# Complex dynamics made simple: colloidal dynamics

Paolo Sibani

Chemnitz, June 2010

Syddansk Universitet

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- Age: time elapsed from the initial quench
- Time homogeneous dynamics: e.g. diffusion, no changes of mesoscopic rates with time.
- Time inhomogeneous dynamics: e.g. aging: rate of significant events goes down with age.

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- Many basins of attraction. E.g. energy minima separated by energy barriers.
- On long enough time scales neglect the details and look at the dynamics at the level of attractor changes.
- The resulting dynamics may be simple: e.g. diffusion in sinusoidal potential
- Or complex: e.g. spin glass dynamics, where averages have power-law or logarithmic dependences.

- Hard sphere colloidal systems-only short range interactions

# Metastability and coarse-graining in hard-sphere colloids

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- Clusters of highly correlated groups of particlesl (spatial coarse graining)
- Dynamics described in terms of the probability per unit of time that a cluster survives the random forces from the rest of the system.

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# What does ‘trivial’ or ‘simple’ mean?

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- For stationary processes:
- Correlation and response functions only depend on time differences
- Can be expanded in eigenvalue expansions

# The simplest stochastic process

- Add random and independent ‘position’ increments

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- Add random and independent ‘position’ increments
- Use linearity of the variance
- $MSD \propto t$ : diffusion

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Record dynamics

- record sized fluctuations
- in the stationary white noise impinging on the system (e.g. thermal noise)
- drive the aging dynamics in marginally stable (glassy) systems
- no. of records is a Poisson process in logarithmic time
- the dynamics appears simple in logarithmic time

- Brief overview of results for *thermally activated* dynamics

# The rest of this talk

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- Experimental[1] colloidal data re-analyzed.

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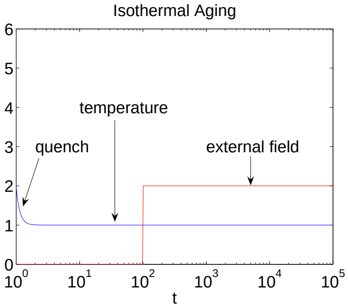
- Experimental[1] colloidal data re-analyzed.
- A cluster model for colloidal behavior numerically analyzed.
- Trees, record dynamics and complexity

# Aging set-up




The age $t$ starts at the 'initial quench', and a field is (possibly) switched on at $t = t_w = 100$.

In a colloid, the system is centrifuged to a high density (quench). No fields are applied.

The Edwards Anderson spin-glass is an Ising spin model where interacting spins $\sigma_i = \pm 1$ are placed on a lattice. The Hamiltonian is

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- The thermal equilibrium properties of the model are *complex*
- The time evolution, as given by a MC algorithm (Metropolis acceptance rule or equivalent) is *complex* and very similar to experimental data

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The p-spin model is an Ising spin model, The Hamiltonian is

- $\mathcal{H} = -\sum_{Plaq.} \sigma_i \sigma_j \sigma_k \sigma_l$
- The thermal equilibrium properties of the model are *trivial*
- The time evolution is nevertheless complex, featuring metastability and aging.

# ROM model

The Restricted Occupancy Model is a lattice model describing vortex creep in type II superconductors in terms of the number $n_i =$ of vortices on site $i$.

- the energy includes repulsive interactions between vortex lines on neighbor sites

see L. P. Oliveira et al. *Phys. Rev. B* 104526 (2005) and references therein.

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- the energy includes repulsive interactions between vortex lines on neighbor sites
- pinning to random sites
- the configuration is updated with Metropolis dynamics

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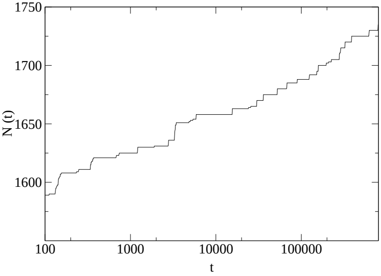
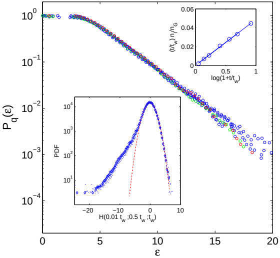
# ROM model intermittency

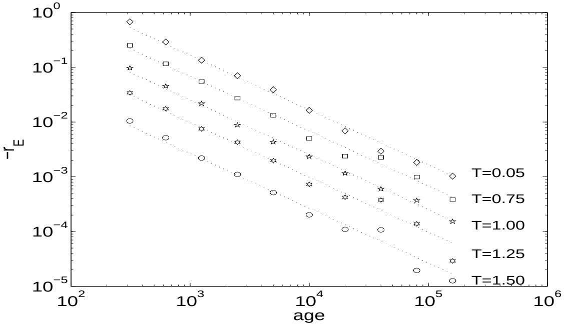



The time variation of the total number of vortices $N(t)$ on the system for a single realization of the pinning potential and the thermal noise in a $8 \times 8 \times 8$ lattice for $T = 0.1$.

