

# Global optimization in chemistry: from abstract tests to practical usefulness



Bernd Hartke

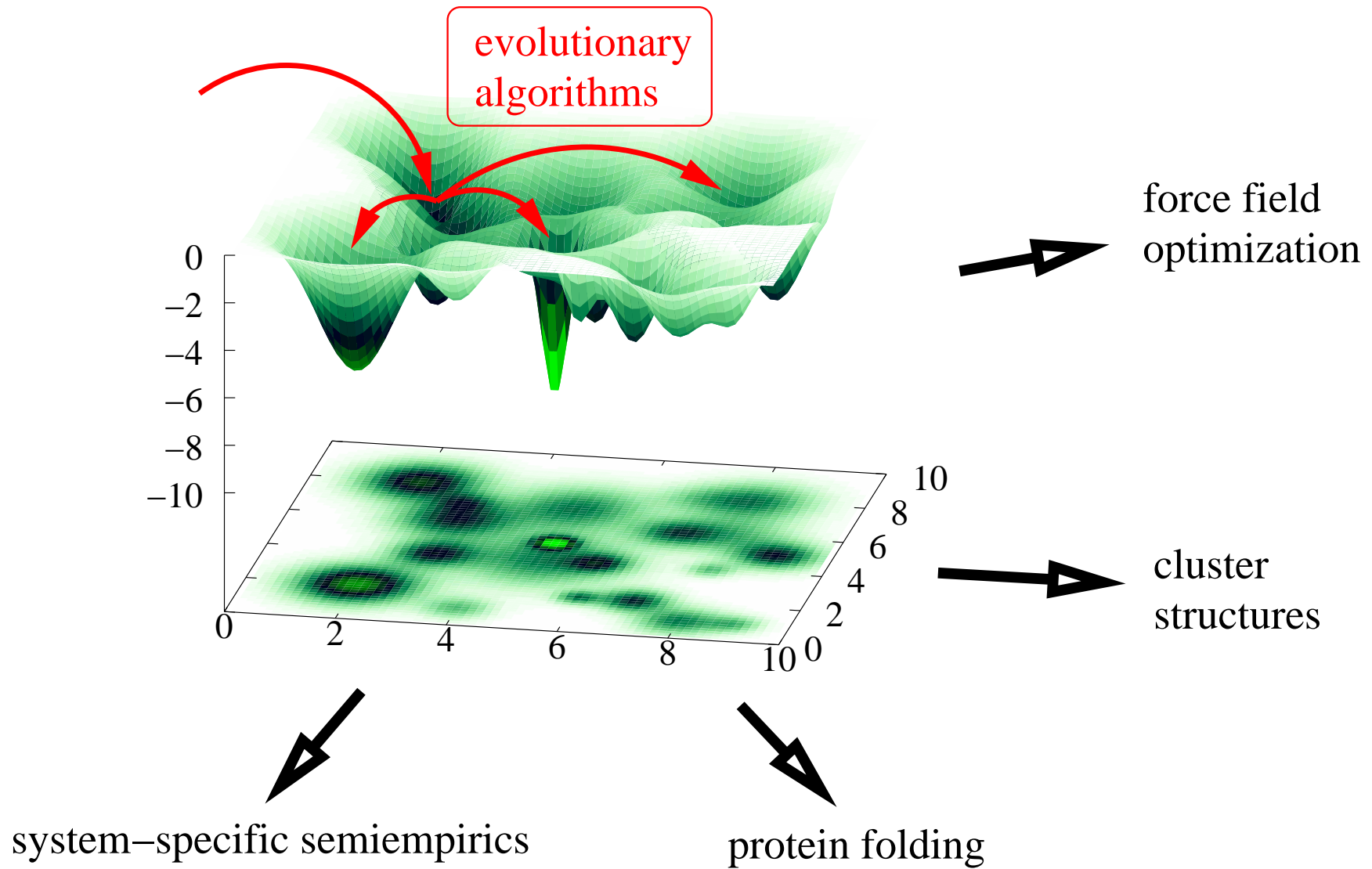
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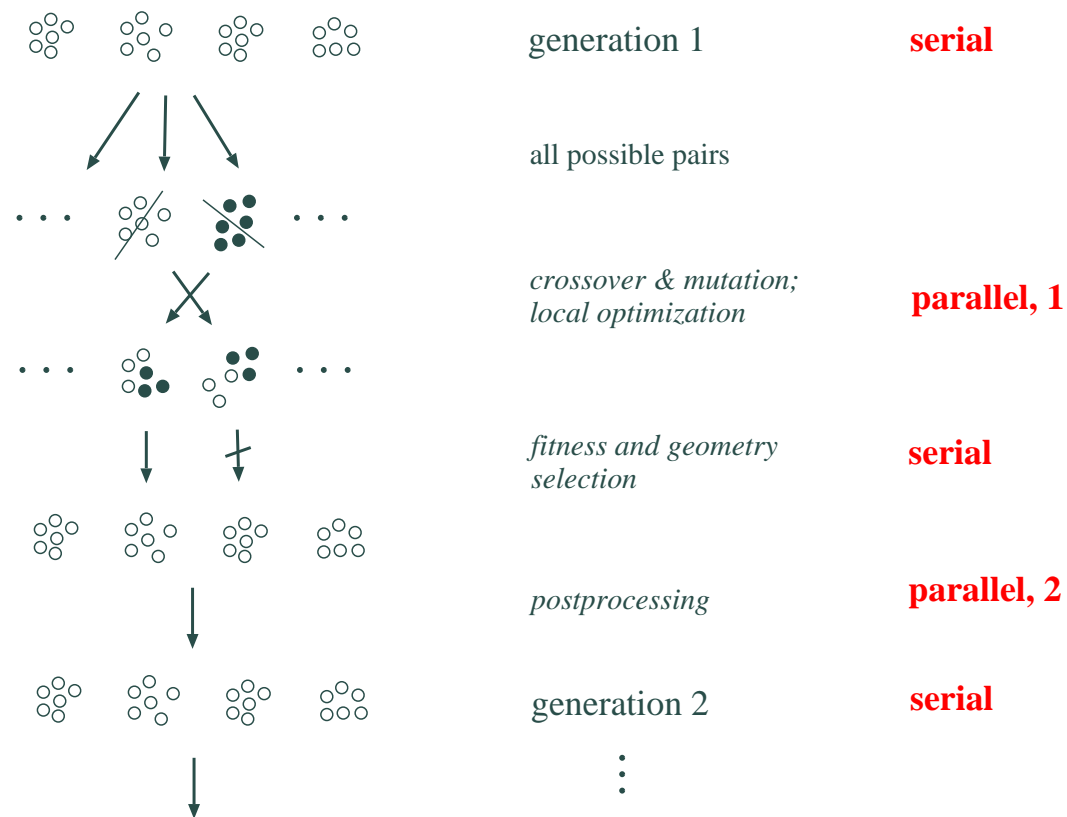
e-mail: [hartke@phc.uni-kiel.de](mailto:hartke@phc.uni-kiel.de)

# Global optimization

# minima grows exponentially with dimensionality



# Global optimization by Evolutionary Algorithms <sup>1 2 3</sup>



## important aspects:

- design problem-specific crossover, exploiting near-separability
- local optimization vital but expensive; use loose thresholds initially
- “directed mutation” scans structurally similar minima
- strongly deceptive landscapes need secondary selection criteria: problem-specific niches

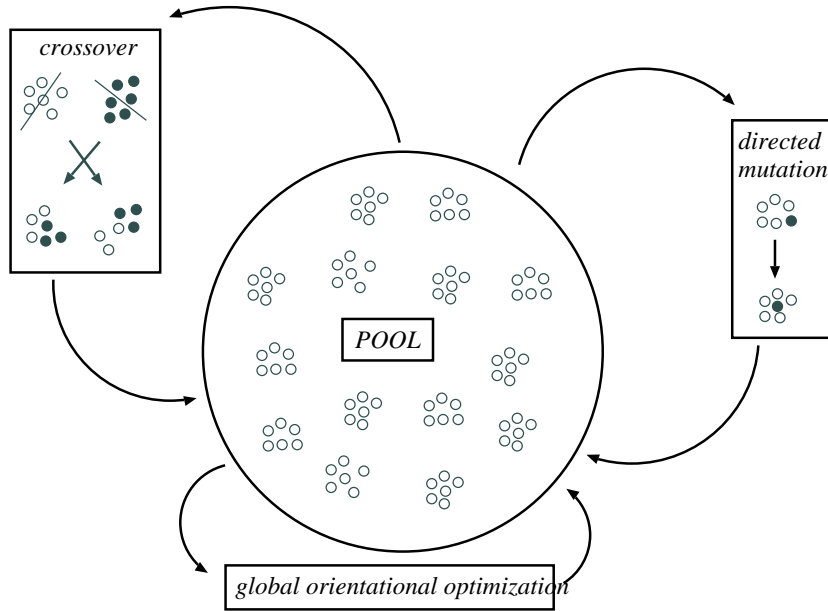
<sup>1</sup> B. Hartke, J. Phys. Chem. 97 (1993) 9973.

<sup>2</sup> D. M. Deaven and K. M. Ho, Phys. Rev. Lett. 75 (1995) 288.

<sup>3</sup> B. Hartke, J. Comput. Chem. 20 (1999) 1752.

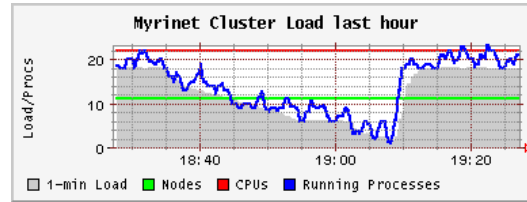
# Parallel implementation<sup>4</sup>

replace generational model by pool model:

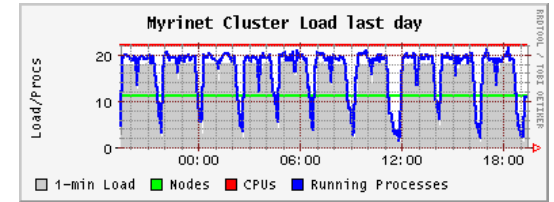


load on a 24-processor PC cluster:

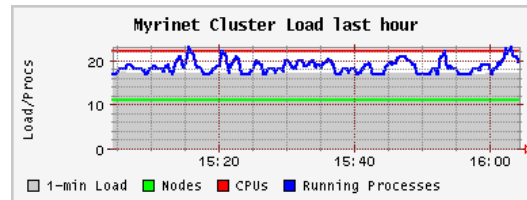
old parallel generational model  
load / 1 hour



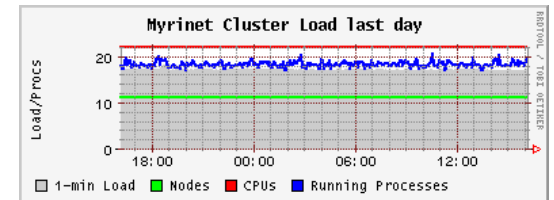
load / 1 day



new parallel pool model  
load / 1 hour



load / 1 day



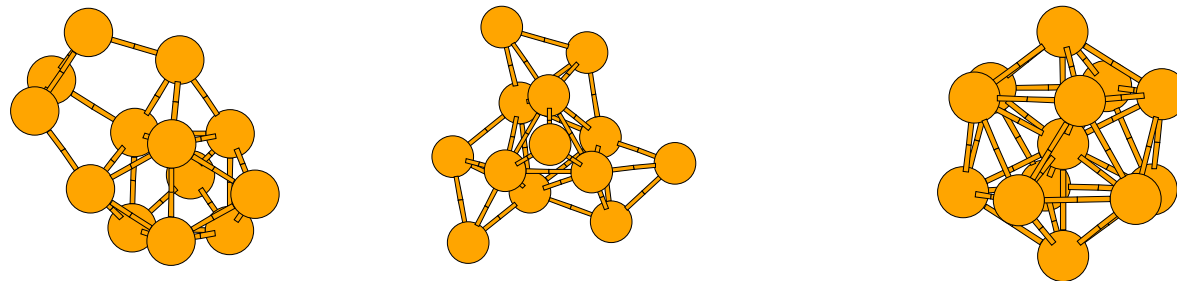
- no serial bottlenecks  $\Rightarrow$  perfectly even load balance during the whole run
- parallel efficiency independent of choices population size vs. # processes

<sup>4</sup> B. Bandow and B. Hartke, J. Phys. Chem. A 110 (2006) 5809.

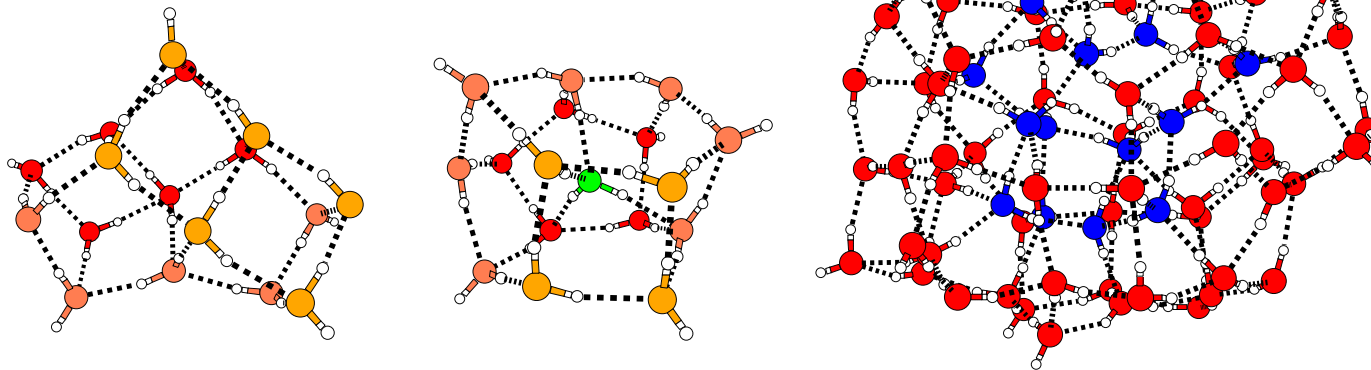


# Global optimization of cluster structures: old stuff...

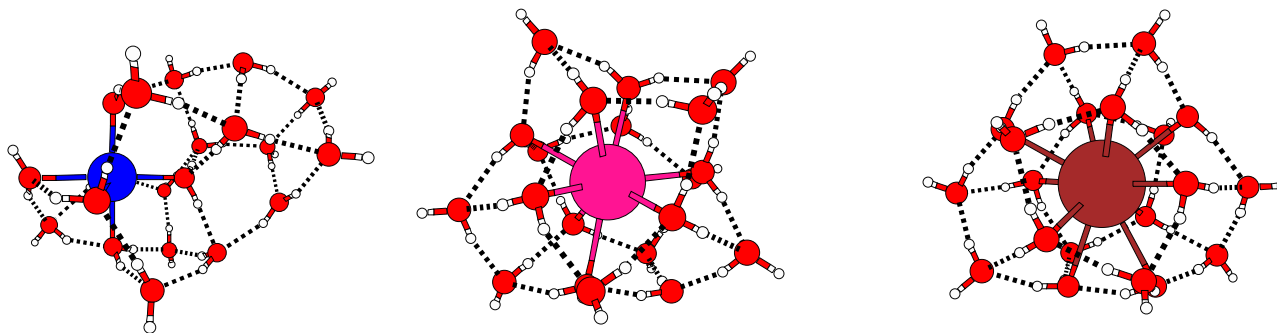
$\text{Hg}_{13}$ :



$(\text{H}_2\text{O})_n$ :



$(\text{Na}^+/\text{K}^+/\text{Cs}^+)(\text{H}_2\text{O})_{20}$ :



# Challenges for global optimization in chemistry:

- larger clusters  $\Leftarrow$  algorithmic improvements needed
- strongly/arbitrarily mixed clusters
- of larger, flexible molecules
- easy access to force-calculation backends: from force-fields to ab-initio quantum chemistry
- prove to be useful in real-life contexts

$\Rightarrow$  development of the general global optimization program suite <sup>5</sup>



(by Johannes Dieterich)

- object-oriented Java
- thread-based SMP parallelism
- MPI-based MPP parallelism
- collision detection
- dissociation detection (Warshall)
- active/passive internal/external coords
- some internal force fields
- with option for their system-specific global reparametrization
- force backends e.g. for
  - Molpro, Orca
  - MNDO, MOPAC, DFTB+
  - AMBER, NAMD, Tinker

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<sup>5</sup> J. M. Dieterich and B. Hartke, Mol. Phys. 108 (2010) 279.

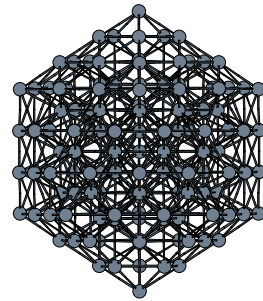
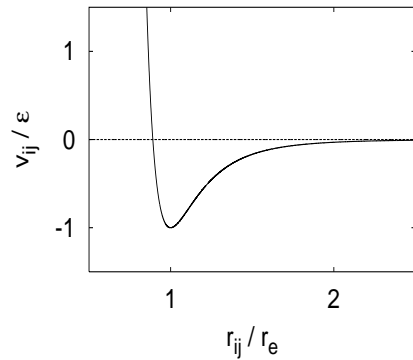
# Strongly mixed clusters: from homogeneous to quinary LJ

reminder: homogeneous LJ clusters

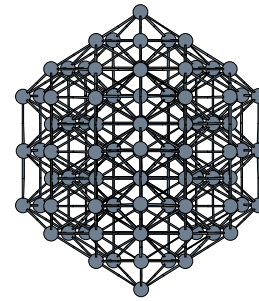
4 structural types known as global minima:

Lennard-Jones potential  
(imperfect model for  
rare gases/benchmark):

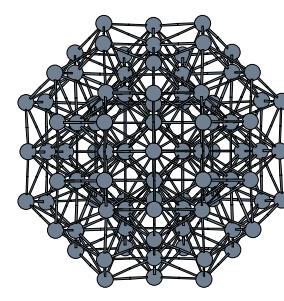
$$v_{ij} = r_{ij}^{-12} - 2r_{ij}^{-6}$$



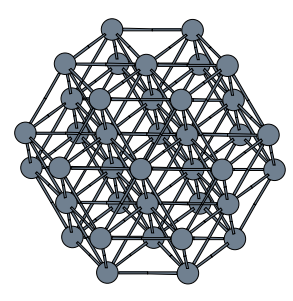
almost all  $n \leq 309$   
icosahedral



$n = 76 \pm 1, 103 \pm 1$   
decahedral



$n = 98$   
tetrahedral<sup>6</sup>

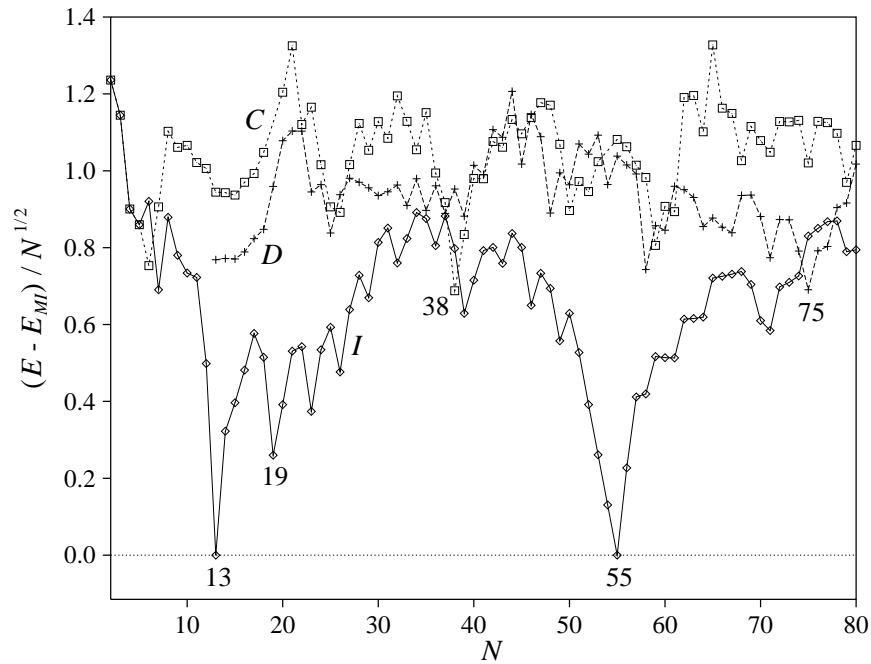


$n = 38, n \geq 750(?)$   
fcc

<sup>6</sup> R. H. Leary and J. P. K. Doye, Phys. Rev. E 60 (1999) R6320.

