Tutorial 5

1. Measurement of the band gap of a semiconductor.

(a) If a semiconductor is transparent to light with a wavelength longer than 0.87 μm, what is its band-gap energy Eg? Useful relationship:

$$h\nu\ (\text{eV}) = \frac{1.24}{\lambda\ (\mu\text{m})}$$

$$h\nu = h\frac{c}{\lambda} = \frac{6.63\times10^{-34}\,(\text{J}\cdot\text{s})\times3\times10^{8}\,\text{m/s}}{0.87\ \mu\text{m}} = \frac{1.99\times10^{-19}\,(\text{J}\cdot\mu\text{m})}{0.87\ \mu\text{m}}$$

$$= \frac{1.99\times10^{-19}\,(\text{eV}\cdot\mu\text{m})}{1.6\times10^{-19}\times0.87\ \mu\text{m}} = \frac{1.24\ (\text{eV}\cdot\mu\text{m})}{0.87\ \mu\text{m}} = 1.42\ \text{eV}$$

Semiconductor	**InSb**	**Ge**	**Si**	**GaAs**	**GaP**	**ZnSe**	**Diamond**
E_g **(eV)**	0.18	0.67	1.12	1.42	2.25	2.7	6.0

1. Measurement of the band gap of a semiconductor.

(b) Which semiconductor is this? To which range of the electromagnetic spectrum corresponds its band gap?

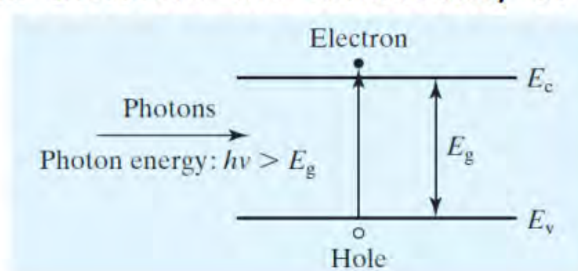
(b) According to the table, the semiconductor with such a bandgap is GaAs; a band gap with energies in the transition from IR to visible-Red. Therefore it will tend to absorb visible light strongly and are opaque in this wavelength range.

(c) Provide an application for the previous method of characterizing a semiconductor?

(c) Can be used as Light Detector. Putting two electrodes on the semiconductor and applying a voltage between the electrodes, one can measure the change in the semiconductor conductance and thus detect changes in light intensity.