L. P. Oliveira et al. Phys. Rev. B 104526 (2005)

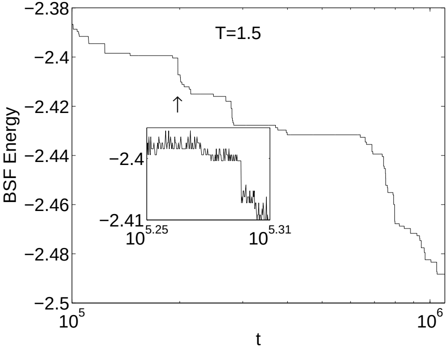



Heat transfer PDF for a spin glass model (PS & H J Jensen, Europhysics Lett. 2005) Heat transfer $H$ over small time $\delta t$ in the E-A spin glass model has a Gaussian part and an intermittent tail. Six different ages are considered with $\delta t/t_w = .01$.



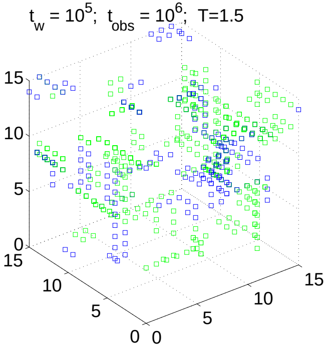

The average rate of energy flow is plotted versus the age for the temperatures shown. The full line has the form $y = C(T)t_w^{-1}$.

# p-spin model, intermittent energy decay




S. Christiansen & PS, *New J. of Physics*, to appear

# p-spin model, quakes in real space

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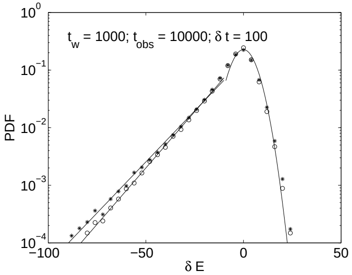
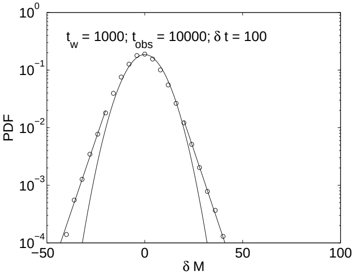
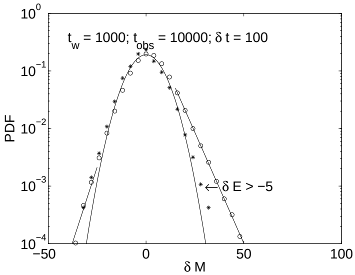



The PDF of the heat exchanged between system and thermal bath over a time $\delta t = 100$. $T = 1.5$.




The PDF of the spontaneous magnetic fluctuations over a time $\delta t = 100$ and $T = 1.5$.

# p-spin model, magnetic fluctuations $H = 0.3$




PDF of the spontaneous magnetic fluctuations over a time $\delta t = 100$. $T = 1.5$.

The magnetic response is SUBORDINATED to the quakes

# Take-home message from thermally activated aging

- The evolution of aggregate variables (i.e. the energy and magnetization) is controlled by extremely rare and irreversible events quakes

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# Take-home message from thermally activated aging

- The evolution of aggregate variables (i.e. the energy and magnetization) is controlled by extremely rare and irreversible events quakes
- Reversible fluctuation of zero average with Gaussian PDF's describe the dynamics in quasi-equilibrium.

# Experimental data

- Tracking data obtained by confocal microscopy

Courtland and Weeks,J. Phys.: Condens. Matter 15, S359, (2003)

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- 'glass formers' $\rho < 0.62$ diffusive behavior, time homogeneous