## Competition between LJ structural types<sup>7</sup>

lowest-energy structure of each type,  
at each cluster size  $N$ :



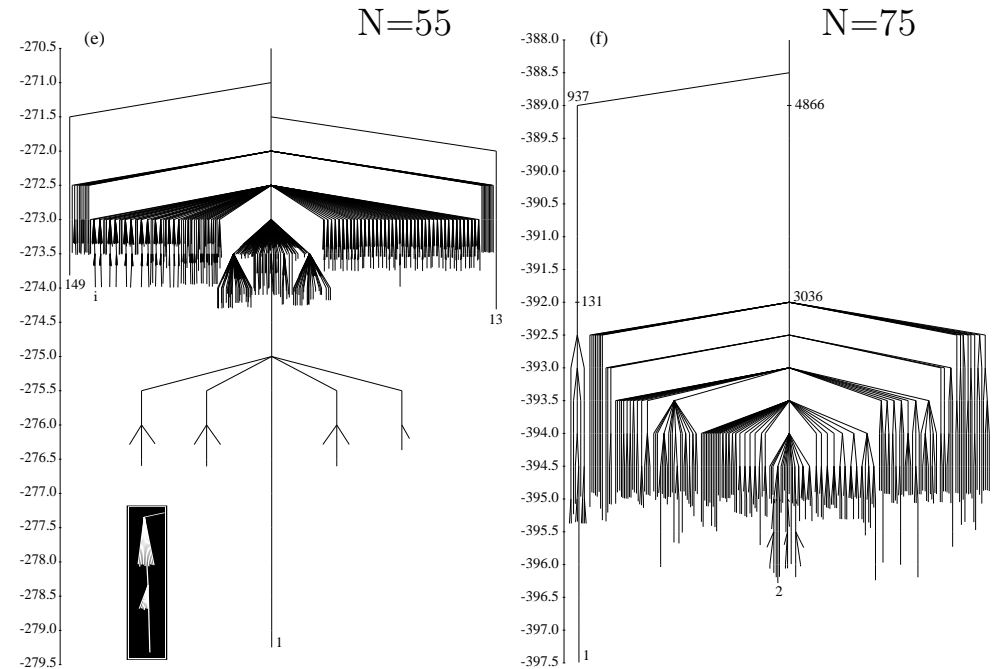
(I: icosahedral, D: decahedral, C: fcc)

⇒ many LJ global minima are easy to find, but a few are very hard.

⇒ expectations for mixed LJ clusters:

sensitivity to structural transitions upon mixing may change with cluster size.

disconnectivity graphs of the lowest-energy minima:



<sup>7</sup> J. P. K. Doye, M. A. Miller and D. J. Wales, J. Chem. Phys. 111 (1999) 8417.

# Ab-initio fitted mixed LJ pair potentials <sup>8</sup>

generalized LJ(6,16,2) ansatz:

$$v_{ij} = 4\epsilon_{ij} \sum_{k=6,k+2}^{16} \text{sign}(\sigma_{k,ij}) \cdot \left(\frac{\sigma_{k,ij}}{r_{ij}}\right)^k \quad (1)$$

with parameters globally fitted to CCSD(T)/aug-cc-pV5Z data.

Comparison: standard LJ(6,12,6) potentials

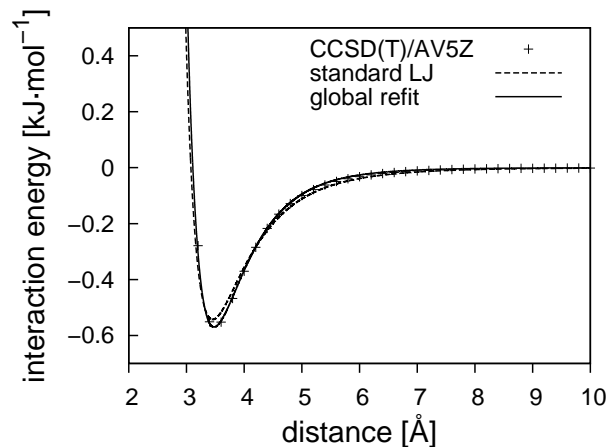
$$v_{ij} = 4\epsilon_{ij} \left[ \left(\frac{\sigma_{ij}}{r_{ij}}\right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}}\right)^6 \right] \quad (2)$$

with Lorentz-Berthelot mixing rules:

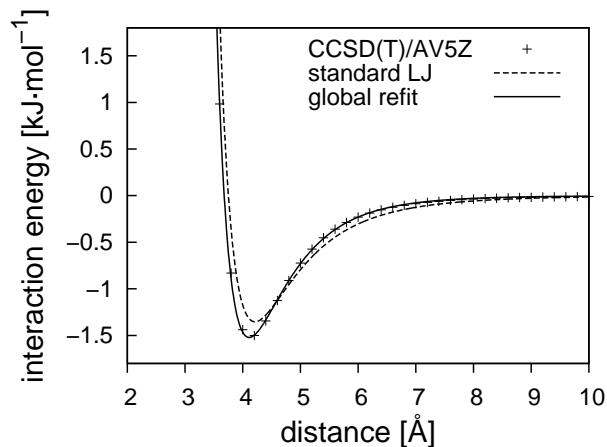
$$\epsilon_{ij} = \sqrt{\epsilon_i \cdot \epsilon_j} \quad (3)$$

$$\sigma_{ij} = \frac{\sigma_i + \sigma_j}{2} \quad (4)$$

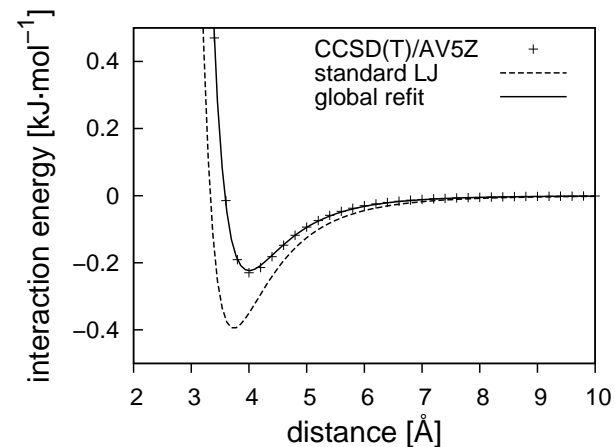
exemplary comparisons for some pair potentials:



Ne-Ar



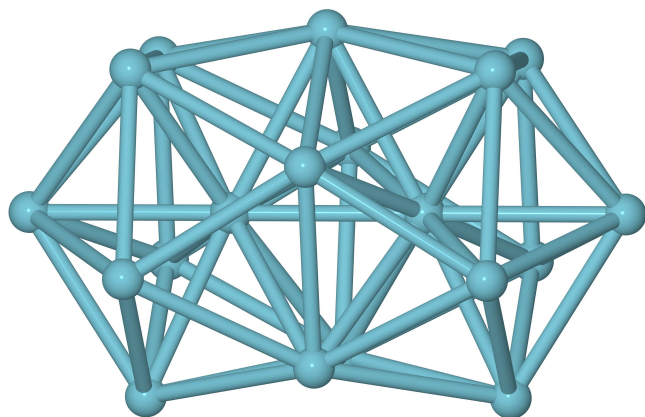
Ar-Xe



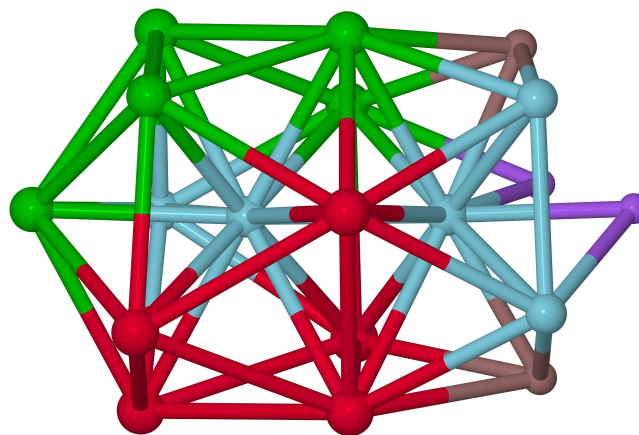
He-Xe

<sup>8</sup> J. M. Dieterich and B. Hartke, manuscript in preparation.

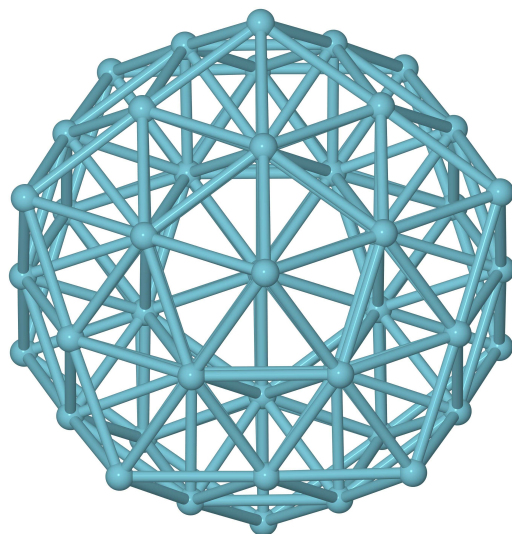
## Structural stability under strong mixing <sup>9</sup>



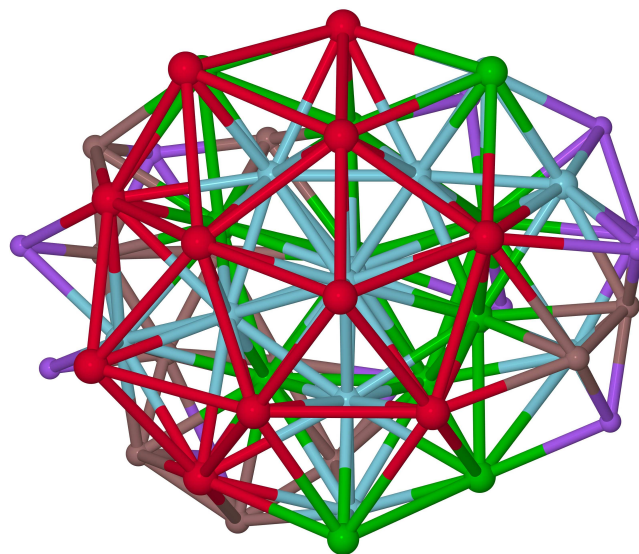
Ar<sub>19</sub>



Ar<sub>5</sub>Kr<sub>5</sub>Xe<sub>5</sub>He<sub>2</sub>Ne<sub>2</sub>



Ar<sub>55</sub>



Ar<sub>11</sub>Kr<sub>11</sub>Xe<sub>11</sub>He<sub>11</sub>Ne<sub>11</sub>

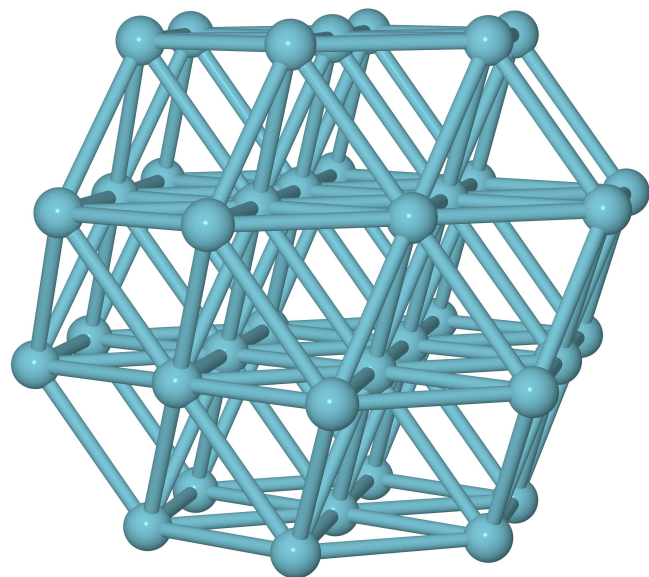
(global minima)

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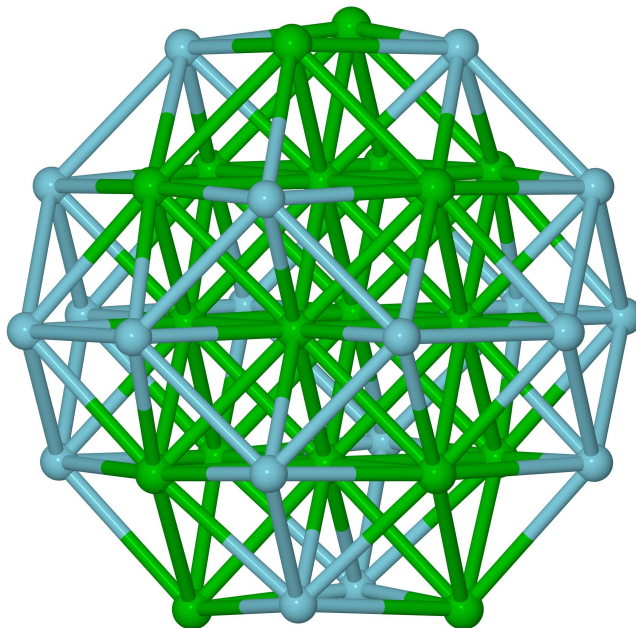
<sup>9</sup> J. M. Dieterich and B. Hartke, manuscript in preparation.



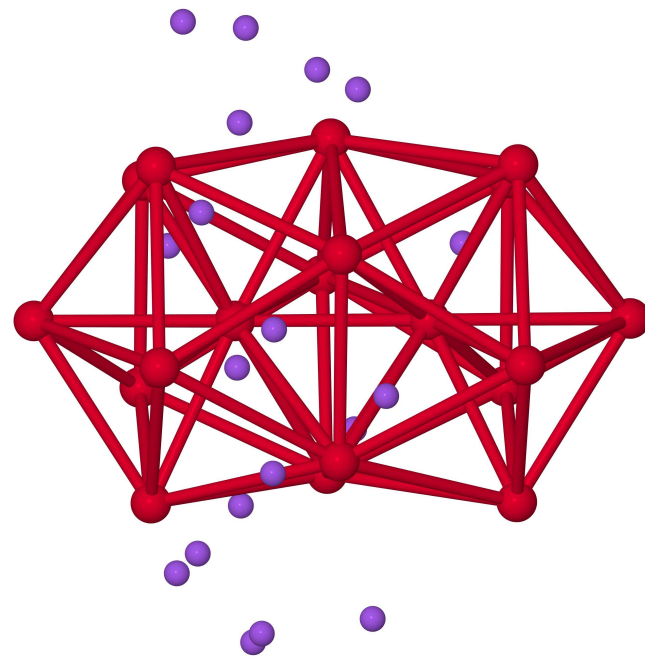
# Mixing-induced structural transitions in $\text{LJ}_{38}$ <sup>10</sup>



$\text{Ar}_{38}$   
fcc



$\text{Ar}_{19}\text{Kr}_{19}$   
still fcc



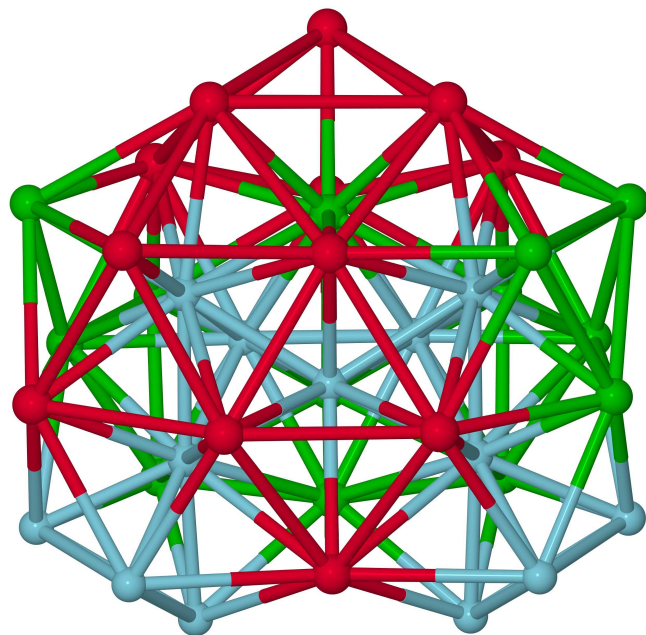
$\text{He}_{19}\text{Xe}_{19}$   
core-shell icosahedral

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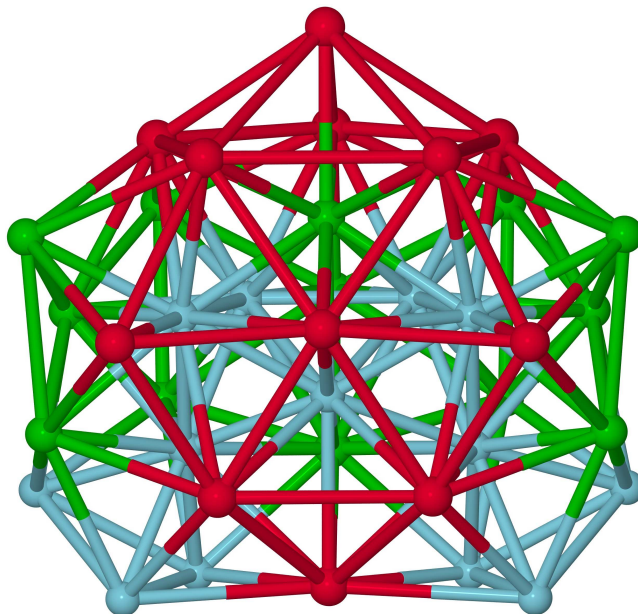
<sup>10</sup> J. M. Dieterich and B. Hartke, manuscript in preparation.