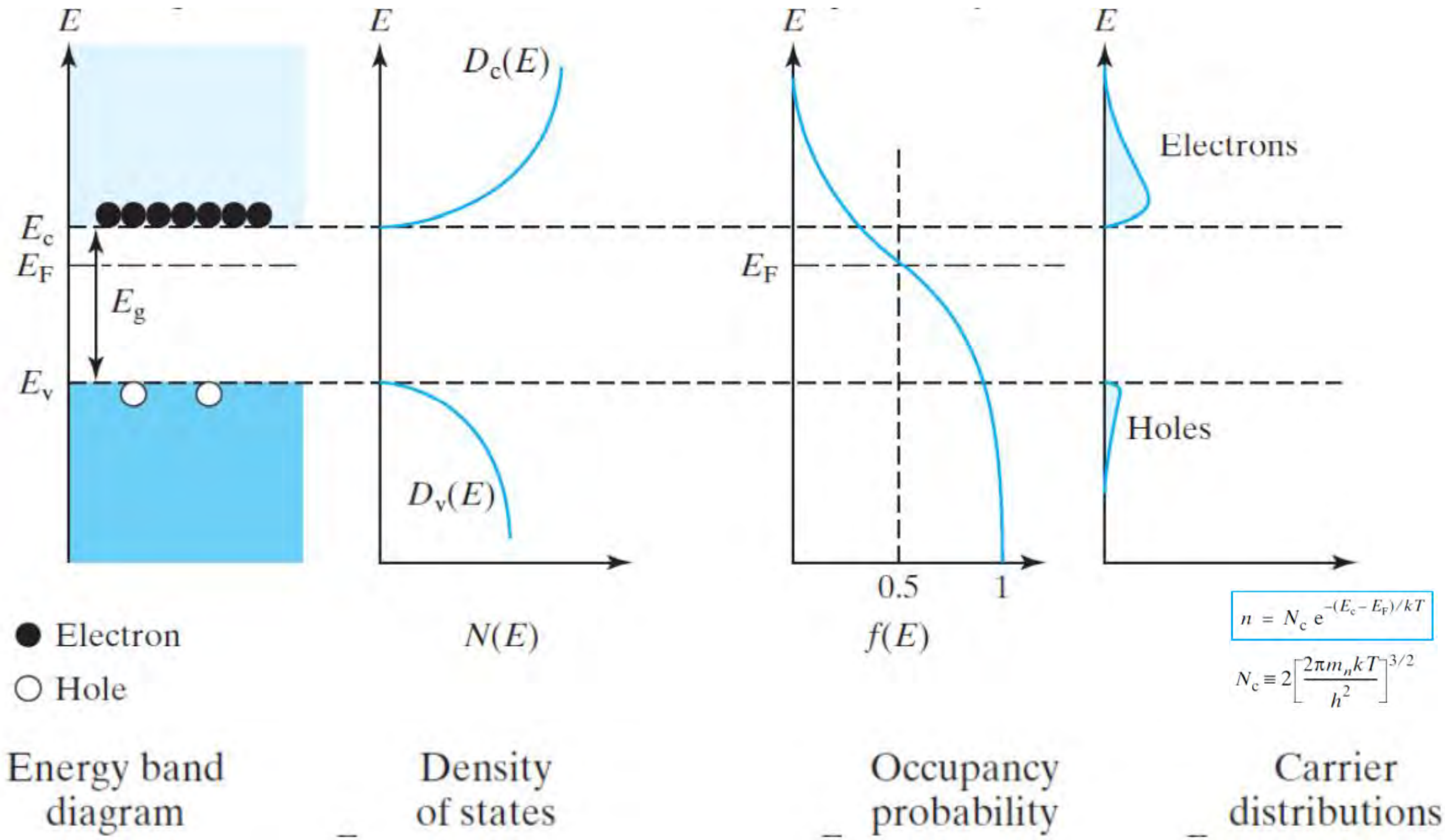
(d) Describe what happens inside a semiconductor when we flash radiation of the same energy (or higher) of its band gap energy? Draw a diagram.

(d) When light is absorbed by a semiconductor sample and electron–hole pairs are created as shown, the number of electrons and holes and therefore the conductivity of the semiconductor) is proportional to the light intensity.



2. (a) Name the mathematical functions plotted in each graph.

(b) Write the mathematical function corresponding to to each graphical representation.



$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

$$D_c(E) = \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3}, \quad E \geq E_c$$

$$D_v(E) = \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3}, \quad E \leq E_v$$

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

2. (c) What are the variables: E_c, E_v, E_g, E_F, kT, N_c, N_v, m_n, m_p and h in these equations?

$$\boxed{f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}}$$

$$D_c(E) = \frac{8\pi m_n \sqrt{2m_n(E-E_c)}}{h^3}, \quad E \geq E_c$$

$$D_v(E) = \frac{8\pi m_p \sqrt{2m_p(E_v-E)}}{h^3}, \quad E \leq E_v$$

$$\boxed{p = N_v\, e^{-(E_F-E_v)/kT}}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

$$\boxed{n = N_c\, e^{-(E_c-E_F)/kT}}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

E_c, energy of conduction band

E_v, energy of valence band

E_g, energy of the band gap

E_F, fermi energy

kT, thermal energy (k=Boltzmann const; T= temperature)

N_c, effective density of states in the conduction band

Why *effective?*

N_v, effective density of states in the valence band

m_n, effective mass of electrons in a material

Why *effective?*

M_p, effective mass of holes in a material

h, Plank const.

3. Is the position of the Fermi level E_F always the same? Explain.

- A very important fact to remember about E_F is that there is only one Fermi level in a system at thermal equilibrium
  - the Fermi level E_F is determined by the available electrons and states in the system.

* E_F (Fermi level) is related to the density of electrons and holes in the following manner:

$$n = N_c\, e^{-(E_c - E_F)/kT}$$

$$p = N_v\, e^{-(E_F - E_v)/kT}$$

- In doped semiconductors, p-type and n-type, the Fermi level is shifted by the impurities, illustrated by their band gaps.

