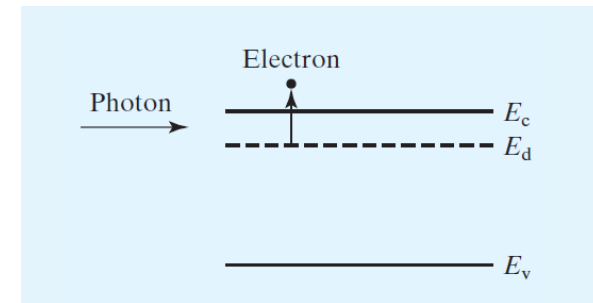
Option:

- use doped Si operating in the freeze-out mode;
- conduction electrons are created when the infrared photons provide the energy to ionize the donor atoms, which are otherwise frozen-out.
- the result is a lowering of the detector's electrical resistance, i.e., photoconductivity.
- at long enough wavelength or low enough photon energy $h\nu$, light will no longer be absorbed by the specimen.

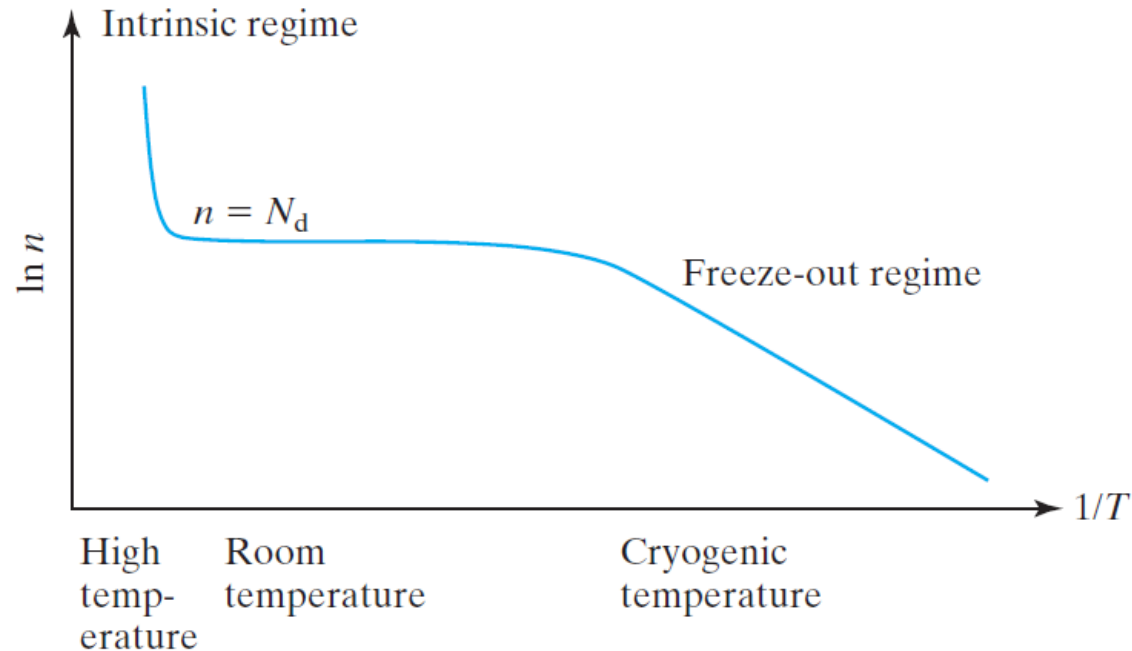
critical $h\nu$ corresponds to $E_c - E_d$



method for measuring the dopant ionization energy, $E_c - E_d$



The following figure summarizes the temperature dependence of majority carrier concentrations.



- The slope of the curve in the intrinsic regime is $E_g/2kT$
- The slope in the freeze-out portion is $(E_c - E_d)/2kT$, according to the previous equation.
- These facts may be used to determine E_g and $E_c - E_d$.