# Mixing-induced structural transitions in $\text{LJ}_{38}$ <sup>11</sup>

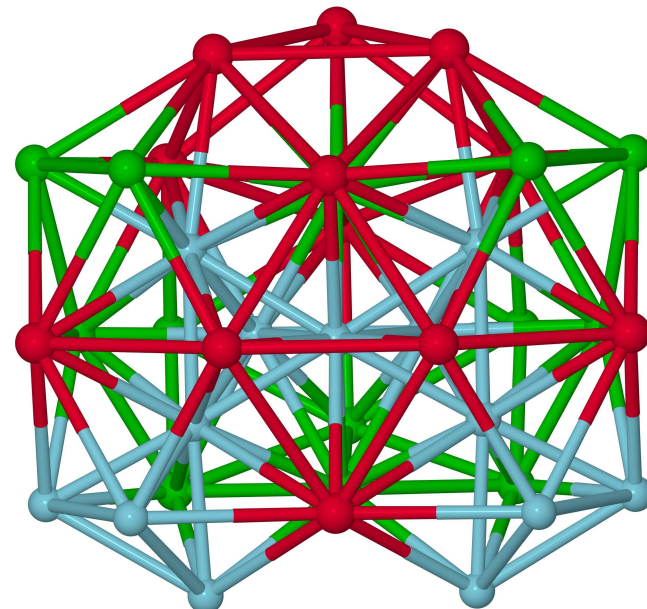
$\text{Ar}_{13}\text{Kr}_{13}\text{Xe}_{12}$ : strongly icosahedrally dominated



-280.4455 kJ/mol  
(global minimum, rank 0)



-280.2024 kJ/mol  
(rank 1)



-280.1620 kJ/mol  
(rank 2)

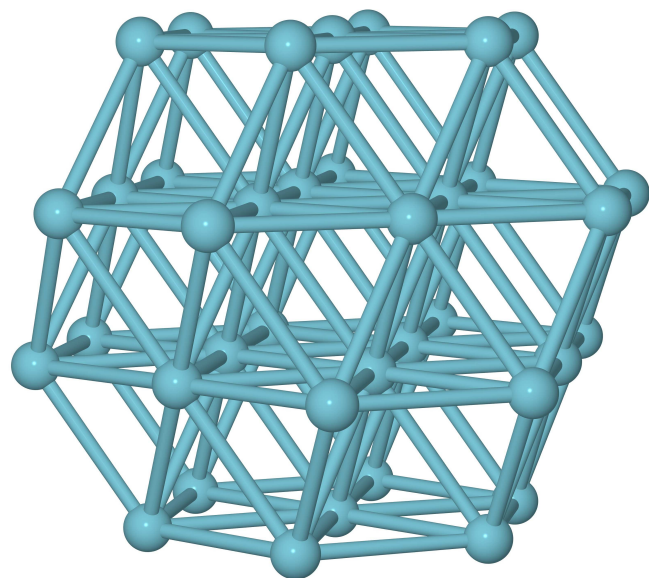
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<sup>11</sup> J. M. Dieterich and B. Hartke, manuscript in preparation.

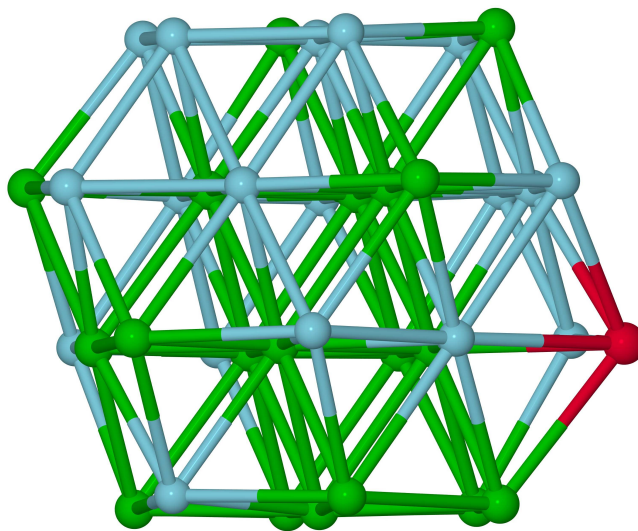


# Mixing-induced structural transitions in $\text{LJ}_{38}$ <sup>12</sup>

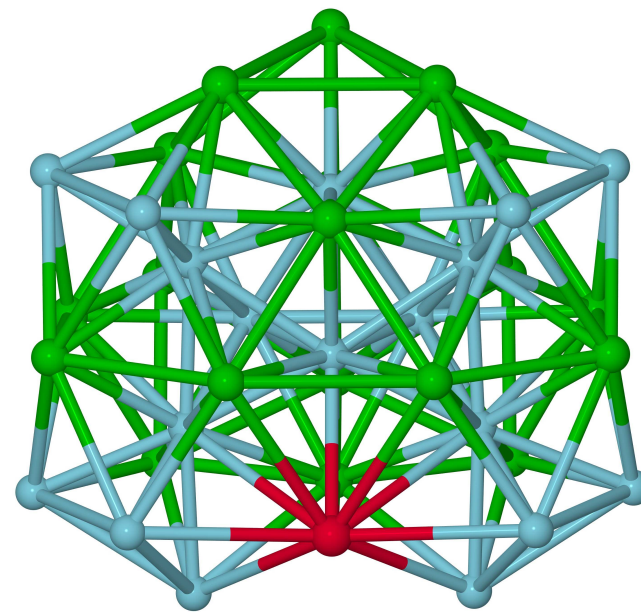
$\text{Ar}_{18}\text{Kr}_{19}\text{Xe}$ : global minimum icosahedral, but low-energy fcc present



$\text{Ar}_{38}$



-242.5240 kJ/mol  
(best fcc)



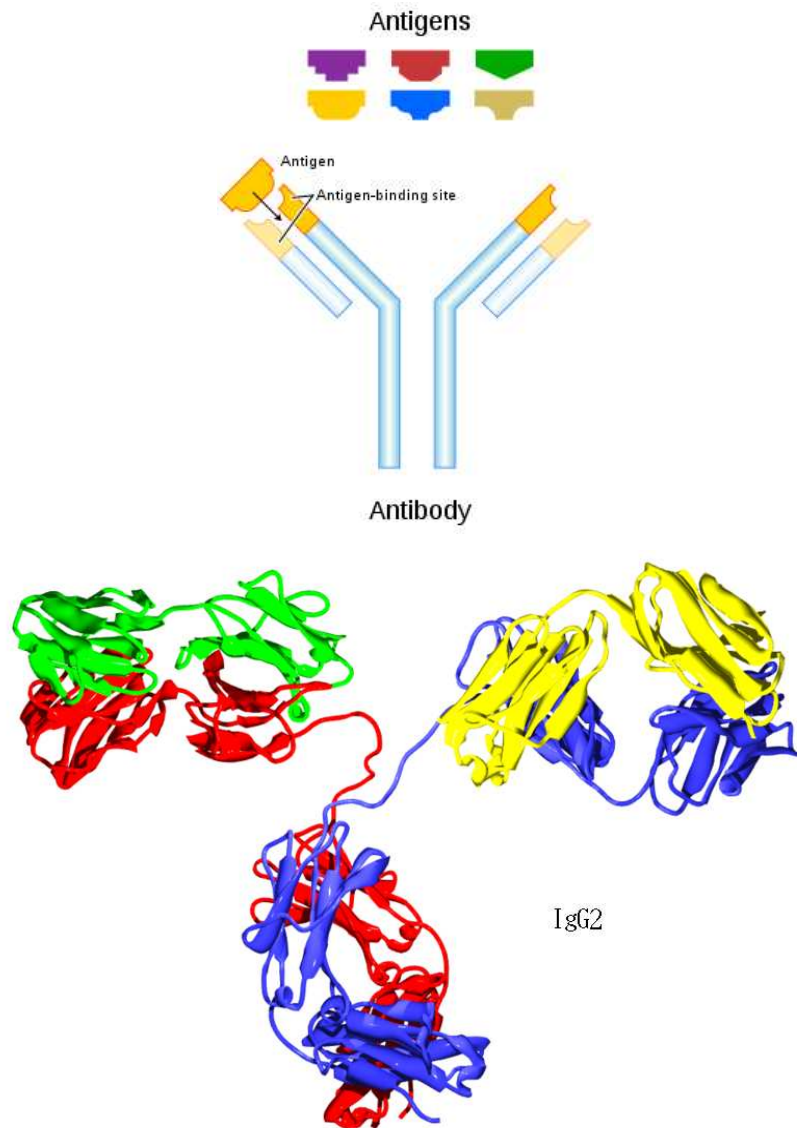
-248.8864 kJ/mol  
(global minimum)

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<sup>12</sup> J. M. Dieterich and B. Hartke, manuscript in preparation.

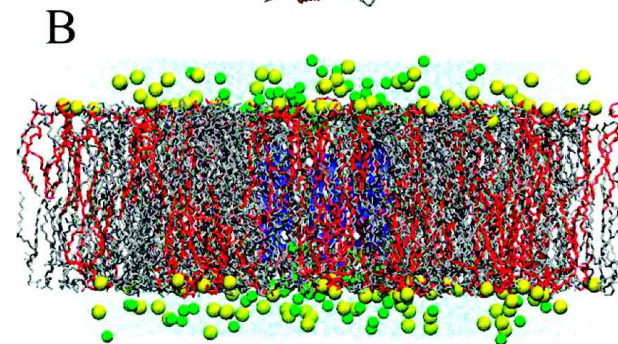
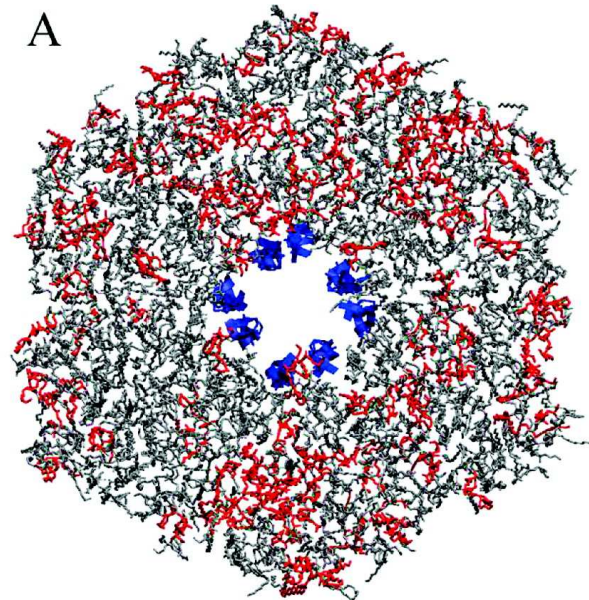
# Mixed clusters of flexible molecules: Theoretical chemistry in medicine

adaptive immune system:



innate immune system:

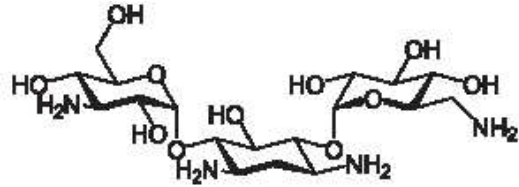
pathogen-associated molecules (PAMs) →  
antimicrobial peptides (AMPs) → cell wall damage:



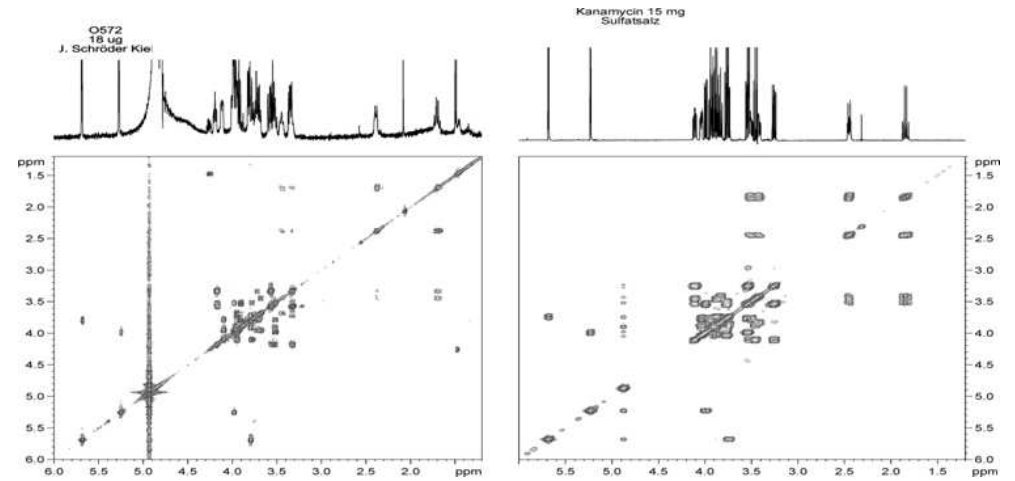
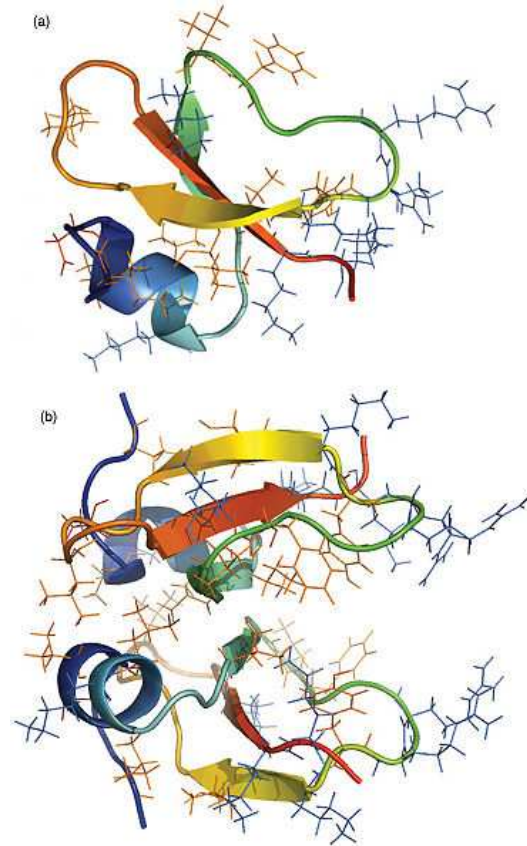
protegrin-1 octamer pore in lipid bilayer

*Pseudomonas aeruginosa*  $\Rightarrow$  unknown PAM

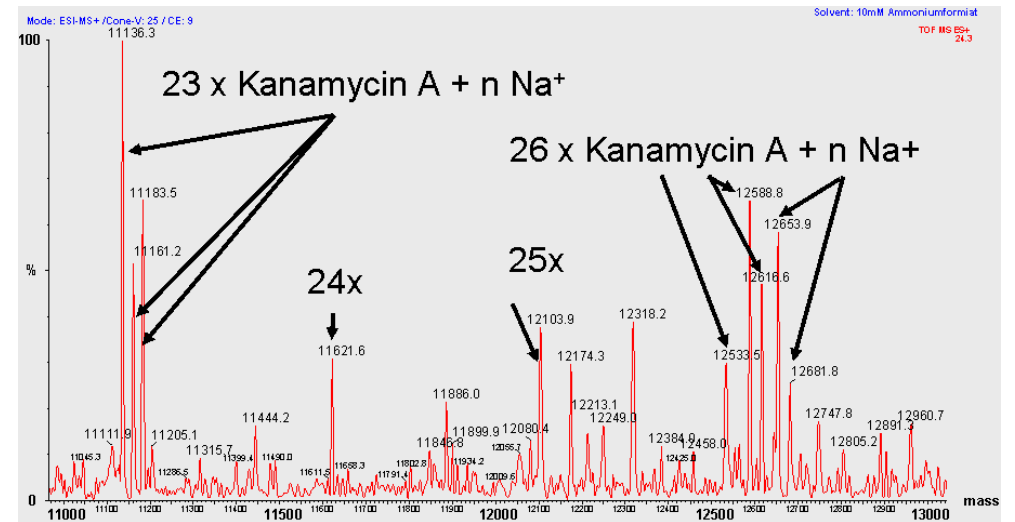
J.-M. Schröder (dermatology, Univ Kiel):  
clusters of Kanamycin A (with  $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Cu}^{2+}$ )



induce human  $\beta$ -defensin in epithelial cells



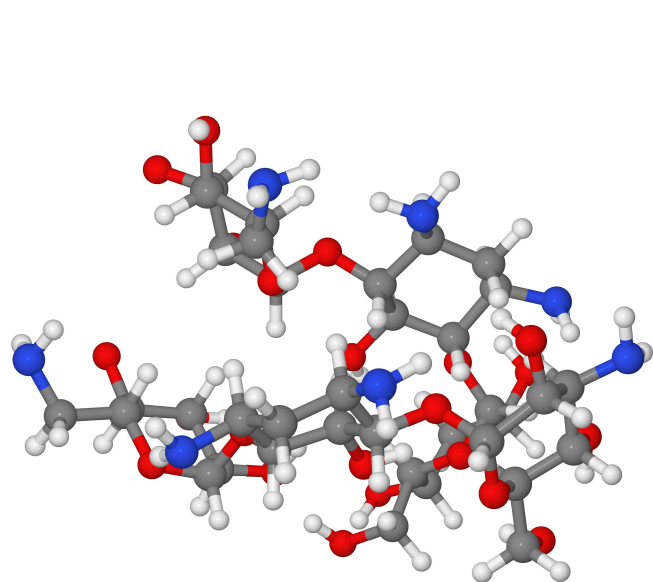
COSY- $^1\text{H}$ -NMR of active PAM (left), Kanamycin A (right)



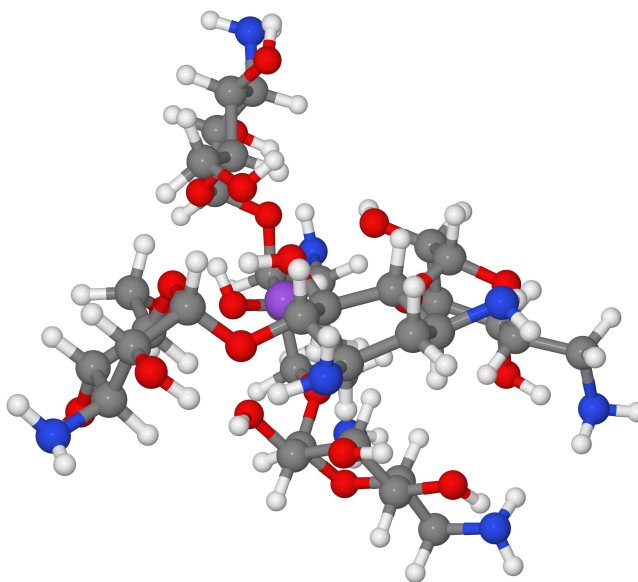
ESI-MS analysis of active PAM (with some assignments)