In an **intrinsic semiconductor** (with no doping at all), the Fermi level is **lying exactly at the middle of the energy bandgap** at T=0 Kelvin.

- With **increasing/decreasing temperature T>0** Kelvin the Fermi energy **remains at this midgap position** if conduction and valence bands have exactly the same dispersion energy or more simply the **same effective masses** for electrons and holes.

Or it will move towards the band with the **smaller/higher effective mass**, in case they are different.

The **position of the Fermi level** with respect to valence and or conduction bands depends on various parameters.

temperature
effective masses of electrons and holes
number of free electrons and holes.

This variation of the Fermi level obeys two conservation conditions :

mass & charge

- the "mass action law" which states that the number of particles of each type as well as the overall number of the particles must conserve whatever is their distribution on the available energy levels.
- the neutrality equation which states that the electrical neutrality has to be fulfilled, i.e. the number of negative charges must be counterbalanced excatly by the same number of positive charges.

In an **extrinsic semiconductors** (with added doping), in order to conserve the number of particles (mass action law) and to fulfill the overall electrical charge neutrality (neutrality equation), the Fermi level has to move away from the midgap position.

- It shifts towards **conduction band in an n-type semiconductor** (extrinsic semiconductors with added doping impurities which are donors i.e. impurities which give additional electrons to the system) where the number of electrons n is higher than the number of holes (n>p).

- It shifts towards **valence band in a p-type semiconductor** (extrinsic semiconductors with added doping impurities which are acceptors i.e. impurities which trap electrons from the system giving rise to a deficit of electrons or an excess of holes) where the number of electrons n is lower than the number of holes (n<p).

4. Find the Fermi Level in Si.

Where is E_F located in the energy band of silicon, at 300 K with $n = 10^{17}\text{cm}^{-3}$? And for $p = 10^{14}\text{cm}^{-3}$? Draw diagram. ($N_c$=2.8X$10^{19}$, N_v=1.04X10^{19} , kT=26 meV)

From Eqs. $\boxed{n = N_c\, e^{-(E_c - E_F)/kT}}$ & $\boxed{p = N_v\, e^{-(E_F - E_v)/kT}}$

For $n = 10^{17}\text{cm}^{-3}$:

$$\begin{aligned} E_c - E_F &= kT \cdot \ln(N_c/n) \\ &= 0.026 \ln(2.8 \times 10^{19}/10^{17}) \\ &= 0.146 \text{ eV} \end{aligned}$$

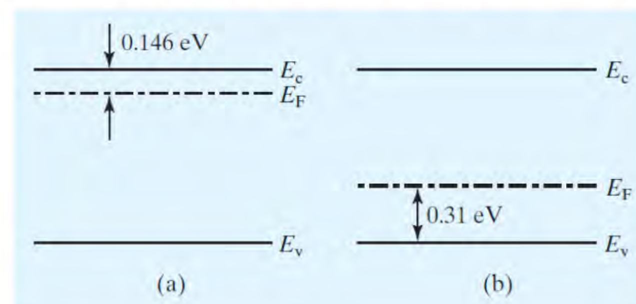
Therefore, E_F is located at 146 meV below E_c

For $p = 10^{14}\text{cm}^{-3}$:

$$\begin{aligned} E_F - E_v &= kT \cdot \ln(N_v/p) \\ &= 0.026 \ln(1.04 \times 10^{19}/10^{14}) \\ &= 0.31 \text{ eV} \end{aligned}$$

Therefore *EF* is located at 0.31 eV above *Ev*.

Location of E_F when $n = 10^{17}\text{cm}^{-3}$ (a), and $p = 10^{14}\text{cm}^{-3}$ (b).



5. Carrier Concentrations

What is the hole concentration at 300 K in an N-type Si semiconductor with $10^{15}cm^{-3}$ of donors? (for Si *ni* at room temperature is roughly $10^{10}cm^{-3}$)

(a) For each ionized donor, an electron is created.