We are now ready to study how **electrons** and **holes**
move and **produce current!**



Motion and Recombination of Electrons and Holes



Even without an applied electric field, carriers are not at rest but possess finite kinetic energies. The average **kinetic energy** of electrons, $E - E_c$, can be calculated in the following manner:

$$\text{Average electron kinetic energy} = \frac{\text{total kinetic energy}}{\text{number of electrons}}$$

$$= \frac{\int f(E) D(E) (E - E_c) dE}{\int f(E) D(E) dE}$$

Integrating over the conduction band, and using the same approximations from lecture 7.

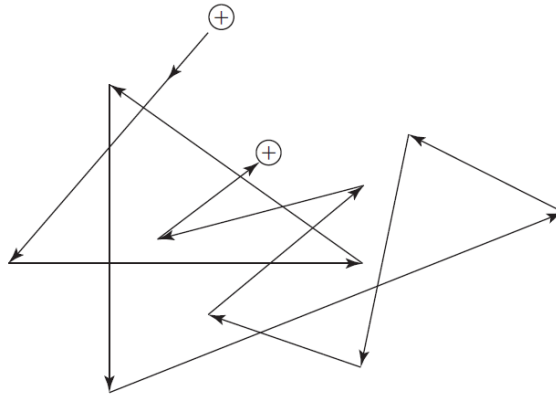
$$\text{Average kinetic energy} = \frac{3}{2} kT$$

Thermal velocity

$$v_{th} = \sqrt{\frac{3kT}{m}}$$

Notes

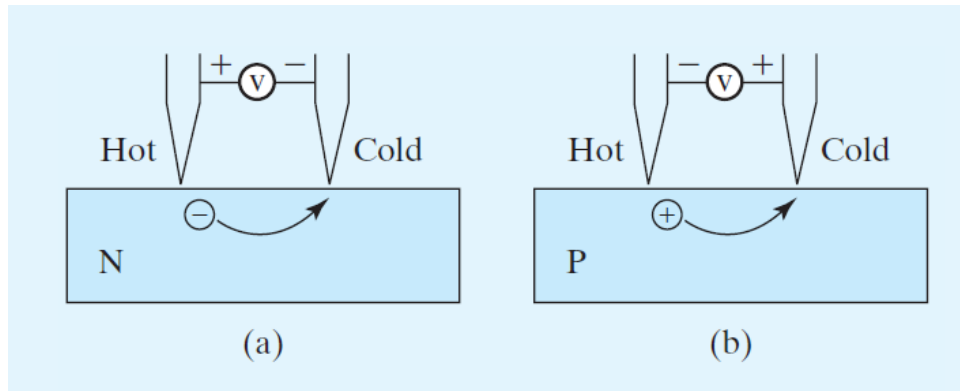
- Previous equations hold for both electrons and holes with E_c expressed as a function of m_n or m_p respectively.
- Electrons and holes move at the thermal velocity but not in a simple straight-line fashion. Their directions of motion change frequently due to **collisions** or **scattering**. The carriers move in a zigzag fashion:



- The **mean free time** between collisions is typically 10^{-13}s or 0.1 ps (picosecond), and the **distance between collisions** is a few **tens of nanometers** or a few hundred angstroms.
- The **net thermal velocity** (averaged over time or over a large number of carriers at any given time) is **zero**. Thus, **thermal motion does not create a steady electric current**, but it does **introduce a thermal noise**.

Hot-point probe test:

Thermal motion can create a net current flow when there is a temperature difference.