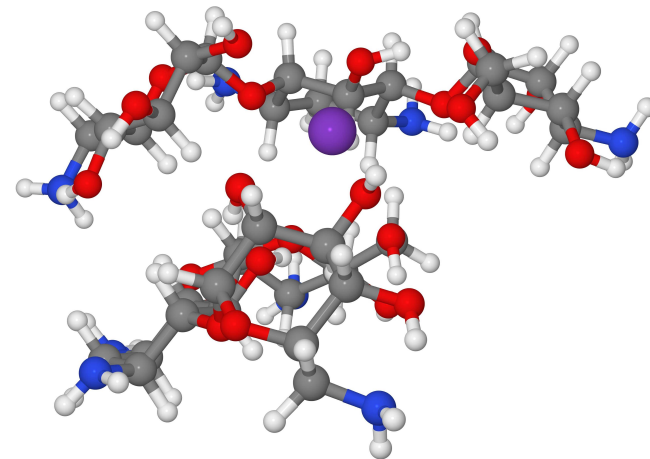
## global structure optimization of Kanamycin-A dimers <sup>13</sup>



(Kanamycin A)<sub>2</sub>



(Kanamycin A)<sub>2</sub>Na<sup>+</sup>



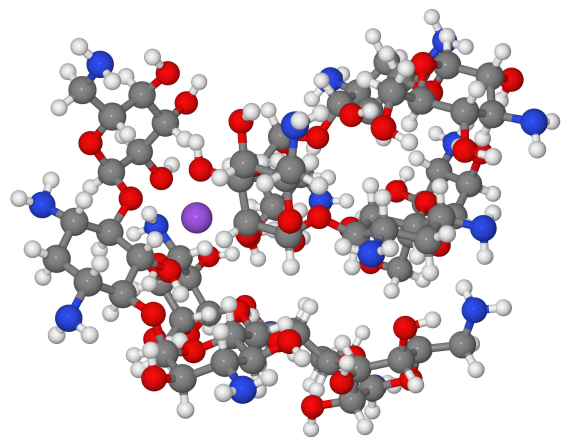
(Kanamycin A)<sub>2</sub>K<sup>+</sup>

- exp. and theory agree: preference of Na<sup>+</sup> over K<sup>+</sup>
- predicted IR and NMR spectra show experimentally accessible signatures of aggregation

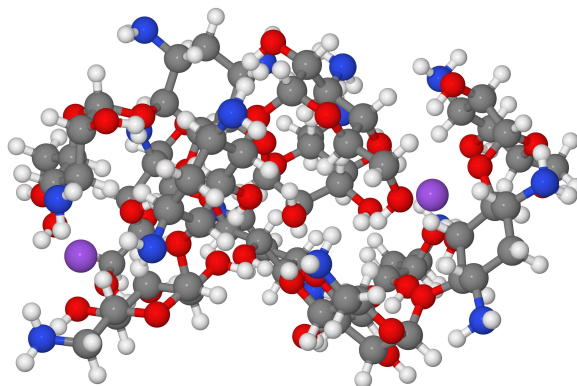
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<sup>13</sup> J. M. Dieterich, U. Gerstel, J.-M. Schröder and B. Hartke, manuscript in preparation.

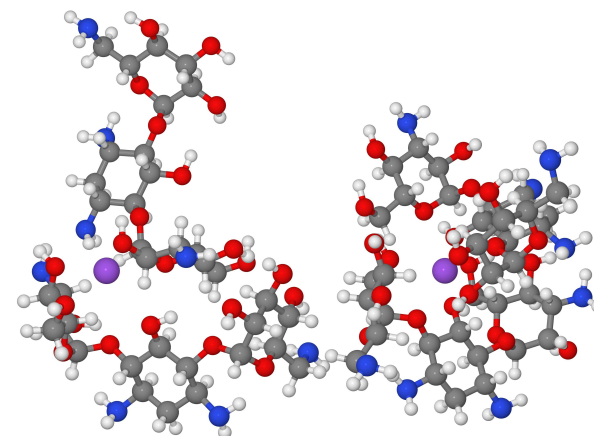
## larger Kanamycin-A clusters <sup>14</sup>



$(\text{Kanamycin A})_4\text{Na}^+$   
very stable



$(\text{Kanamycin A})_2\text{Na}_2^+$   
barely stable



$(\text{Kanamycin A})_2\text{Na}_2^+$   
dissociated

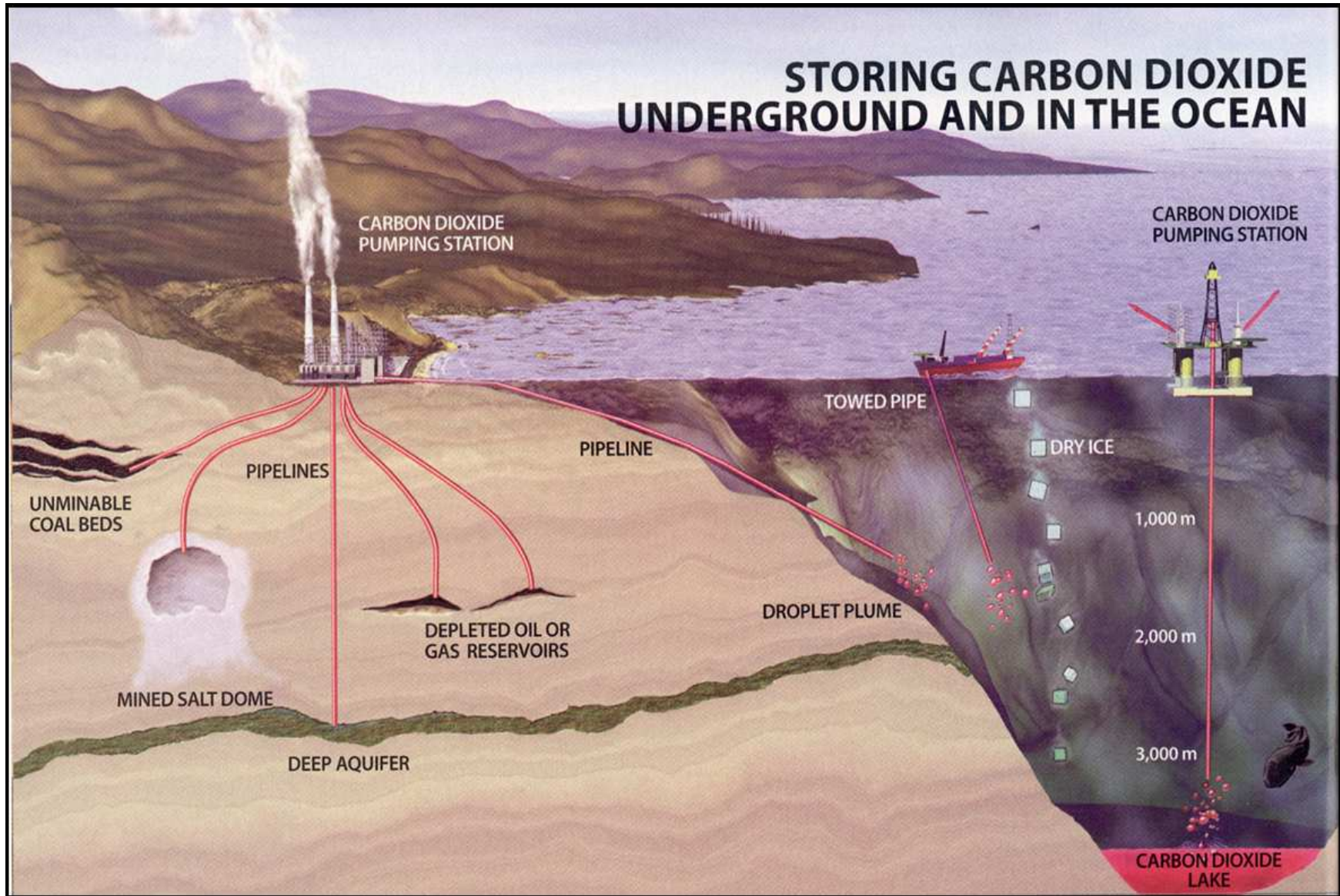
future work:

- larger clusters, with more/different ions
- prediction of which minor constitutional isomer is likely to be present experimentally
- design of low-molecular weight AMP-inducing agents

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<sup>14</sup> J. M. Dieterich and B. Hartke, Mol. Phys. 108 (2010) 279.

## Cluster of Excellence: Future Ocean, project A5: CO<sub>2</sub> sequestration



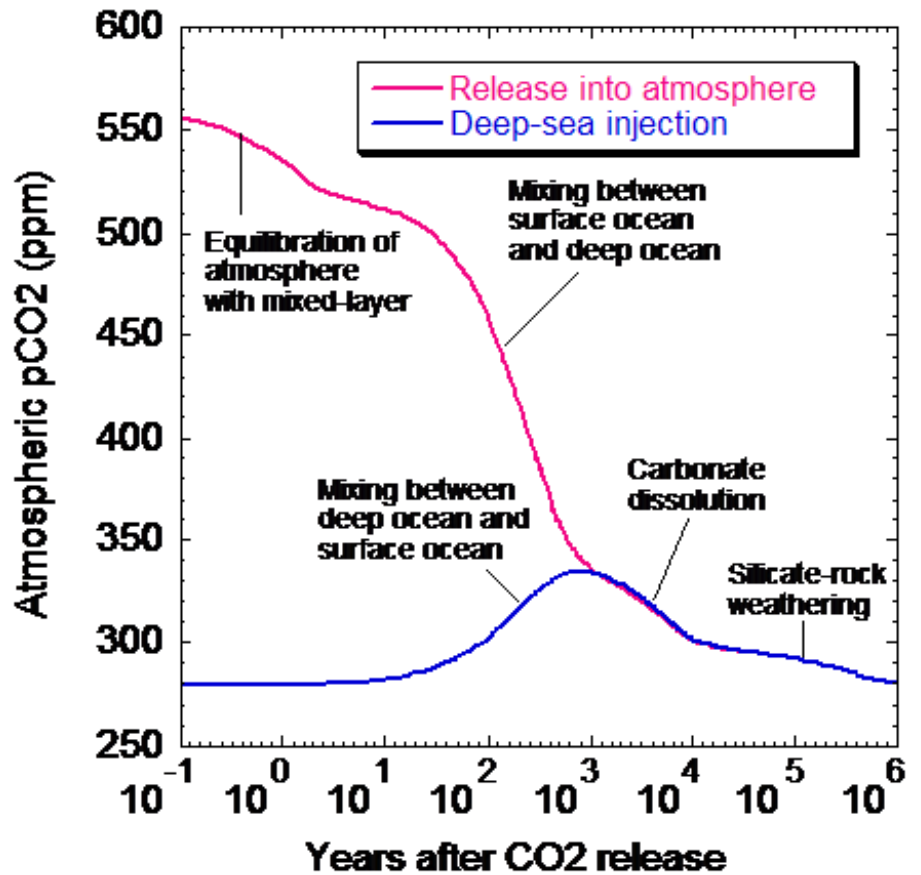
## Future Ocean A5: CO<sub>2</sub> sequestration

Global simulation of CO<sub>2</sub> distribution

(Ken Caldeira, Lawrence Livermore National Lab):

instantaneous addition of a mass of CO<sub>2</sub> equal to the pre-industrial amount, introduced either to the atmosphere or the deep ocean.

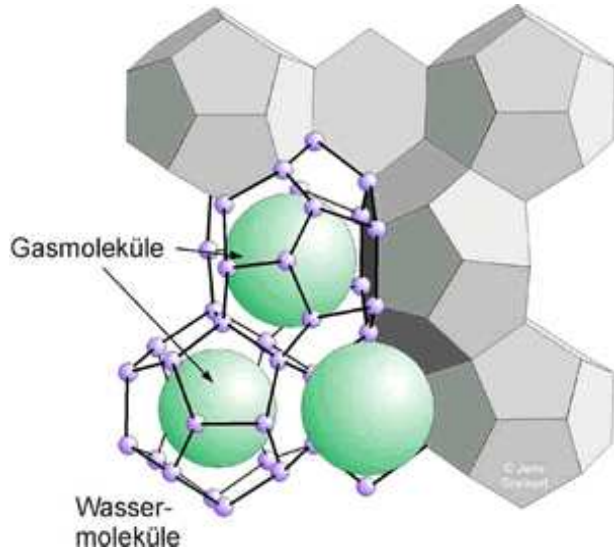
- equilibrium CO<sub>2</sub> uptake by the ocean 90% anyway: same state after relaxation of the system (note time scale: 10<sup>6</sup> years!)
- *but “benefit”*: ocean sequestration avoids atmospheric “detour” of CO<sub>2</sub> and hence (hopefully) most of its adverse climatic effects (on a time scale of 100 years)



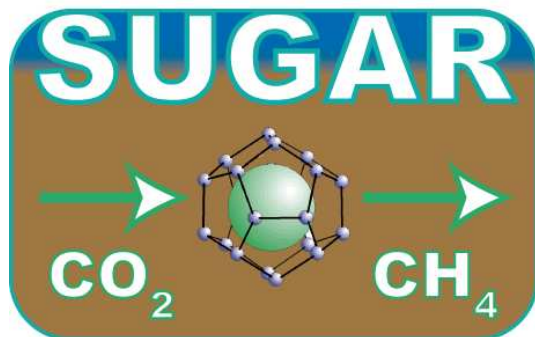
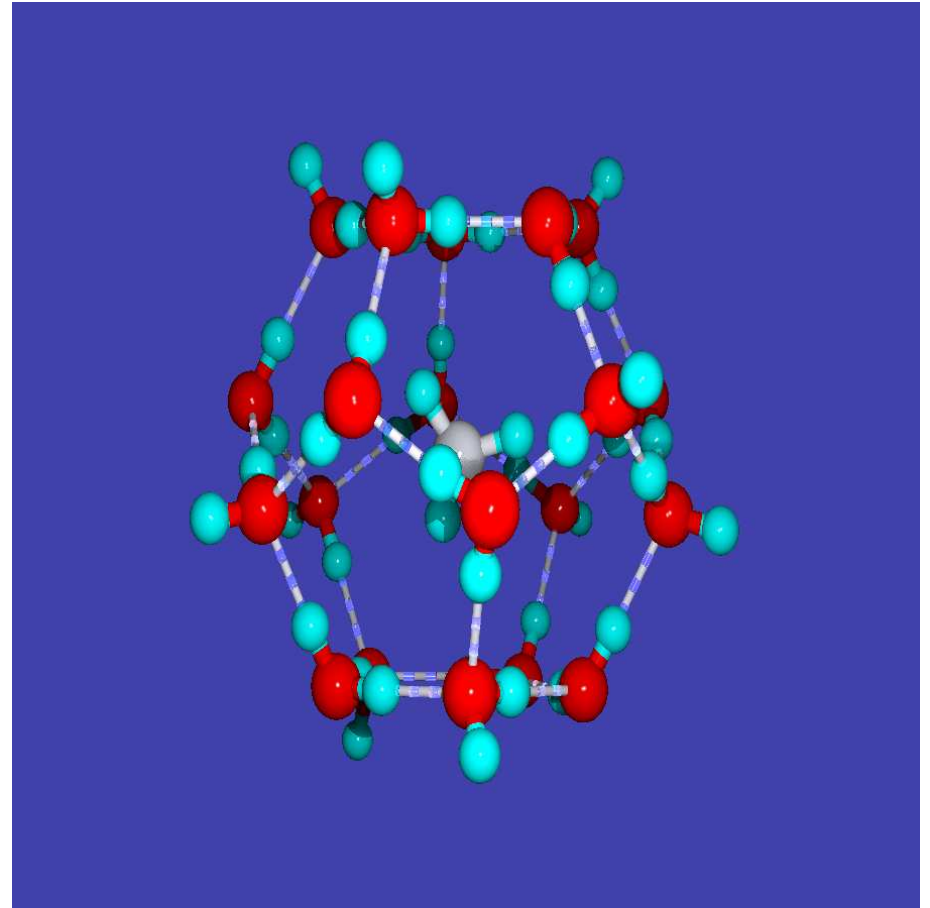


# Global optimization in marine/climate research: clathrate hydrates

- water cages around  $\text{CH}_4$ ,  $\text{CO}_2$ ,...

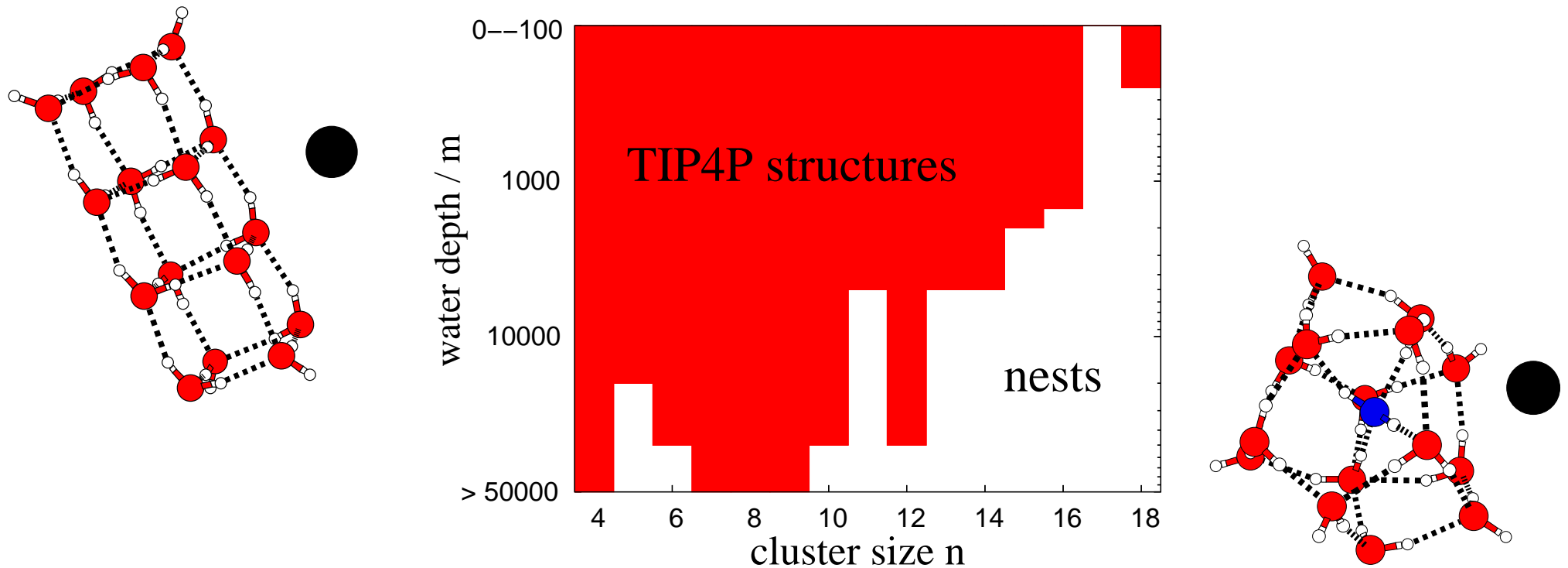


- stable at low T & high p  $\Rightarrow$  large deposits of  $\text{CH}_4$  hydrate on the seafloor (continental shelves)
- carbon content  $\approx$  fossil fuels
- possibility to get rid of  $\text{CO}_2$  (sequestration, CCS)
- $\Rightarrow$  current commercial dream:

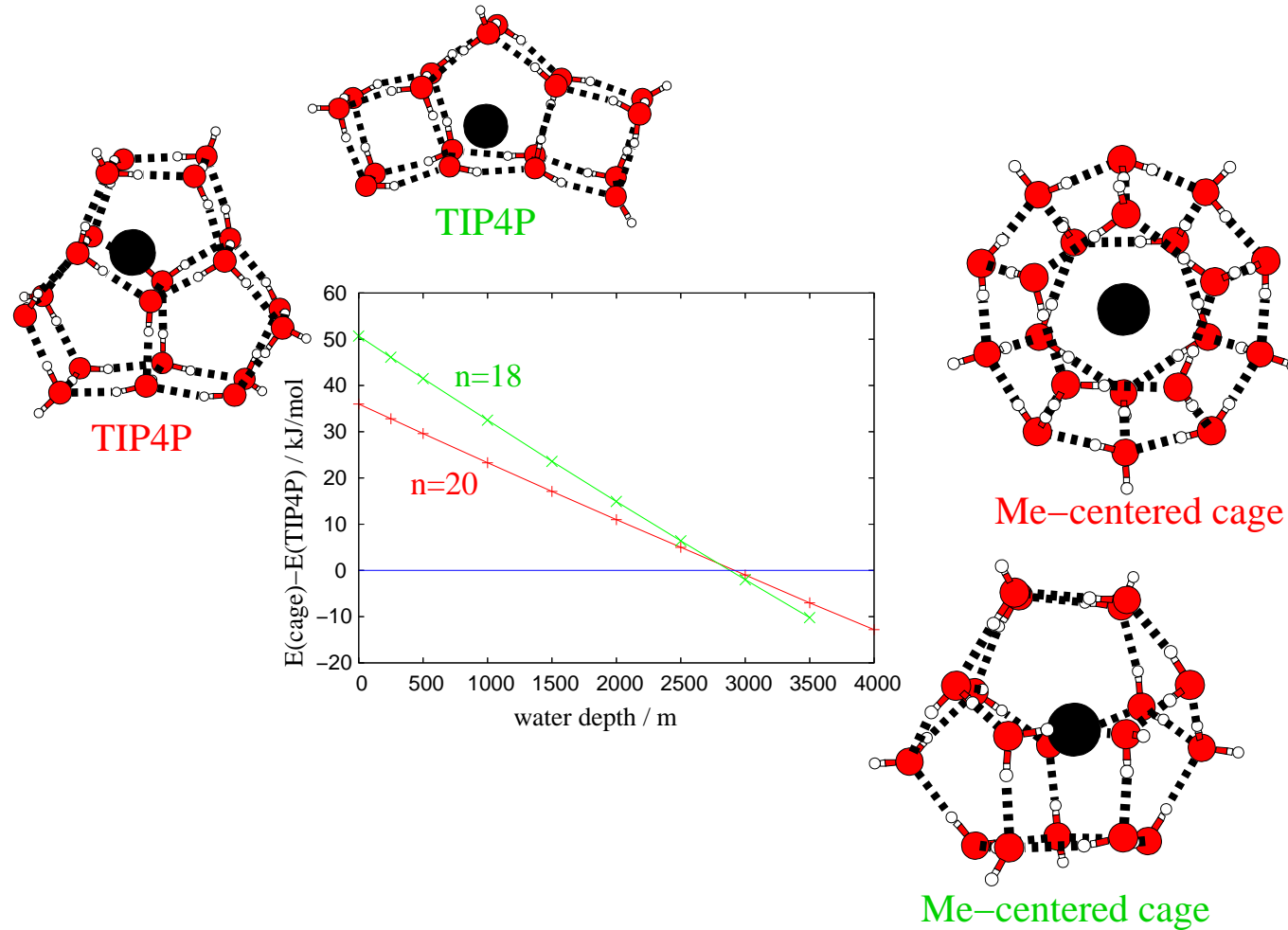




# Methane hydrate clusters: global optimization under pressure <sup>15</sup>



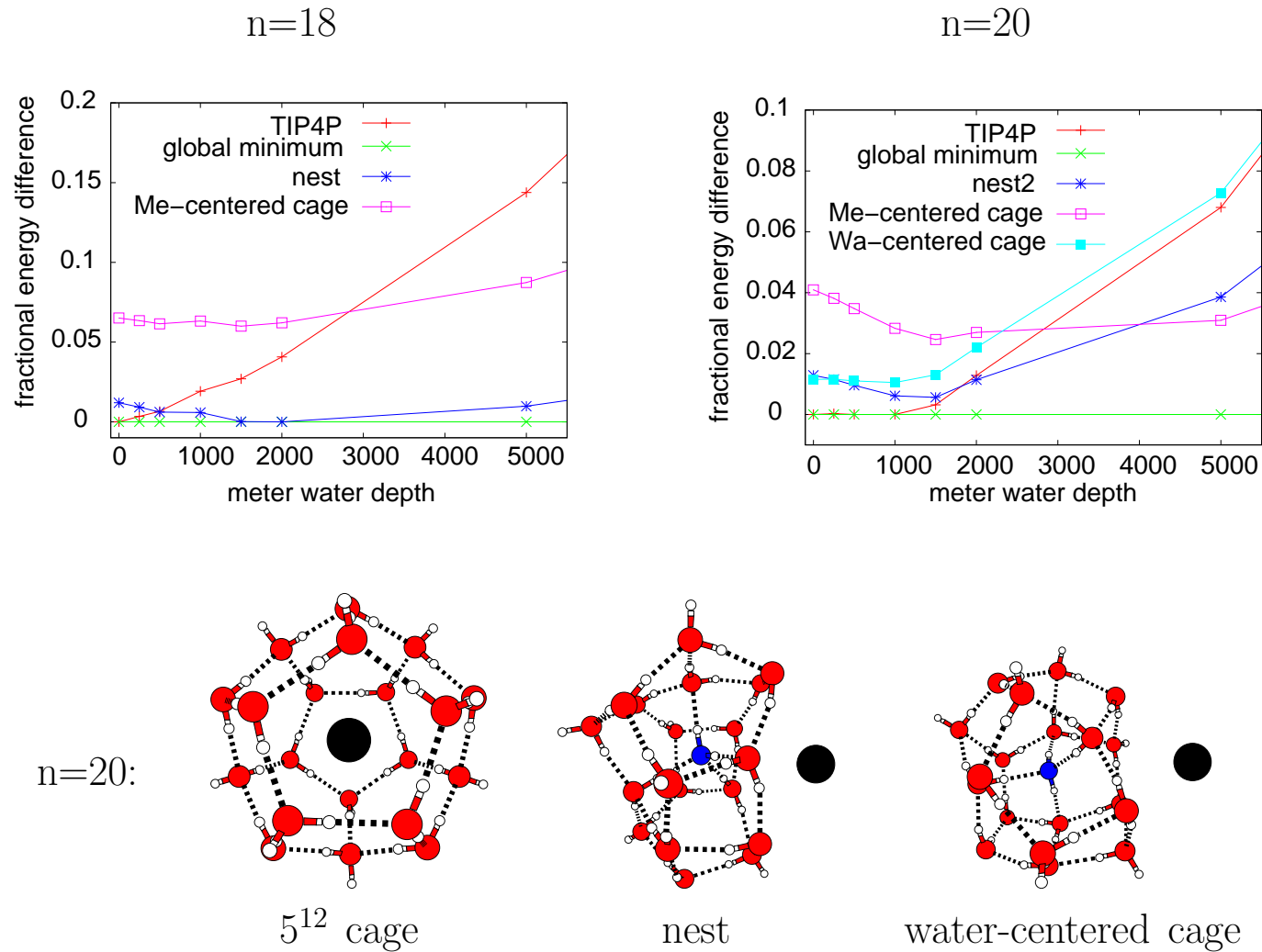
outlook to larger clusters:



transition to Me-centered cages expected for  $n \approx 20$  and 3000m

**but this is wrong!** (local optimization of given structures, no global optimization)

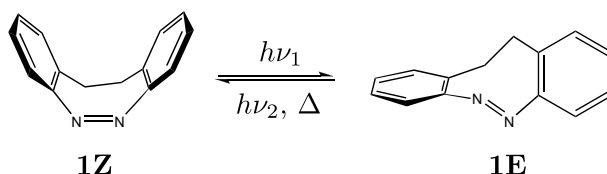
what really happens at  $n=18,20$  after global optimization:



There *is* a crossover between the TIP4P structures and clathrate-like Me-centered cages at 3000m — but global optimization finds many better nests.

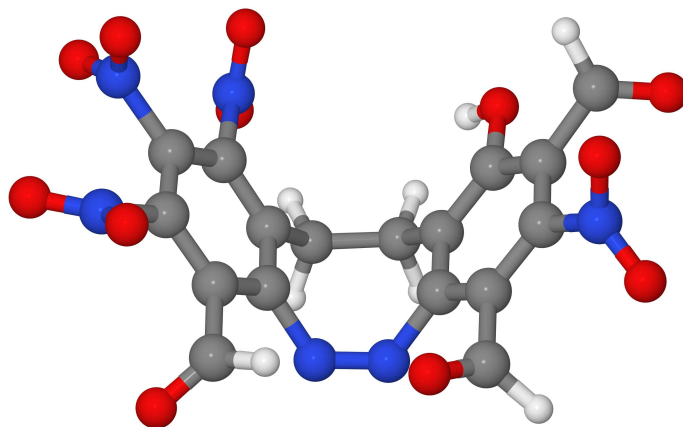
# Global optimization for the design of photochemically switchable molecules

In recent work <sup>16</sup>, a bridged azobenzene derivative was found to have superior photophysical properties compared to the parent compound:



Application of OGOLEM to globally optimize substituent patterns (8 allowed types);  
aim: move absorption maxima from 425nm (cis) / 579nm (trans)  
to the easily accessible 405nm / 532nm (commercial laserpointers).

Optimized compound <sup>17</sup>: multiple nitro and carbonyl groups, absorption maxima: 405nm / 532nm

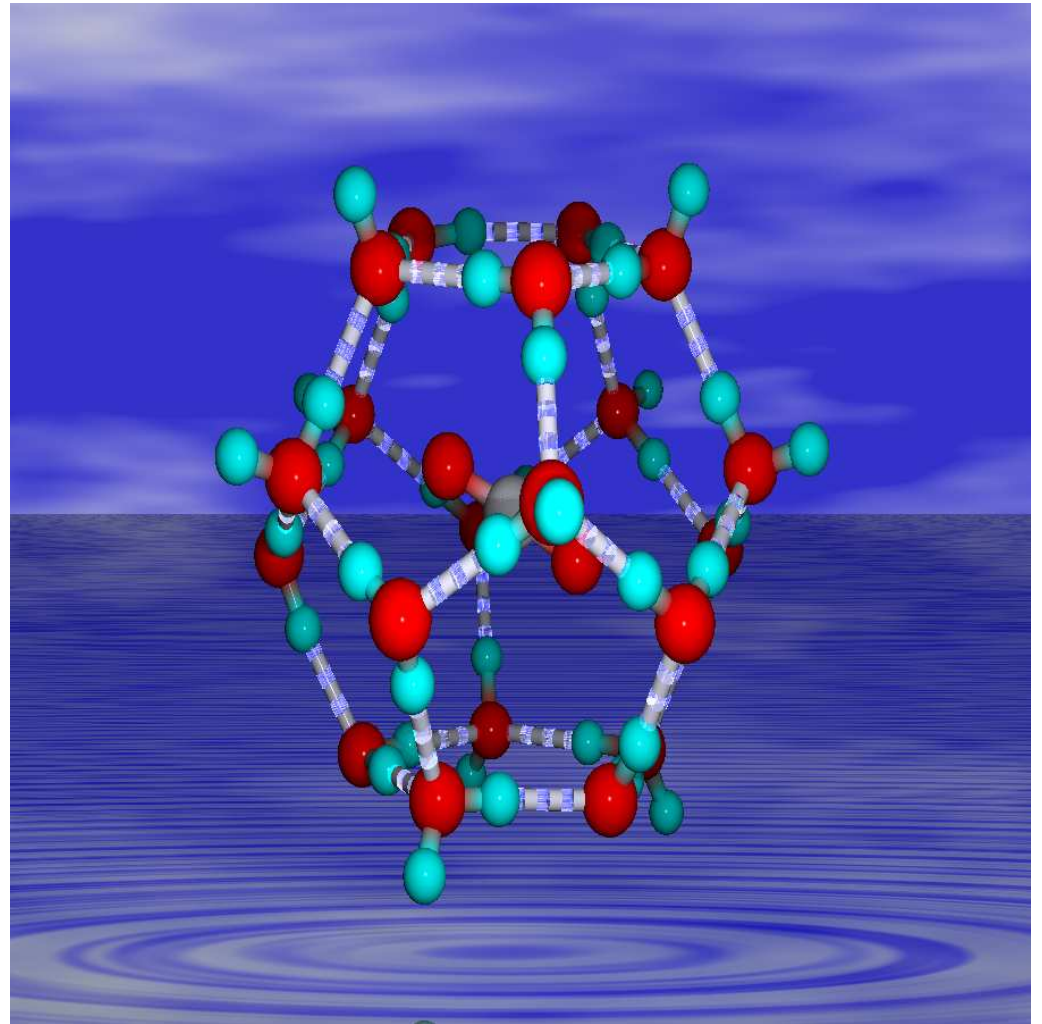


<sup>16</sup> R. Siewertsen, H. Neumann, B. Buchheim-Stehn, R. Herges, C. Näther, F. Renth, and F. Temps, J. Am. Chem. Soc 131 (2009) 15594.

<sup>17</sup> N. O. Carstensen, J. M. Dieterich and B. Hartke, manuscript in preparation.

## acknowledgements:

- J. Dieterich, Ole Carstensen (AK Hartke)
- J.-M. Schröder (dermatology, Univ Kiel)
- A. Neumaier (Mathematics, Univ Wien)
- S. T. Bromley (Univ Barcelona)
- B. Abel (MPI and Univ Göttingen)
- A. Lüchow (RWTH Aachen)
- V. Buch † (Jerusalem)
- J. W. Ponder (St.Louis/Missouri)
- J. M. Lisy (Urbana-Champaign/Illinois)
- U. H. E. Hansmann (Houghton/Michigan)



**cpu time:** computing center Univ Kiel, North-German supercomputing alliance (HLRN Berlin/Hannover)

**€€€:** DFG (Normalverfahren, SFB, Exzellenzcluster);

Innovationsfonds des Landes Schleswig-Holstein (Numerik-Zentrum Uni Kiel)

# Basic problem of global cluster structure optimization: computational complexity

## atomic clusters:

possibly exponential increase in number of local minima with cluster size:

- empirical scaling formula for number of minima in Lennard-Jones clusters (Hoare/McInnes):

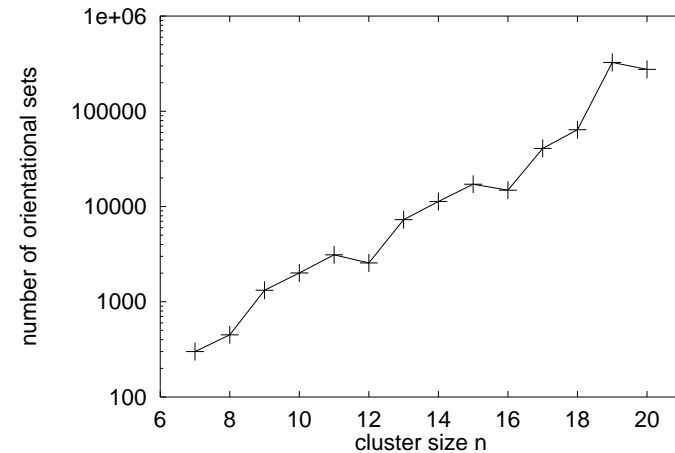
$$\exp(0.028 n^2 + 0.3572 n - 2.5176) \quad (5)$$

$\Rightarrow$  98 LJ atoms have  $10^{130}$  local minima

- other estimates vary wildly, e.g.:  
98 LJ atoms have  $10^{40}$  local minima
- abstract proof: NP-hard problem<sup>18</sup>,  
but proof not quite correct<sup>19</sup>

## molecular clusters:

3 additional orientational degrees of freedom per particle.  
example: purely orientational subspace of water clusters<sup>20</sup>



$\Rightarrow$  combination of two interdependent exponentially scaling problems

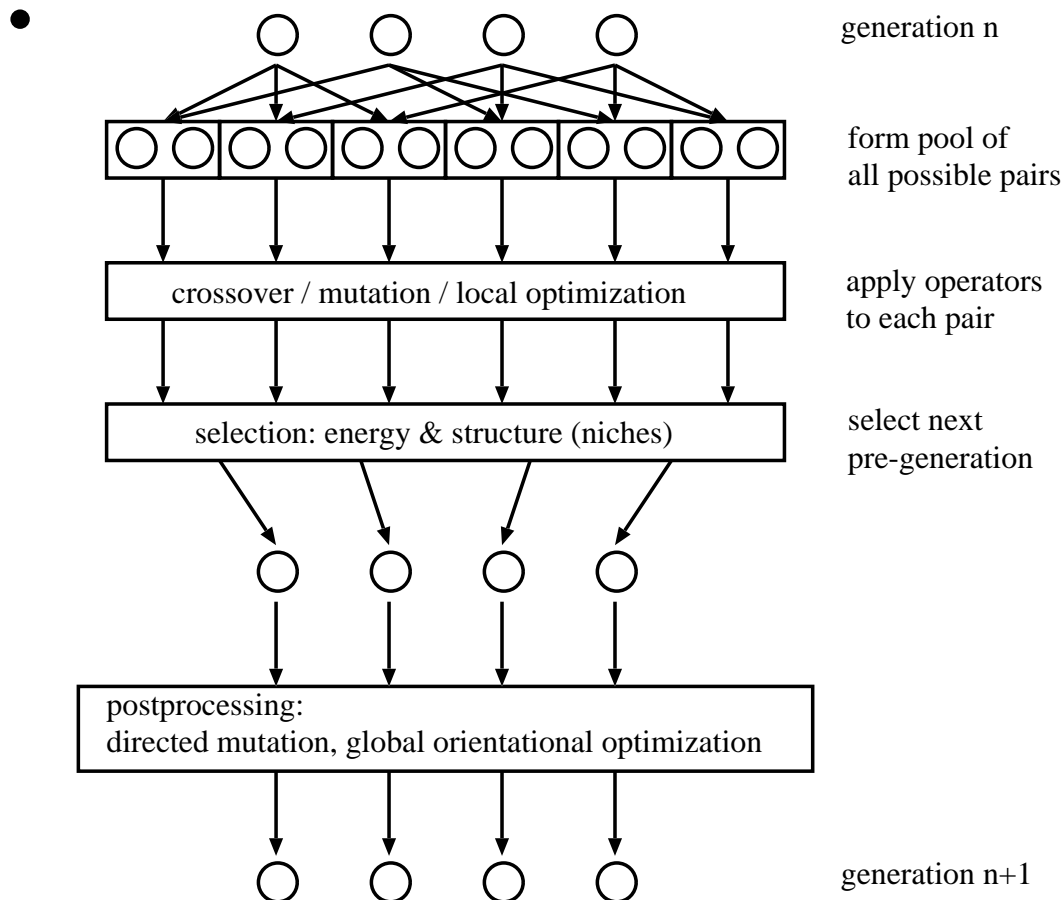
- deterministic global optimization is catching up, but still not applicable to clusters of nontrivial size  
 $\Rightarrow$  alternative: stochastic-heuristic algorithms
- $\Rightarrow$  clusters of interesting sizes can be studied,  
but guarantee for finding global minima is lost.