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# Experimental procedure

- Centrifuge the sample to desired density

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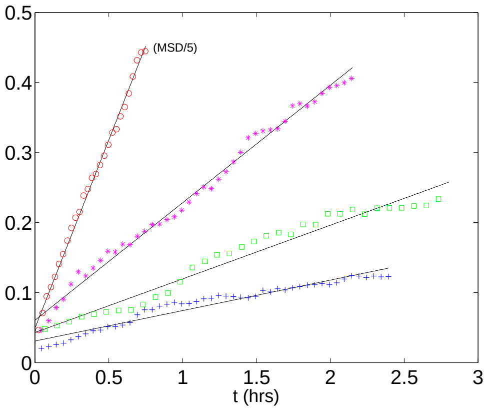
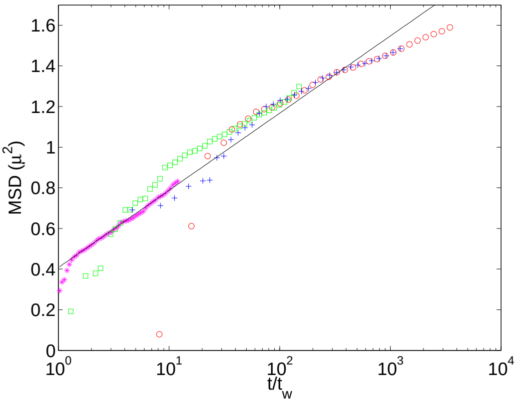
- Start tracking. Scanning through sample $\approx 20$ s

# MSD vs time. Glass-former



# MSD vs time. Dense



# Persistence analysis

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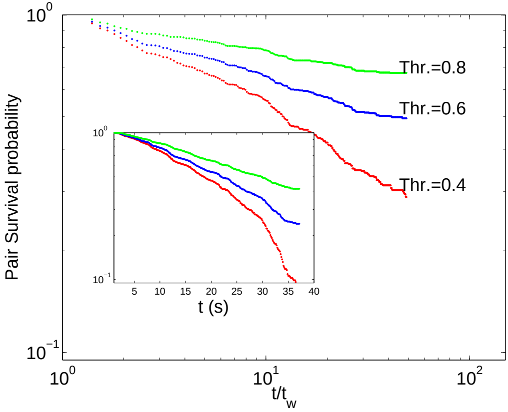
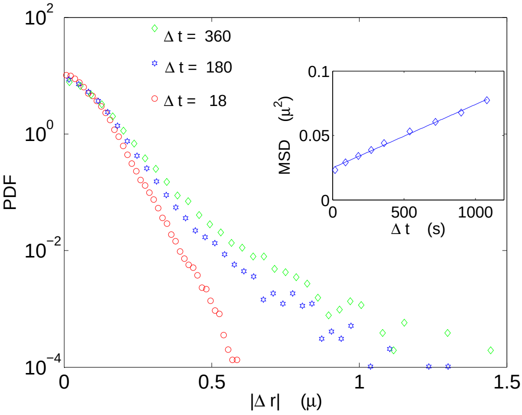
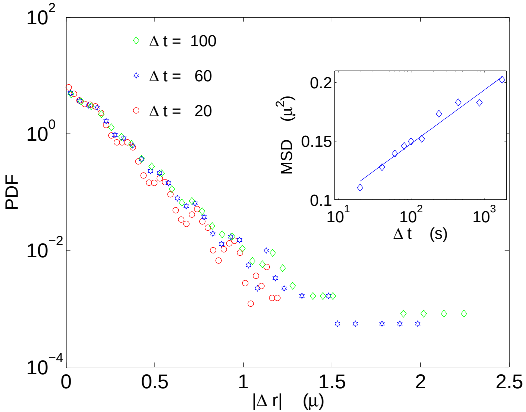
- Identify the time at which each pair splits
- Calculate the fraction of pairs which survive at time $t$.

# Persistence curves






'Glass former'




Dense colloid

# Conclusion from colloid data analysis

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- The persistence probability is exponential in $t$ (glass formers) and $\ln(t) - \ln(t_w)$ (dense)
- Transformation $t \rightarrow \ln(t/t_w)$ trivializes the dynamics of dense colloids
- Highly intermittent and correlated motion

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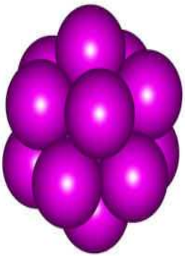
- Particle displacement over observation time of the order of particle radius
- Particle motion is jerky and spatially correlated

# A cluster model



particles in a cluster moves
in fully correlated fashion
on average zero CM displacement
as long as the cluster persists

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- for all $k$: $P_k(h \to 0)$ for $h \to \infty$

# The algorithm

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# The algorithm

- Markov chain, $L$ clusters
- pick a cluster at random
- destroy it with probability $P_k(h)$
- or choose to another cluster
- iff cluster is destroyed:
  - partition its particles randomly in two groups
  - each sub-group joins neighboring cluster
  - particles move one random unit step on the lattice