Therefore $n = 10^{15}cm^{-3}$:

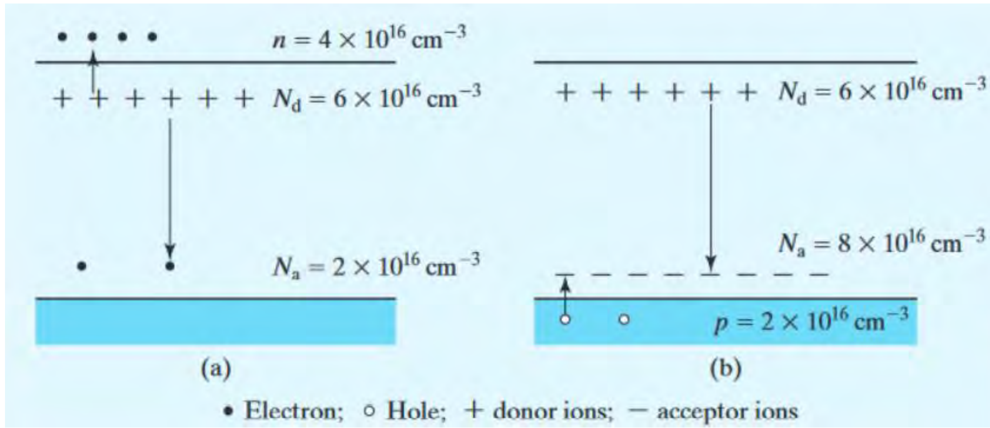
$$p = \frac{n_i^2}{n} \approx \frac{10^{20}\ \mathrm{cm}^{-3}}{10^{15}\ \mathrm{cm}^{-3}} = 10^{5}\mathrm{cm}^{-3}$$

$$np = n_i^2$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

6 **Dopant Compensation**

What are n and p in a Si sample with $N_d = 6 \times 10^{16} cm^{-3}$ and $N_a = 2 \times 10^{16}\ cm^{-3}$? What happens if we add $6 \times 10^{16}\ cm^{-3}$ of acceptors?



$$n = N_d - N_a = 4 \times 10^{16} \text{cm}^{-3}$$

$$p = n_i^2/n = 10^{20}/4 \times 10^{16} = 2.5 \times 10^3 \text{cm}^{-3}$$

With the additional acceptors, Na= $2 \times 10^{16} + 6 \times 10^{16} = 8 \times 10^{16}$cm–3, holes become the majority:

$$p = N_a - N_d = 8 \times 10^{16} - 6 \times 10^{16} = 2 \times 10^{16} \text{cm}^{-3}$$

$$n = n_i^2/p = 10^{20}/(2 \times 10^{16}) = 5 \times 10^3 \text{cm}^{-3}$$

The addition of acceptors converted the Si to P-type as shown in (b).

7 Thermal Velocity

What are the approximate thermal velocities of electrons and holes in silicon at room temperature? (Assume T= 300 K and recall $m_n = 0.26\ m_0$)

$$\text{Kinetic energy} = \frac{1}{2} m_n v_{th}^2 = \frac{3}{2} kT$$

$$\begin{aligned} v_{th} &= \sqrt{\frac{3kT}{m}} \\ &= (3 \times 1.38 \times 10^{-23}\,\text{J/K} \times (300\ \text{K}/0.26 \times 9.1 \times 10^{-31}\,\text{kg}))^{1/2} \\ &= 2.3 \times 10^{5}\,\text{m/s} = 2.3 \times 10^{7}\,\text{cm/s} \end{aligned}$$

Note that the typical thermal velocity of electrons and holes is 2.5×10^7cm/s, which is about 1000 times slower than the speed of light and 100 times faster than the speed of sound.

8 Drift Velocity, Mean Free Time, and Mean Free Path

Given μ_p= 470 cm²/V·s for Si, what is the hole drift velocity at E= 10^3V/cm? What is τ_{mp} and what is the average distance traveled between collisions, i.e.,the mean free path?