<sup>18</sup> L. T. Wille and J. Vennik, J. Phys. A 18 (1985) L419.

<sup>19</sup> A. Srivastav, Univ. of Kiel, personal communication.

<sup>20</sup> B. Hartke, Z. Phys. Chem. 214 (2000) 1251.

# Global optimization by Evolutionary Algorithms<sup>21 22 23</sup>



## important aspects:

- design problem-specific crossover, exploiting near-separability
- local optimization vital but expensive; use loose thresholds initially
- “directed mutation” scans structurally similar minima
- strongly deceptive landscapes need secondary selection criteria: problem-specific niches

<sup>21</sup> B. Hartke, J. Phys. Chem. 97 (1993) 9973.

<sup>22</sup> D. M. Deaven and K. M. Ho, Phys. Rev. Lett. 75 (1995) 288.

<sup>23</sup> B. Hartke, J. Comput. Chem. 20 (1999) 1752.

# Parallel implementation<sup>24</sup>

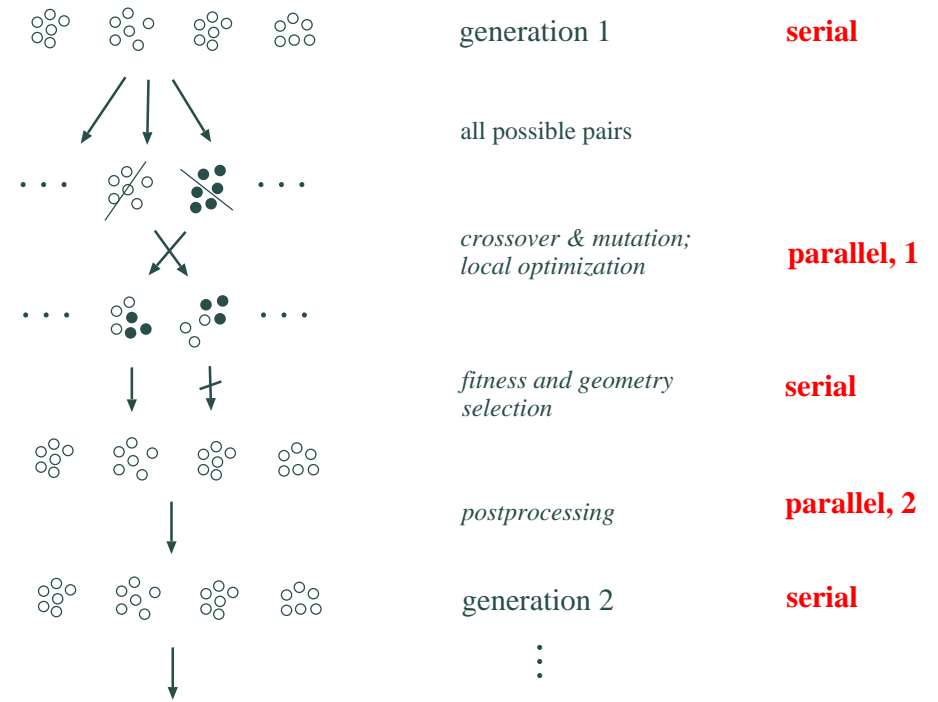
properties of the algorithm

- local optimizations need >90% of total CPU time;
- but they are independent of each other, and
- data communication demands are very low.

⇒ “embarrassingly parallel”:

realization:

- Fortran90 with simple, explicit standard-MPI calls;
- “master-slave” model
- time for local optimizations varies  
→ implicit load-balancing by  $\# \text{ locOpt} \gg \# \text{ processes}$



Problem: lengths of local optimization tasks vary, but serial bottlenecks in generational scheme need synchronization of all processes ⇒ significant idle times in most processes

<sup>24</sup> project at the national computer center HLRN (Berlin/Hannover)



# Global geometry optimization of clusters on ab-initio potentials, guided by empirical potentials

## Problem:

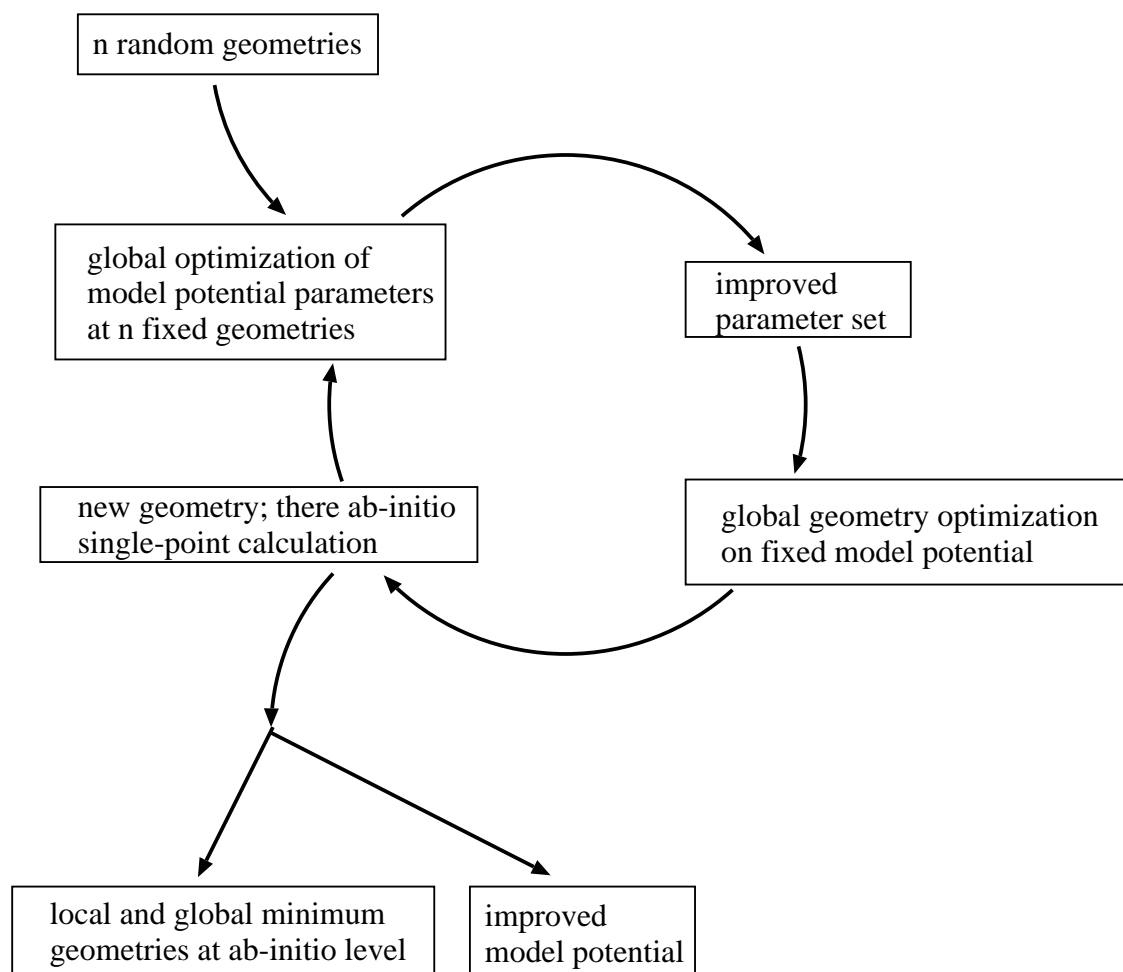
*Every* known heuristic is too expensive on ab-initio potentials.

## Solution:<sup>25</sup>

Globally optimize cluster geometries on a model potential that is simultaneously fit to ab-initio single-point calculations (*surrogate function optimization*):

*work in progress:*

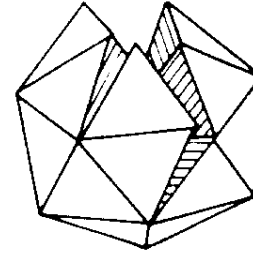
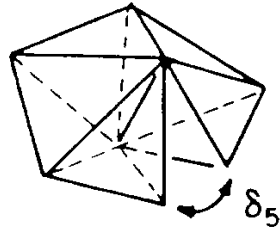
- inclusion of gradient information;
- surrogate function with smooth interpolation between ab-initio data and model data;
- universal surrogate functional forms.



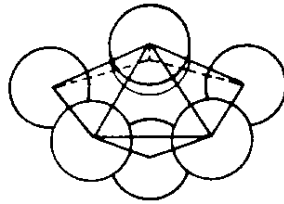
<sup>25</sup> B. Hartke, Chem. Phys. Lett. 258 (1996) 144.

# Hard sphere packing

packing of *a few* hard spheres  
→ tetrahedron at  $n = 4$ .

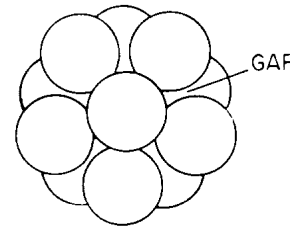


Packing of regular tetrahedrons  
*almost* leads to larger regular forms:



$n = 7$

pentagonal bipyramid



$n = 13$

icosahedron

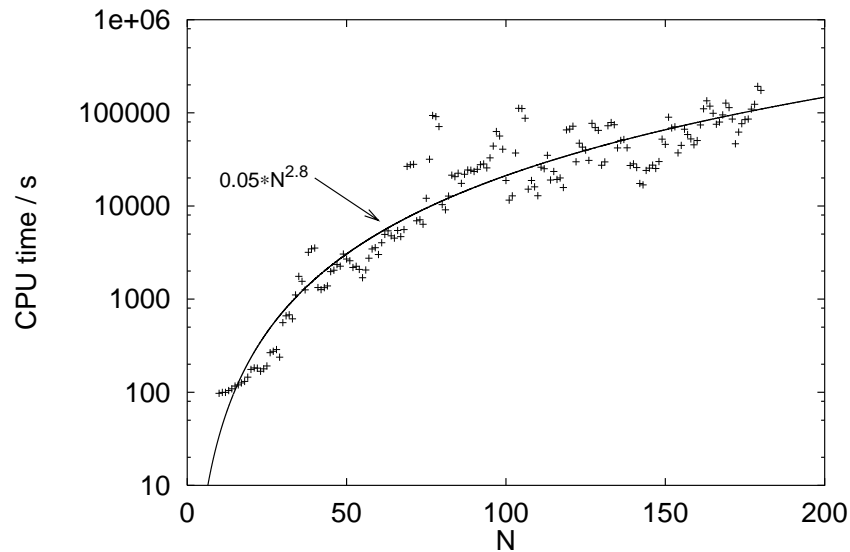
- “frustration” → polytetrahedral<sup>26</sup> quasi-crystals, glasses, ...
- five-fold symmetry axes; icosahedral growth
- impossible in periodic crystals  $\Rightarrow$   $n$ -dependent structural transitions

---

<sup>26</sup> F. C. Frank, Proc. Royal Soc. London 215A (1952) 43; H. S. M. Coxeter, Illinois J. Math. 2 (1958) 746; J. D. Bernal, Proc. Royal Soc. London Ser. A280 (1964) 299.

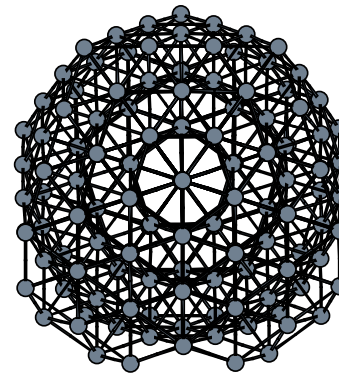
# Performance in the LJ cluster benchmark:<sup>27</sup>

- all 4 structural types found without prior information
- *cubic* scaling of CPU time for reaching global minima:

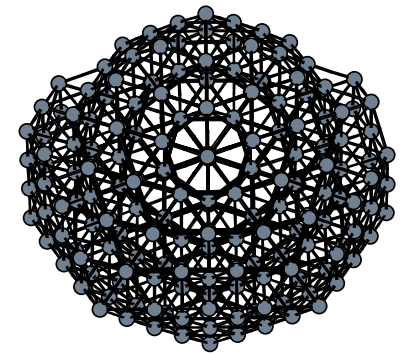


⇒ *large* systems can be treated:

- improved global minima found for  $n = 185, 186, 187$ ;
- published global minima confirmed up to  $n = 250$ :



(LJ)<sub>186</sub>,  $E = -1132.669966$

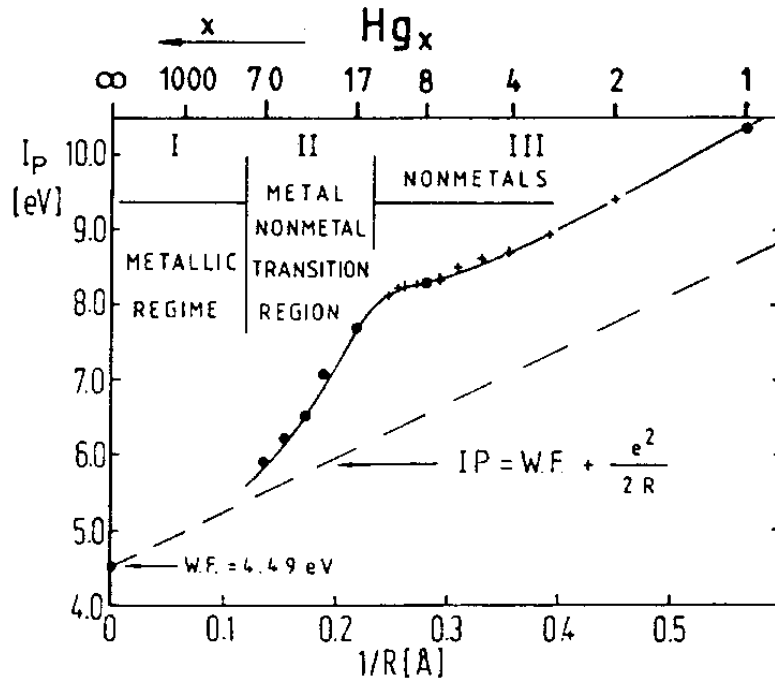


(LJ)<sub>250</sub>,  $E = -1579.794975$

open problem: structural transition from dominance of icosahedral structures to bulk-like fcc;  
suspected at approx.  $n = 750 - 1500$

<sup>27</sup> B. Hartke, J. Comput. Chem. 20 (1999) 1752.

# Application example: Mercury clusters



hypothesis:  
two size-dependent transitions in bond type

hybrid model (Dolg & Flad, Phys.Rev.B 61 (2000) 2362.):

$$E(\text{Hg}_n) = E_{HF,ECP,CP}(Hg_n) - \sum_{i<j}^n \frac{C_6 f_6(R_{ij})}{R_{ij}^6}$$

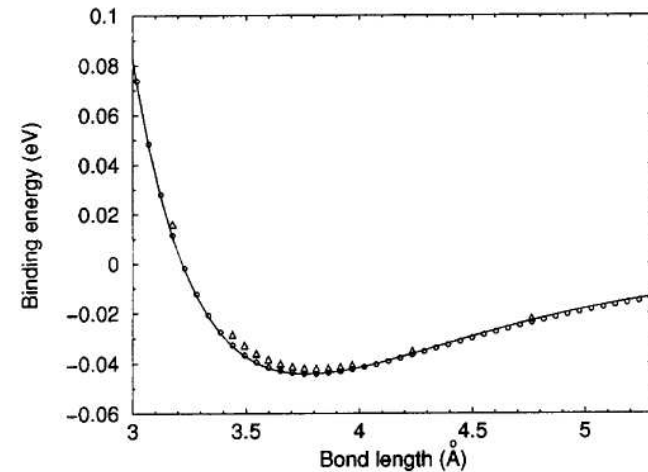
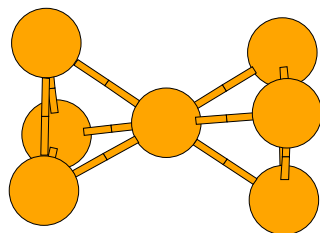


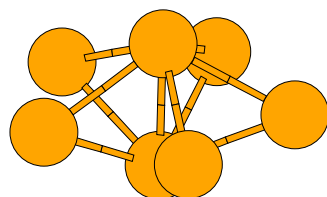
FIG. 1. Potential energy curve of  $\text{Hg}_2$  obtained from CCSD(T) calculations using a 2-valence-electron pseudopotential plus core-polarization potential  $\circ$  and a 20-valence-electron pseudopotential  $\triangle$ . The solid line represents the hybrid model adjusted to the aforementioned type of calculation.

# Mercury cluster structures in the hybrid model: <sup>28</sup>

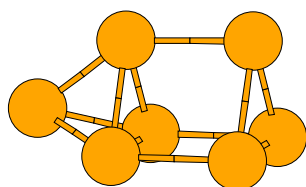
**Hg<sub>7</sub>**



$$\frac{E_{coh}}{atom} = 12.3 \text{ kJ/mol}$$



$$10.7 \text{ kJ/mol}$$

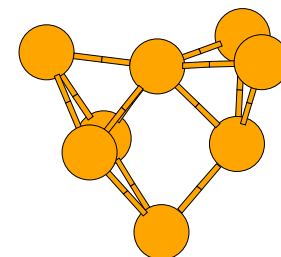


$$9.0 \text{ kJ/mol}$$

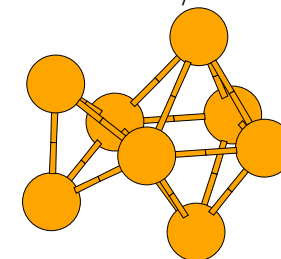
3-body forces compress the  
pentagonal bipyramid:

	dihedral angle/ degrees
Lennard-Jones	73.1382
Morse	72.8817
this potential	67.1189
pot. w/o 3-body	85.6078

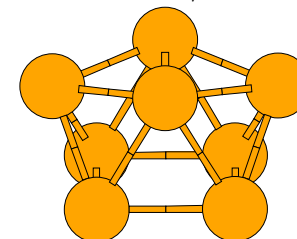
**Hg<sub>8</sub>**



$$\frac{E_{coh}}{atom} = 12.8 \text{ kJ/mol}$$



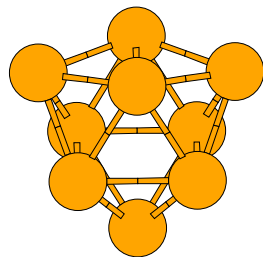
$$10.6 \text{ kJ/mol}$$



$$10.5 \text{ kJ/mol}$$

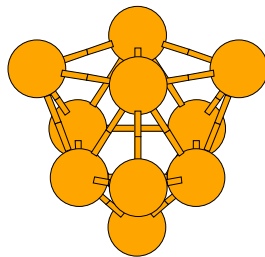
<sup>28</sup> B. Hartke, H.-J. Flad and M. Dolg, Phys. Chem. Chem. Phys. 3 (2001) 5121.