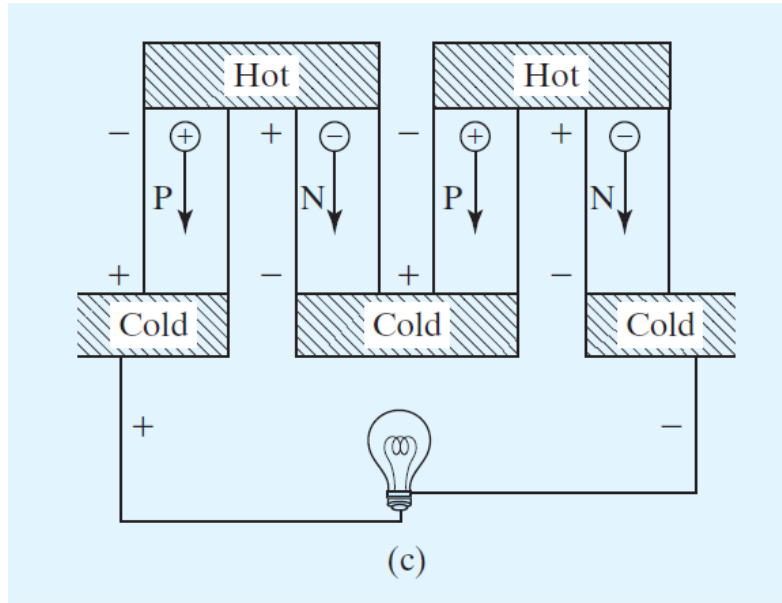
(a, b) **Hot-point probe test** can determine the doping type of a sample.

Example:

- ✓ (a) N-type semiconductor sample ($-V$ on the cold side)
- ✓ (b) P-type semiconductor sample ($+V$ on the cold side)
- ✓ cold (room temperature) metal probe is placed on the sample close to a hot probe (f.e. soldering iron)
- ✓ The electrons around the hot probe have higher thermal velocity and therefore on average move toward the cold side at a higher rate than the electrons on the cold side move to the hot side.
- ✓ The imbalance causes the electrons to accumulate on the cold side and build up a negative voltage, which is detected with a voltmeter.

Thermoelectric Generator

A thermoelectric generator converts heat into electric power



Powered some early space satellites using the radioactive decay of radioactive materials as the heat source.

If instead of extracting power from the device, **current is fed into it**, one set of the junctions become cooler than the other. The device is then a **thermoelectric cooler**. It can be used to cool a hot IC in a circuit board or beverages in a battery powered portable cooler.

References

Basic Approach: https://en.wikipedia.org/wiki/Thermoelectric_generator

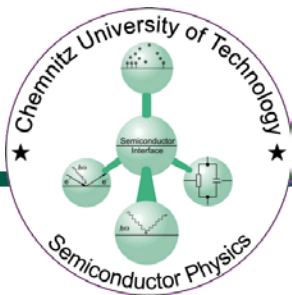
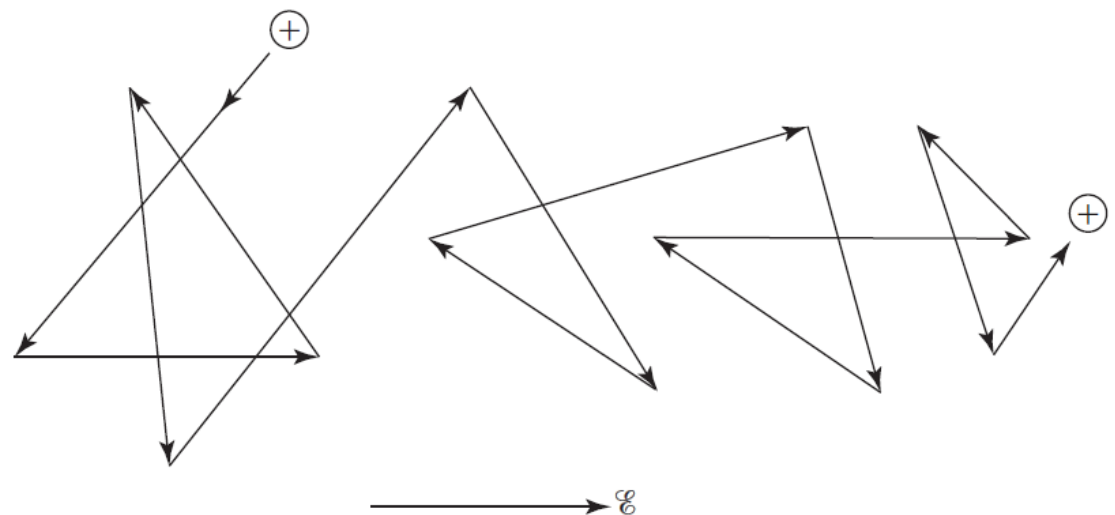
Advanced approach:

<http://www.sciencepublishinggroup.com/journal/paperinfo?journalid=622&doi=10.11648/j.ajpa.20180605.14>



Drift is the motion of charge carriers caused by an electric field. Clearly, drift is usually at play when voltages are applied to a semiconductor device.

The average velocity of the carriers is no longer zero when an electric field is applied to the semiconductor. This nonzero velocity is called the **drift velocity**. The drift velocity is superimposed on the thermal motion:



Mathematical Model

Assume that the *mean free time between collisions* is τ_m and that the carrier transfers its entire *drift momentum*, \mathbf{m}_v , after each collision. The drift momentum gained between collisions is equal to the *force*, \mathbf{q} , times the mean free time.

Equating the loss to the gain, we can find the steady state drift velocity, v for both charge carriers electrons and holes:

holes

$$m_p v = q \mathcal{E} \tau_{mp}$$

$$v = \frac{q \mathcal{E} \tau_{mp}}{m_p}$$

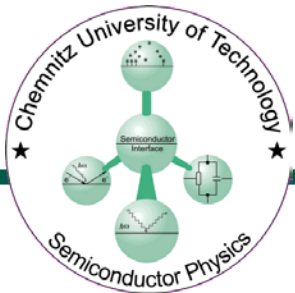


$$v = \mu_p \mathcal{E}$$
$$\mu_p = \frac{q \tau_{mp}}{m_p}$$

electrons

$$v = -\mu_n \mathcal{E}$$

$$\mu_n = \frac{q \tau_{mn}}{m_n}$$



Notes

- The drift velocity is proportional to the electric field
- The proportionality constants μ_p / μ_n are the **holes/electrons mobility**
- For electrons the negative sign means that the electrons drift is in a direction opposite to the field and is negative charged.
- Carrier mobility has the same dimension as v/E , i.e., $\text{cm}^2/\text{V}\cdot\text{s}$.

Electron and hole mobilities at room temperature of selected lightly doped semiconductors

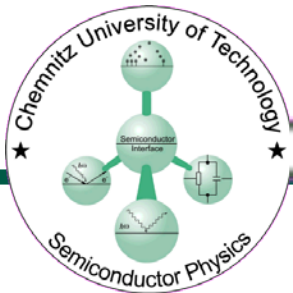
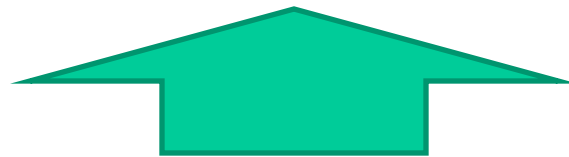
	Si	Ge	GaAs	InAs
μ_n ($\text{cm}^2/\text{V}\cdot\text{s}$)	1400	3900	8500	30, 000
μ_p ($\text{cm}^2/\text{V}\cdot\text{s}$)	470	1900	400	500

* GaAs has a much higher μ_n than Si due to a smaller m_n . Thus, higher-speed transistors can be made with GaAs, which are typically used in communications equipment.

Mechanisms of Carrier Collisions and Scattering

two main causes: **phonon scattering** and **ionized impurity scattering**.

τ_{mn} and τ_{mp} can vary significantly with **temperature** and the **doping concentration**.



Next Lecture !

Semiconductor Physics