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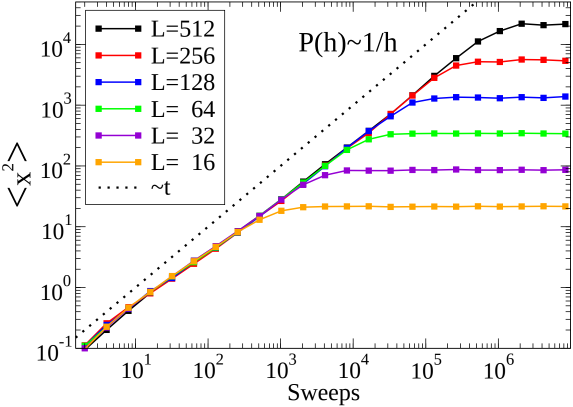
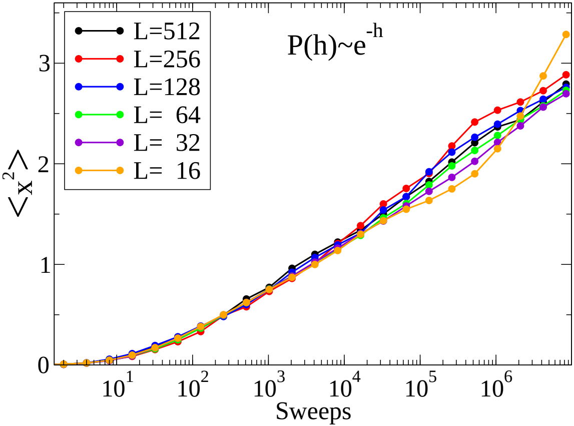
$P_1(h) = \frac{1}{1+h}$ 'glass former', diffusive



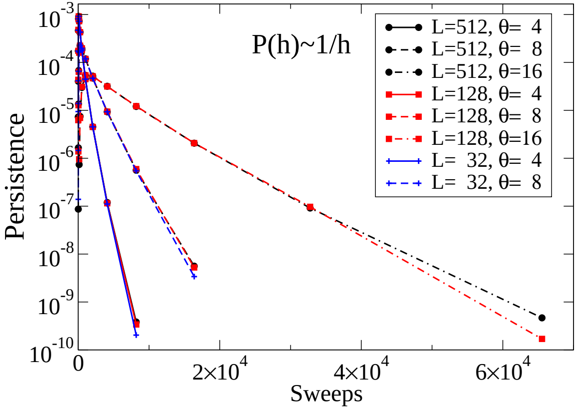
# Diffusive behavior II

$P_\infty(h) = e^{-h}$ dense colloid, log diffusive

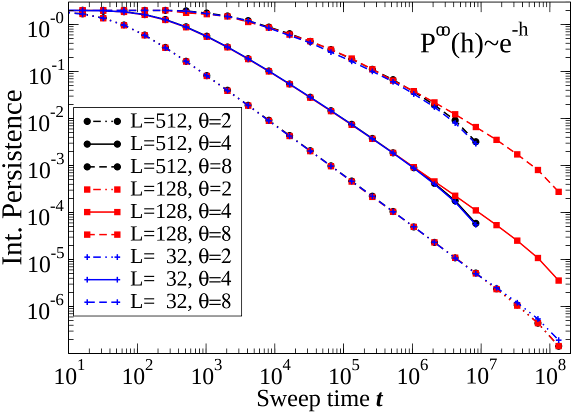


# Persistence I

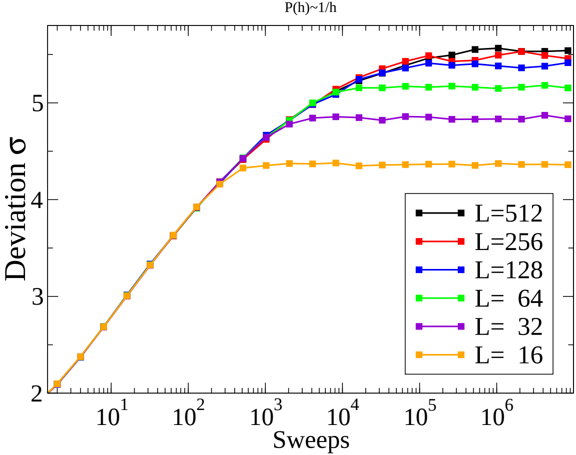
$P_1(h) = \frac{1}{1+h}$ 'glass former', exponential decay