**Hg<sub>9</sub>**



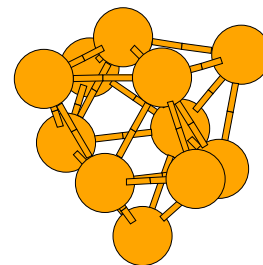
$\frac{E_{coh}}{atom} = 16.1 \text{ kJ/mol}$

**Hg<sub>10</sub>**

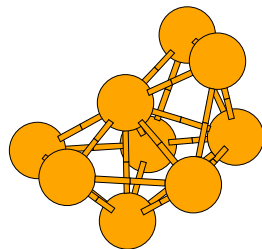


19.1 kJ/mol

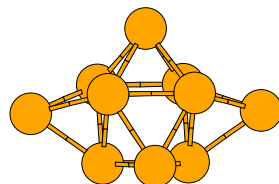
**Hg<sub>11</sub>**



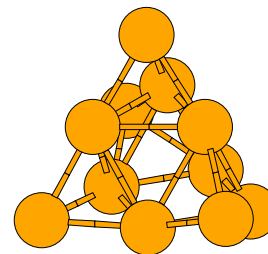
19.7 kJ/mol



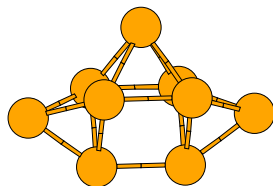
13.7 kJ/mol



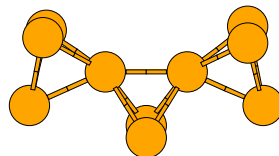
15.4 kJ/mol



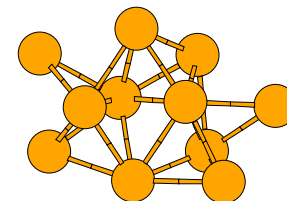
18.9 kJ/mol



12.6 kJ/mol

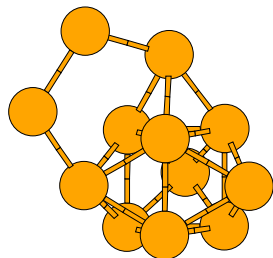


13.6 kJ/mol



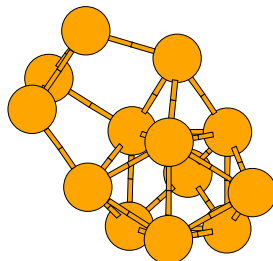
14.5 kJ/mol

**Hg<sub>12</sub>**



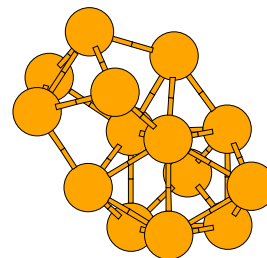
$\frac{E_{coh}}{atom} =$  18.9 kJ/mol

**Hg<sub>13</sub>**

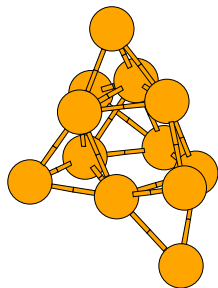


20.2 kJ/mol

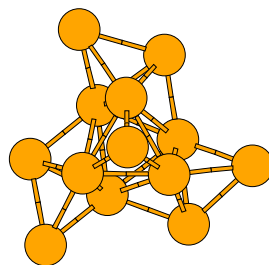
**Hg<sub>14</sub>**



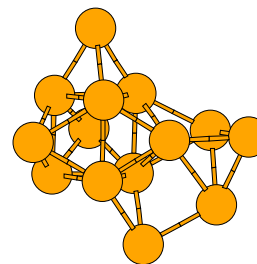
21.6 kJ/mol



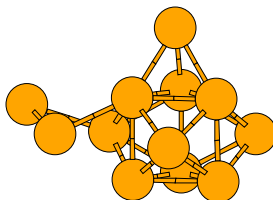
18.5 kJ/mol



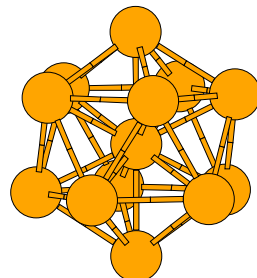
18.2 kJ/mol



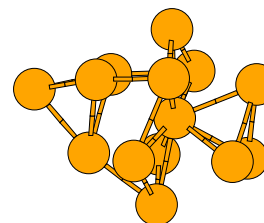
20.1 kJ/mol



18.3 kJ/mol

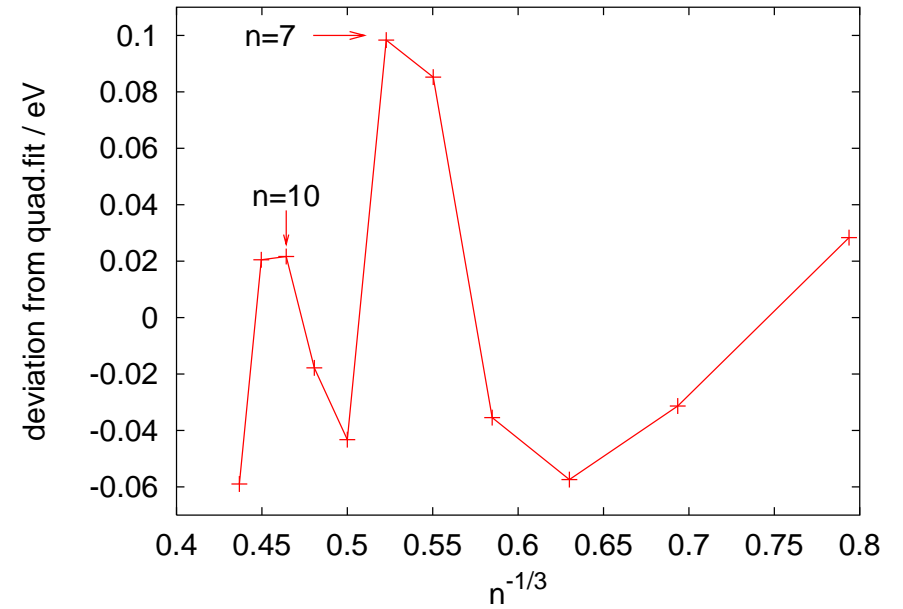
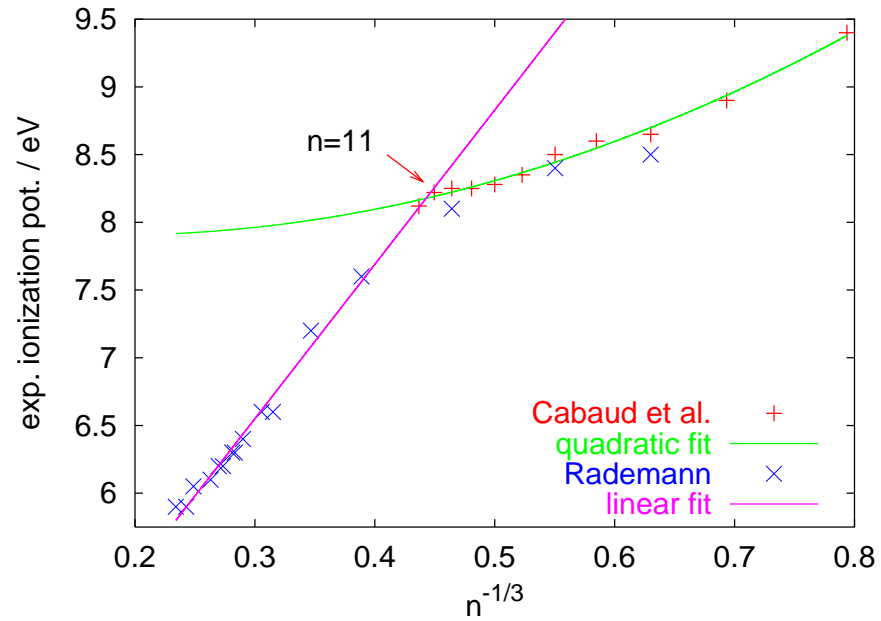


15.1 kJ/mol



16.7 kJ/mol

tentative connection to experiment:



B. Cabaud, A. Horeau and P. Melinon, J. Phys. D: Appl. Phys. 13 (1980) 1831.

K. Rademann, Ber. Bunsenges. Phys. Chem. 93 (1989) 653.



# Application example: pure neutral water clusters

literature results for small clusters  $(\text{H}_2\text{O})_n$ :

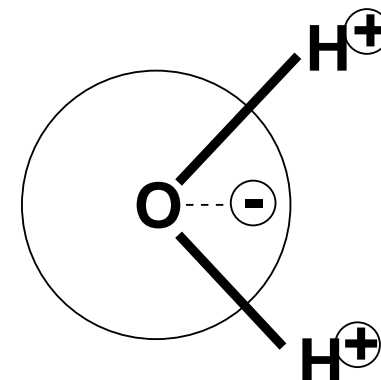
- experimental VRT spectra of size-selected clusters  $n=3-6$  simulated theoretically<sup>a</sup> with excellent agreement;
- IR spectra in the OH-stretch region of size-selected clusters  $n=7-10$  measured and simulated<sup>b</sup>, with good agreement.

larger clusters  $n > 10$ :

- only a few ab-initio/DFT calculations for selected geometries
- global optimization<sup>29</sup> only for TIP4P, up to  $n=22$ .

water in the TIP4P model:

- rigid monomers
- point charges
- not polarizable
- no many-body terms



---

<sup>a</sup> E. M. Maas, R. Bukowski, K. Szalewicz, G. C. Groenenboom, P. E. S. Wormer and A. van der Avoird, J. Chem. Phys. 113 (2000) 6678, 6702.

<sup>b</sup> J. Sadlej, V. Buch, J. K. Kazimirski and U. Buck, J. Phys. Chem. A 103 (1999) 4933.

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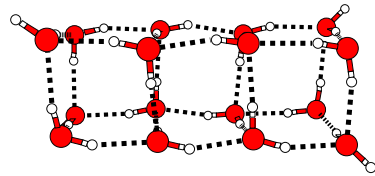
D. J. Wales and M. P. Hodges, Chem. Phys. Lett. 286 (1998) 286; B. Hartke, Z. Phys. Chem. 214 (2000) 1251.

## Results of previous studies with TIP4P:

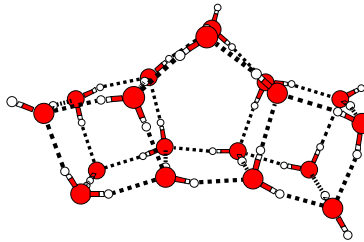
- structures of small clusters  $n \leq 10$  qualitatively correct.

But for larger clusters:

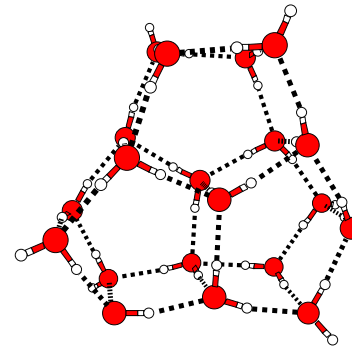
- strange tendency to fused cubes and pentagonal prisms
- no systematic structural trends
- all molecules at the cluster surface, up to at least  $n = 22$ .



$n=16$



$n=18$



$n=20$

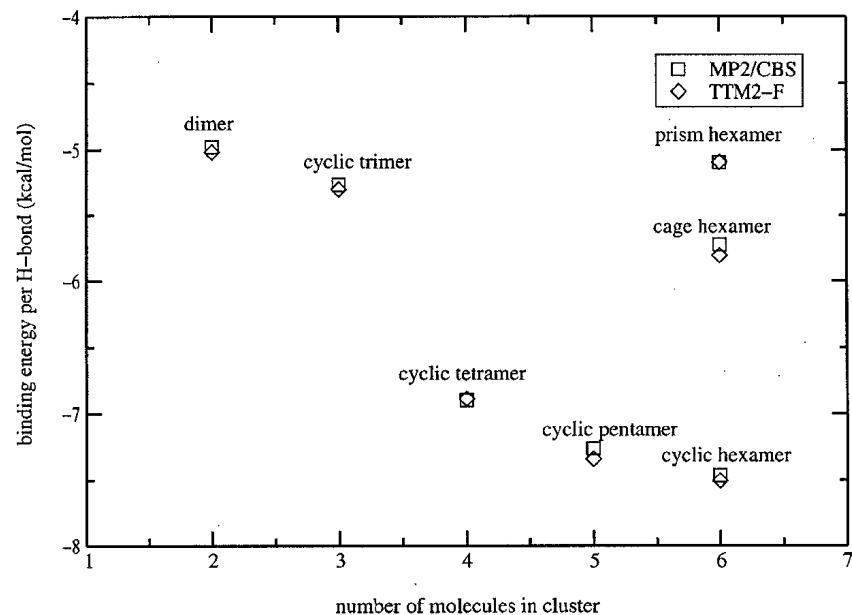
⇒ artifacts of the TIP4P model?

## The TTM2-F model: <sup>30</sup>

important properties:

- flexible monomers, with a
- highly exact, internal ab-initio potential;
- smeared partial charges;
- smeared polarizabilities;
- explicit many-body interactions via iterative induction.

quantitative agreement with MP2-CBS  
for small clusters:



...but computationally more expensive than TIP4P  
by a factor of 20 !

---

<sup>30</sup> C. J. Burnham and S. S. Xantheas, J. Chem. Phys. 116 (2002) 5115.

# Global minimum structures with TTM2-F: <sup>31</sup>

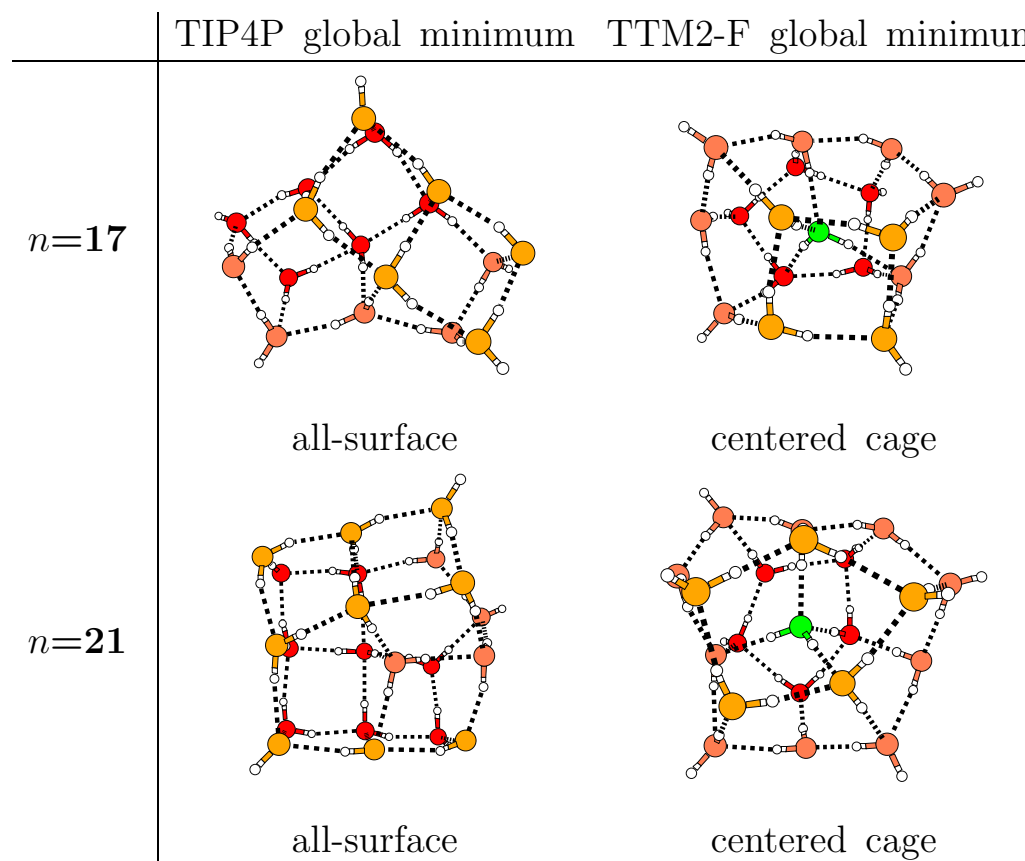
some qualitative agreements between TIP4P and TTM2-F:

- same global and local minima up to  $n=11$ ;
- some global minima identical up to  $n=20$ ;
- tendency to cubes and pentagonal prisms.

but also qualitative differences:

- different orientational isomers already at  $n=12,13$ ;
- big differences at  $n=17,21,22,\dots$ :

Our TTM2-F results were later supported by MP2/aug-cc-pVTZ results <sup>32</sup>

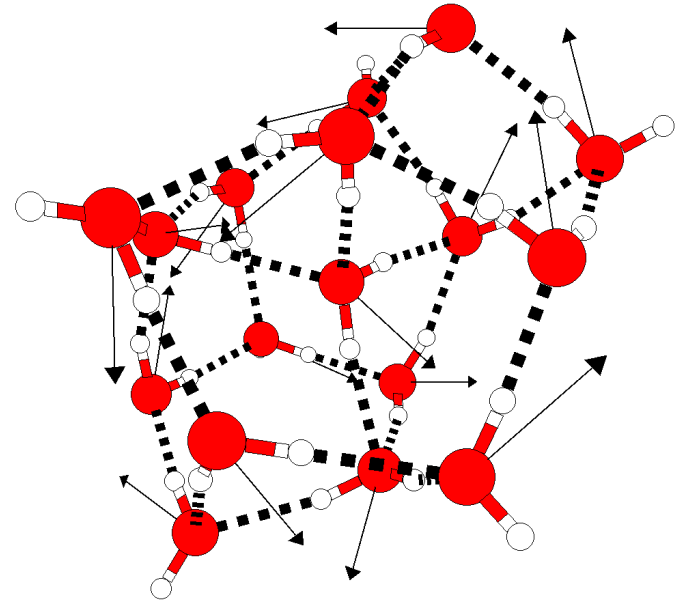


<sup>31</sup> B. Hartke, Phys. Chem. Chem. Phys. 5 (2003) 275.

<sup>32</sup> A. Lagutschenkov, G. S. Fanourgakis, G. Niedner-Schatteburg, S. S. Xantheas, J. Chem. Phys. 122 (2005) 194310.