$v = \mu_p \mathscr{E} = 470\ \text{cm}^2/\text{V·s} \times 10^3\ \text{V/cm} = 4.7 \times 10^5 \text{cm/s}$, much lower than the thermal velocity, ~2.1×10^7 cm/s. Recalling that:

$$\boxed{\begin{aligned} v &= -\mu_n \mathscr{E} \\ \mu_n &= \frac{q\tau_{mn}}{m_n} \end{aligned}}$$

We can then calculate:

$$\begin{aligned} \tau_{mp} &= \mu_p m_p / q = 470\ \text{cm}^2 \times 0.39 \times 9.1 \times 10^{-31}\text{kg}/1.6 \times 10^{-19}\text{C} \\ &= 0.047\ \text{m}^2 \times 2.2 \times 10^{-12}\ \text{kg/C} = 1 \times 10^{-13}\text{s} = 0.1\ \text{ps} \end{aligned}$$

$$\begin{aligned} \text{Mean free path} &= \tau_{mp}\, v_{\text{th}} \sim 1 \times 10^{-13}\text{s} \times 2.2 \times 10^7\text{cm/s} \\ &= 2.2 \times 10^{-6}\text{cm} = 220\ \text{Å} = 22\ \text{nm} \end{aligned}$$

9 Diffusion Constant

What is the hole diffusion constant in a piece of silicon doped with $3 \times 10^{15}\text{cm}^{-3}$ of donors and $7 \times 10^{15}\text{cm}^{-3}$ of acceptors at 300 K?

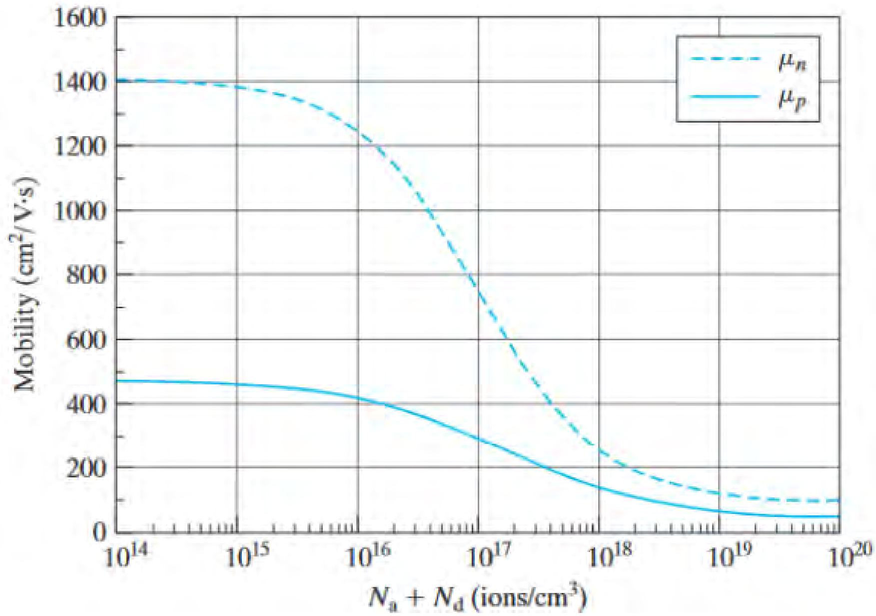


Figure 1: shows the electron and hole mobilities of silicon at 300 K.
At low dopant concentration, the electron mobility is dominated by phonon scattering; at high dopant concentration, it is dominated by impurity ion scattering.

From figure 1 we can read:

$N_a + N_d = 3 \times 10^{15} + 7 \times 10^{15} = 1 \times 10^{16}\text{cm}^{-3}$, μ_p is

about 410 cm²/V·s at 300 K.

$D_p = (kT/q)\mu_p = 26 \text{ mV} \times 410\text{cm}^2/\text{V}\cdot\text{s} = 11\text{cm}^2/\text{s}$.