$P_\infty(h) = e^{-h}$ dense colloid, exponential decay in log $t$

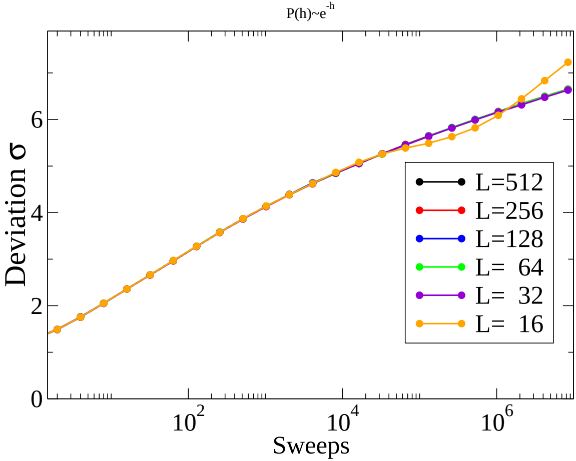


$P_1(h) = \frac{1}{1+h}$ 'glass former', approaches stationary state



# Spatial heterogeneity II

$P_\infty(h) = e^{-h}$ dense colloid, equilibrium is out of reach

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# Quakes in colloidal model

- Collapses of clusters of different sizes are widely separated in time
- All clusters of size $< \sigma(t)$ only provide background of mobile particles
- A quake on a time scale $t$ corresponds to the collapse of the smallest cluster $> \sigma$

# Quakes in thermally activated aging

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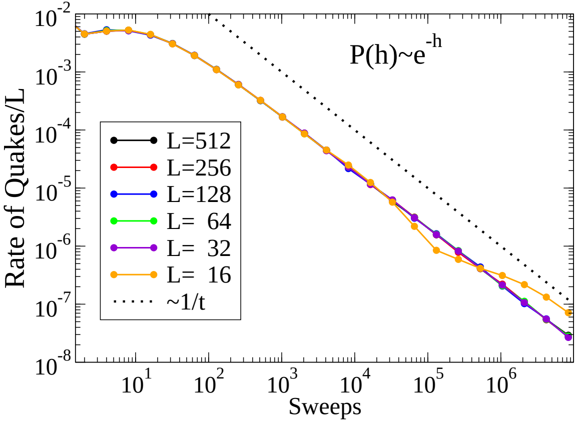
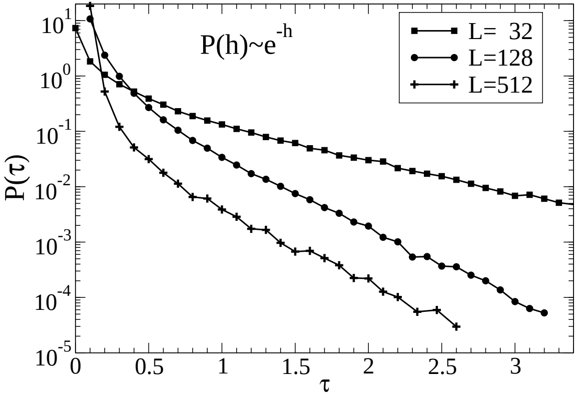
- Size $-\delta e/T >> \log t$

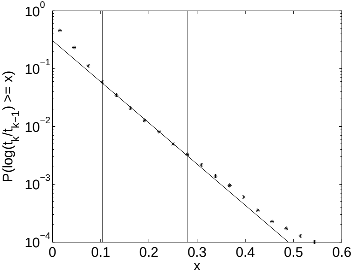
# Temporal statistics of quakes

In the aging regime, each cluster collapse is called a 'quake'



$\tau_k = \log(t_k) - \log(t_{k-1})$: waiting log-times






The distribution of the logarithmic differences $\log(t_k) - \log(t_{k-1})$. is approximately exponential.

# The 'log' Poisson distribution

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- $$P(n, t_1, t_2) = \frac{\mu^n}{n!} \exp(-\mu) \quad \mu(t_1, t_2) = \alpha \log(t_2/t_1) \qquad (1)$$

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- The statistics applies to e.g. spin-glasses, colloids, evolutionary dynamics etc.
- Where can it possibly come from?

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# Marginal increase in stability

- In colloids: The dynamics decelerates once a smallest cluster marginally larger than its predecessor is formed
- This cluster is toppled by a record sized fluctuation on a time scale $t_2 > t_1$...
- Sequence of magnitude records in the probability of toppling the smallest cluster marks the evolution of the dynamics.

# Relation to hierarchies and trees?

- Two different ways to describe the same generic situation

( Fischer Hoffmann & Sibani, PRE 2008 )

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- Two different ways to describe the same generic situation
- Record dynamics needs an underlining hierarchical structure

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# Relation to hierarchies and trees?

- Two different ways to describe the same generic situation
- Record dynamics needs an underlining hierarchical structure
- work with Andreas Fischer and KH elaborates on this point

( Fischer Hoffmann & Sibani, PRE 2008 )

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