10 Temperature Dependence of Resistance

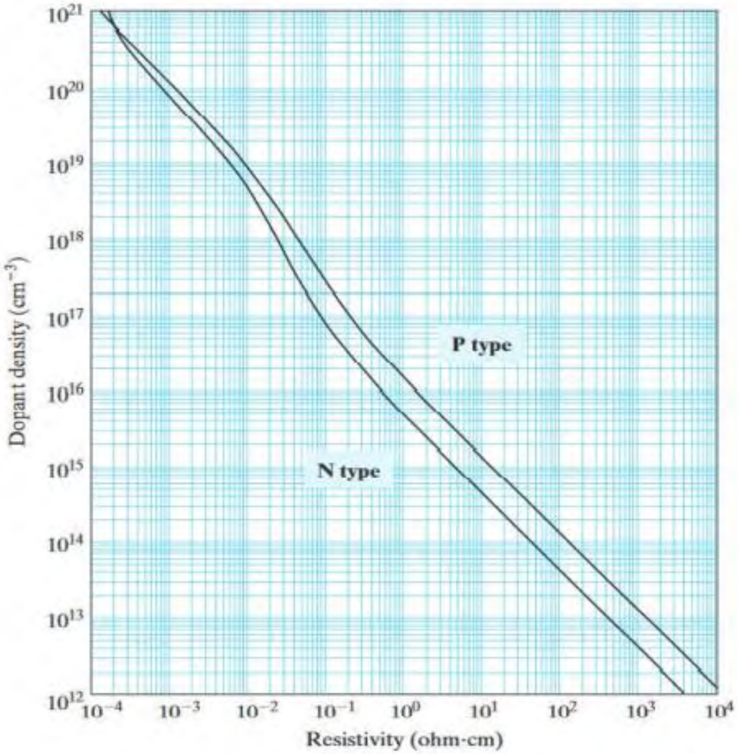


Figure 1: Conversion between resistivity and dopant density of silicon at room temperature.

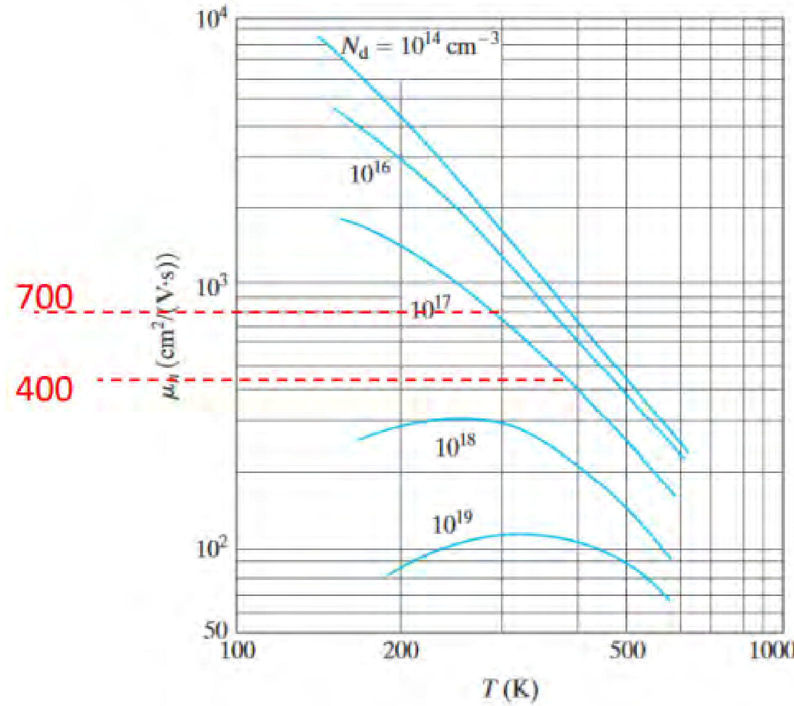


Figure 2: Temperature dependence of the electron mobility in Si.

a. What is the resistivity, ρ, of silicon doped with $10^{17}\mathrm{cm}^{-3}$ of arsenic?

b. What is the resistance, R, of a piece of this silicon material 1 µm long and 0.1 µm^2 in cross-sectional area?

c. By what factor will R increase (or decrease) from $T = 300$ K to T = 400 K?

a. What is the resistivity, ρ, of silicon doped with $10^{17} cm^{-3}$ of arsenic?

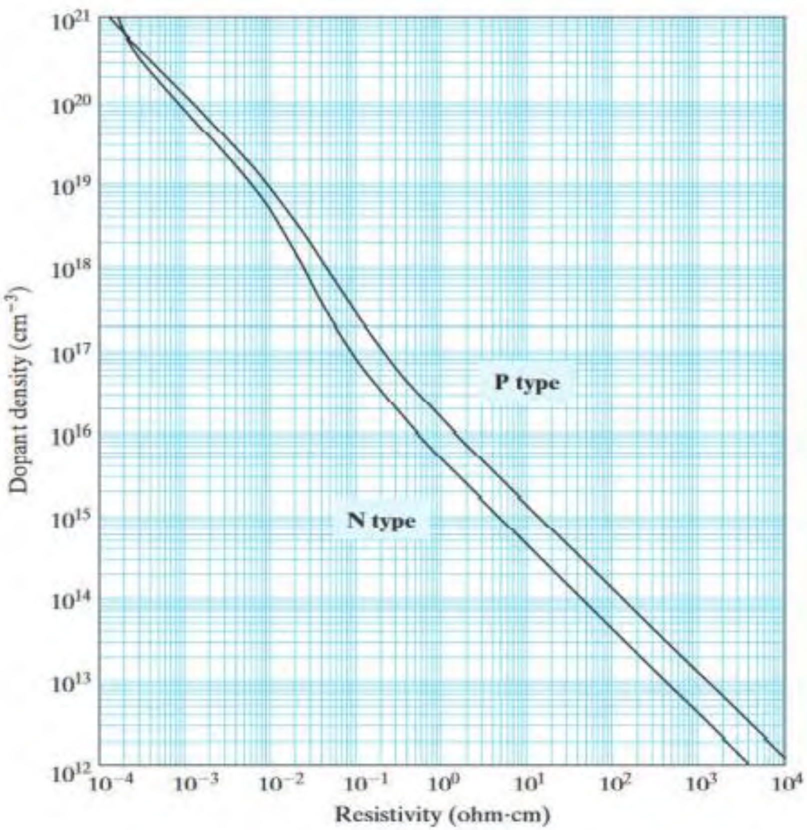


Figure 1: Conversion between resistivity and dopant density of silicon at room temperature.

a. Use N-curve and read the value.

Why N-type? As is a donor ~ 0.09 Ω·cm.

b. What is the resistance, R, of a piece of this silicon material 1 μm long and 0.1 μm^2 in cross-sectional area?

$$R = \rho \times \text{length/area} = 0.084\ \Omega\cdot\text{cm} \times 1\ \mu\text{m} \ /\ 0.1\ \mu\text{m}^2$$
$$= 0.084\ \Omega\cdot\text{cm} \times 10^{-4}\text{cm} \ /10^{-9}\text{cm}^{-2} = 8.4 \times 10^3\Omega.$$

c. By what factor will R increase (or decrease) from $T = 300$ K to T = 400 K?

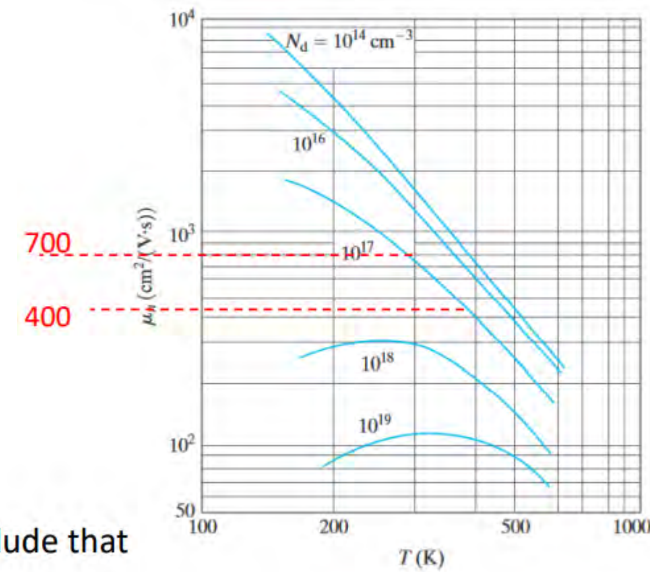


Figure 2: Temperature dependence of the electron mobility in Si.

c. The temperature dependent factor in σ, and therefore in ρ is μ_n:

$$\sigma = qn\mu_n + qp\mu_p \qquad \mu_{\text{impurity}} \propto \frac{T^{3/2}}{N_a + N_d}$$

Figure 2 shows μ_n to decrease from 770 at 300 K to 400 at 400 K. We conclude that R increases by:

$$\frac{770}{400} = 1.93$$

11 Photoconductors

A bar of Si is doped with boron at 10^{15} cm^{-3}. It is exposed to light such that electron–hole pairs are generated throughout the volume of the bar at the rate of 10^{20}/s·cm^3. The recombination lifetime is 10 μs. What are:

a. p_0
b. n_0
c. p'
d. n'
e. p
f. n
g. The np product

a. $p_0 = N_a = 10^{15}\text{cm}^{-3}$ is the equilibrium hole concentration.

b. $n_0 = n_i^2/p_0 \approx 10^5\text{cm}^{-3}$ is the equilibrium electron concentration.

c. In steady state, the rate of generation is equal to the rate of recombination

Recombination rate $= \frac{n'}{\tau} = \frac{p'}{\tau}$

$$10^{20}/\text{s}\cdot\text{cm}^3 = p'/\tau$$

$$\therefore\ p' = 10^{20}/\text{s}\cdot\text{cm}^3\cdot t = 10^{20}/\text{s}\cdot\text{cm}^3\cdot 10^{-5}\text{s} = 10^{15}\text{cm}^{-3}$$

d. $n' = p' = 10^{15}\text{cm}^{-3}$ ($\boxed{n' \equiv p'}$)

e. $p = p_0 + p' = 10^{15}\text{cm}^{-3} + 10^{15}\text{cm}^{-3} = 2\times 10^{15}\text{cm}^{-3}$

f. $n = n_0 + n' = 10^5\text{cm}^{-3} + 10^{15}\text{cm}^{-3} \approx 10^{15}\text{cm}^{-3}$. **The non-equilibrium minority carrier concentration is often much much larger than the small equilibrium concentration.**

g. $np \approx 2\times 10^{15}\text{cm}^{-3}\times 10^{15}\text{cm}^{-3} = 2\times 10^{30}\text{cm}^{-6} >> n_i^2 = 10^{20}\text{cm}^{-6}$. **The *np* product can be very different from n_i^2.**

12 Quasi-Fermi Levels

Consider an Si sample with $N_d = 10^{17} cm^{-3}$, $ni^2 = 10^{20} cm^{-6}$

a. Find the location of E_F

b. Find the location of E_{Fn} and E_{Fp} when excess carriers are introduced such that $n' = p'$ $10^{15} cm^{-3}$.

c. Draw diagram.

a. Using Eq. $n = N_c e^{-(E_c - E_F)/kT}$

$$n = N_d = 10^{17} cm^{-3} = N_c e^{-(E_c - E_F)/kT}$$

$$E_c - E_F = kT \ln \frac{N_c}{10^{17} cm^{-3}} = 26 \text{ meV} \cdot \ln \frac{2.8 \times 10^{19} cm^{-3}}{10^{17} cm^{-3}} = 0.15 \text{ eV}$$

E_F is below E_c by 0.15 eV.

12 Quasi-Fermi Levels

Consider an Si sample with $N_d = 10^{17} cm^{-3}$, $n_i^2 = 10^{20} cm^{-6}$

a. Find the location of E_F

b. Find the location of E_{Fn} and E_{Fp} when excess carriers are introduced such that $n' = p'$

$10^{15} cm^{-3}$.

c. Draw diagram.

b. $n = n_0 + n' = N_d + n' = 1.01 \times 10^{17} \text{cm}^{-3}$

Using Eq. $n = N_c e^{-(E_c - E_{Fn})/kT}$

$$1.01 \times 10^{17} \text{cm}^{-3} = N_c e^{-(E_c - E_{Fn})/kT}$$

$$E_c - E_{Fn} = kT \ln \frac{N_c}{1.01 \times 10^{17} \text{cm}^{-3}} = 26 \text{ meV} \cdot \ln \frac{2.8 \times 10^{19} \text{cm}^{-3}}{1.01 \times 10^{17} \text{cm}^{-3}} = 0.15 \text{ eV}$$

$$p = p_0 + p' = \frac{n_i^2}{N_d} + p' = 10^3 \text{cm}^{-3} + 10^{15} \text{cm}^{-3} = 10^{15} \text{cm}^{-3}$$

Using Eq. $p = N_v e^{-(E_{Fp} - E_v)/kT}$

$$10^{15} \text{cm}^{-3} = N_v e^{-(E_{Fp} - E_v)/kT}$$

$$E_{Fp} - E_v = kT \ln \frac{N_v}{10^{15} \text{cm}^{-3}} = 26 \text{ meV} \cdot \ln \frac{1.04 \times 10^{19} \text{cm}^{-3}}{10^{15} \text{cm}^{-3}} = 0.24 \text{ eV}$$

c