# Reasons for structural differences

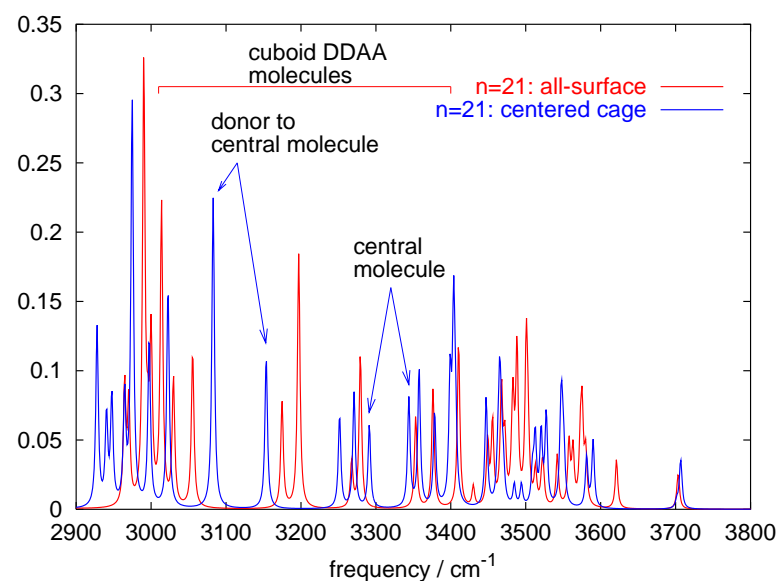
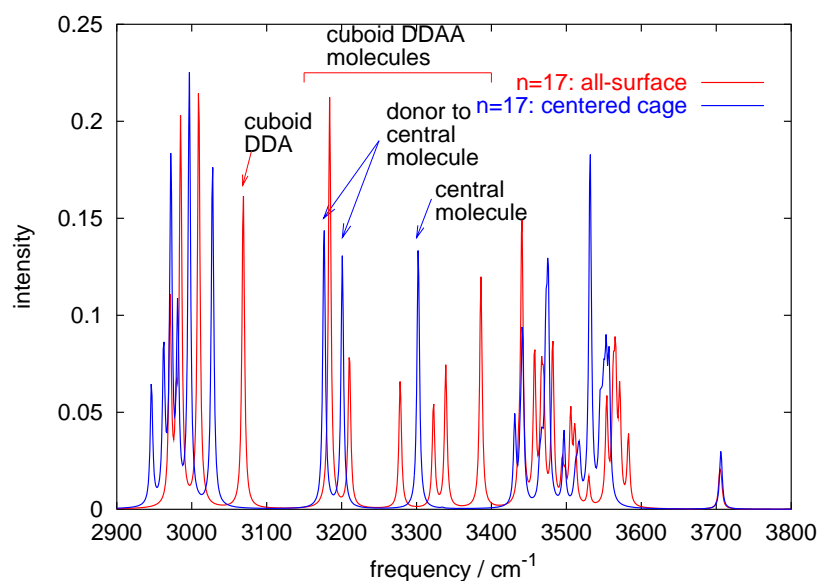
- monomer flexibility is not important (comparison to TTM2-R): at most 16% of the effect, and often even contraproductive.
- adaptive size and orientation of monomer multipoles in TTM2-F leads to
  - antiparallel homodromy in ring stacks is less bad (TIP4P favors parallel homodromy,  $n=12,15,16$ )
  - cages less bad, by deviation of dipoles from HOH-plane and from HOH angle bisector  $\Rightarrow$  better relative orientations of the dipoles



# Connection to experiment

Calculation of IR frequencies and intensities of OH-stretch vibrations, using the anharmonic empirical model of Buch et al. (good agreement with experiment for small clusters<sup>33</sup>)

Characteristic DDAA region allows for differentiation between all-surface structures and centered cage structures:

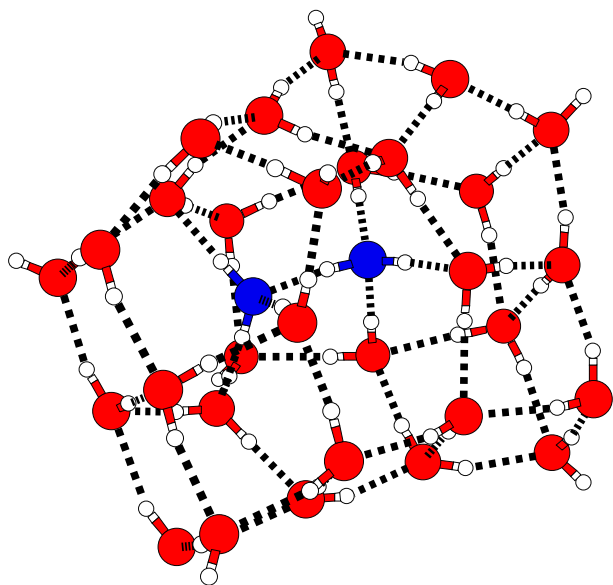


<sup>33</sup> J. Sadlej, V. Buch, J. K. Kazimirski and U. Buck, J. Phys. Chem. A 103 (1999) 4933.

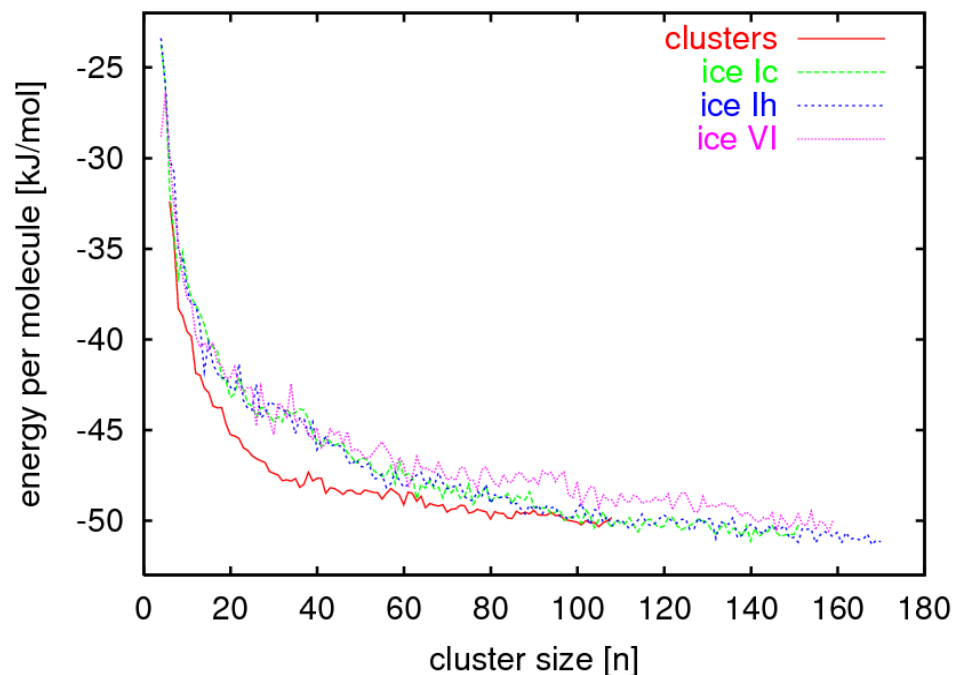
# Larger water clusters on their way to ice<sup>34</sup>

- TTM2-F water potential<sup>35</sup> (20 times more expensive than TIP4P); quantitative agreement with MP2/CBS
- parallel pool model on 8–64 processors (local PC clusters, national high-performance computing center HLRN Berlin/Hannover)
- up to 110 molecules; seeding with spherical ice cut-outs

n=32, cage with  
2 internal mol.



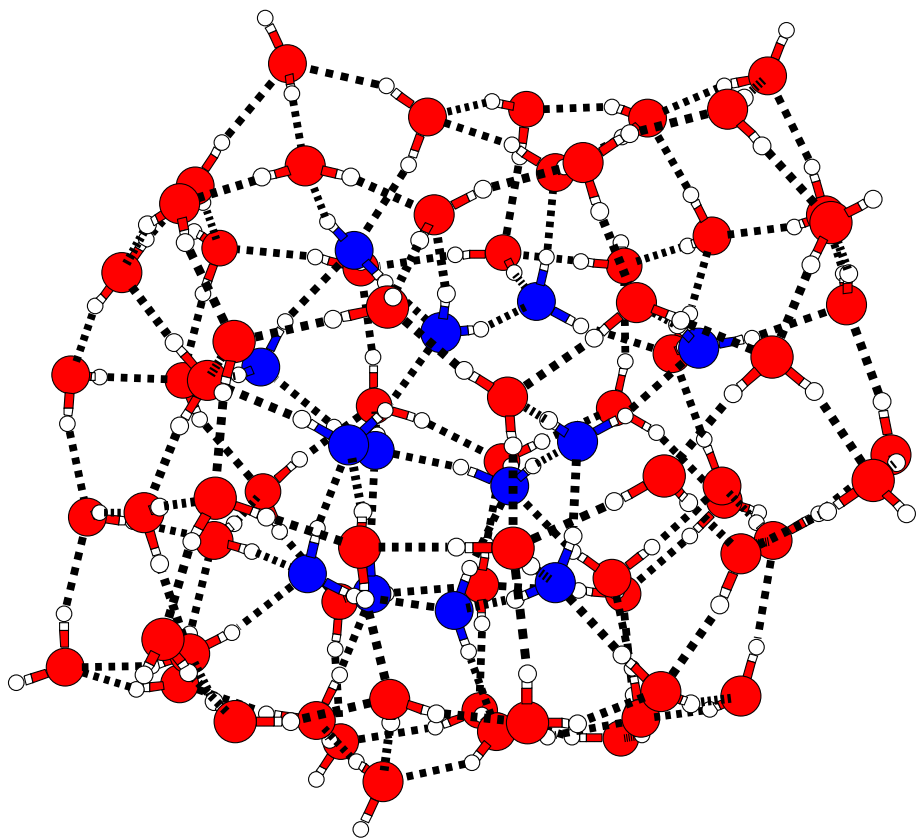
clusters  $\leftrightarrow$  locally relaxed ice cutouts;  
 $\Rightarrow$  isoenergetic at n=90 (??)



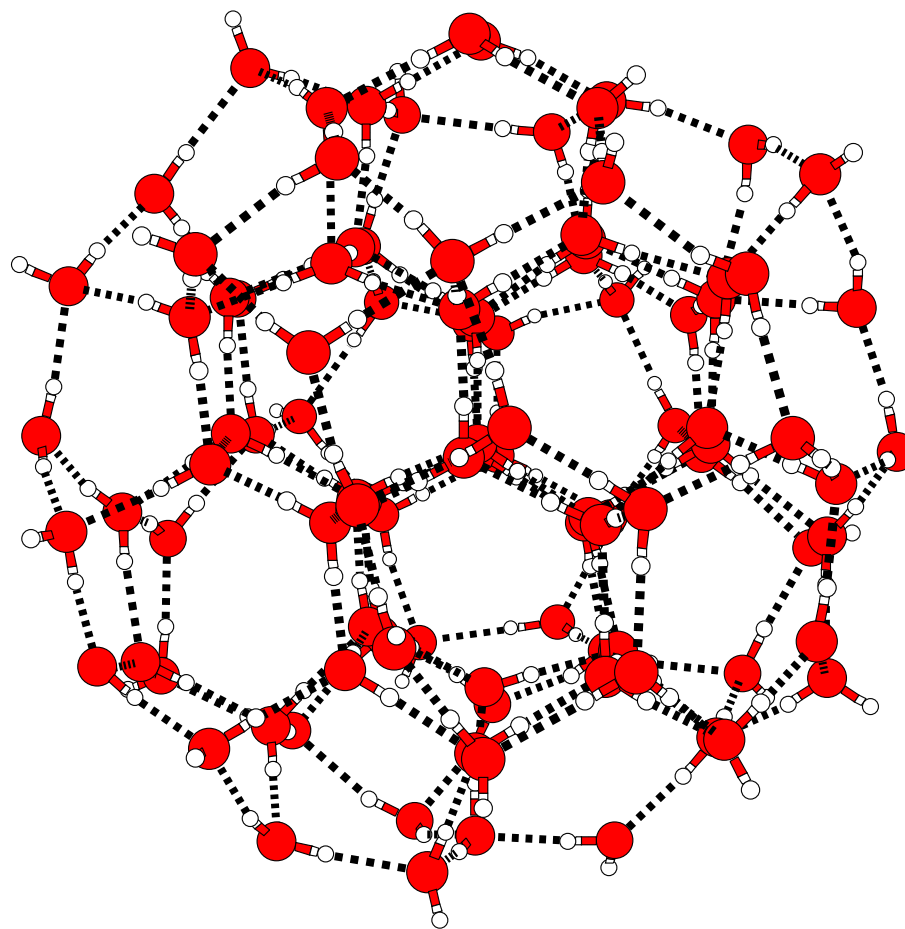
<sup>34</sup> B. Bandow and B. Hartke, J. Phys. Chem. A 110 (2006) 5809.

<sup>35</sup> C. J. Burnham and S. S. Xantheas, J. Chem. Phys. 116 (2002) 5115.

n=80, with 13 internal molecules,  
after 20 generations unbiased globopt



n=100, ice Ih seeded; very likely  
only a locally relaxed seed



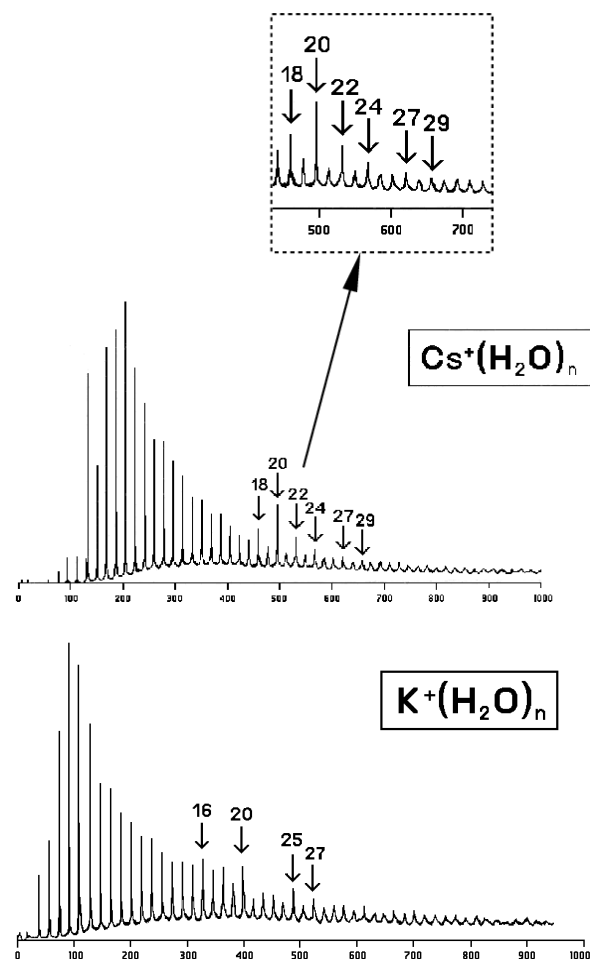
difficulties:

- global optimization at these cluster sizes extremely expensive, even in massively parallel pool mode;
- recognition of a minimal-size ice core non-trivial.

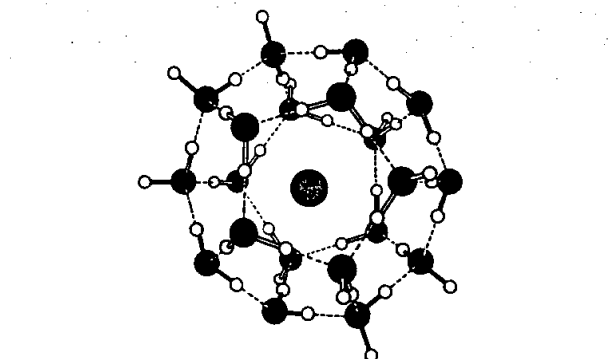


# Application example: microhydration clusters of alkali cations

robust experimental result<sup>36</sup>:  
(weak) magic numbers

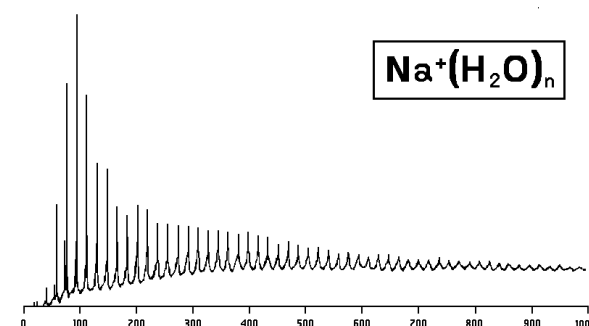


standard explanation<sup>37</sup>:  
“clathrate” formation  
( $n=20$ : dodecahedron)



**Figure 2.** Proposed structure for  $\text{Cs}^+(\text{H}_2\text{O})_{20}$ . This structure comprises 12 pentagons. White spheres, hydrogen; black spheres, oxygen; gray central sphere,  $\text{Cs}^+$ ; dashed lines, hydrogen bonds.

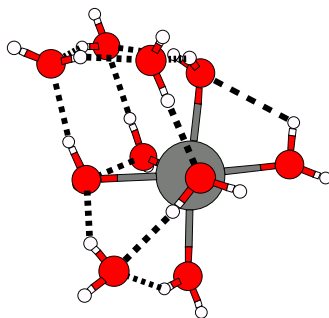
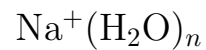
...but not for  $\text{Na}^+$ :



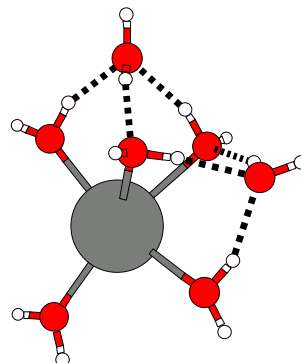
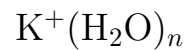
<sup>36</sup> B. Brutschy et al., Int. J. Mass Spectrom. 185/186/187 (1999) 271.

<sup>37</sup> A. Selinger and A. W. Castleman, Jr., J. Phys. Chem. 95 (1991) 8442

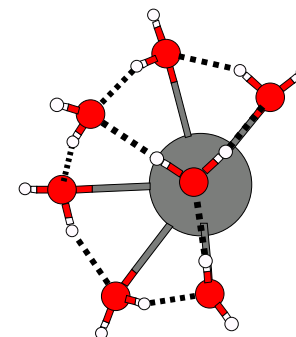
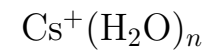
# Unbiased global optimization: <sup>38</sup> structural trends:



- coordination polyhedra around the ion
- competition between
  - ion centered
  - ion off-center
- no cages



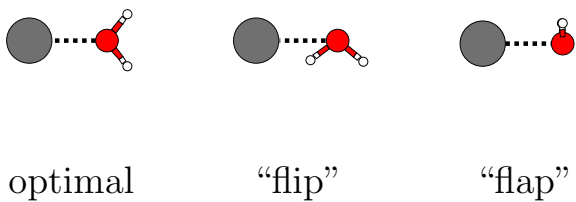
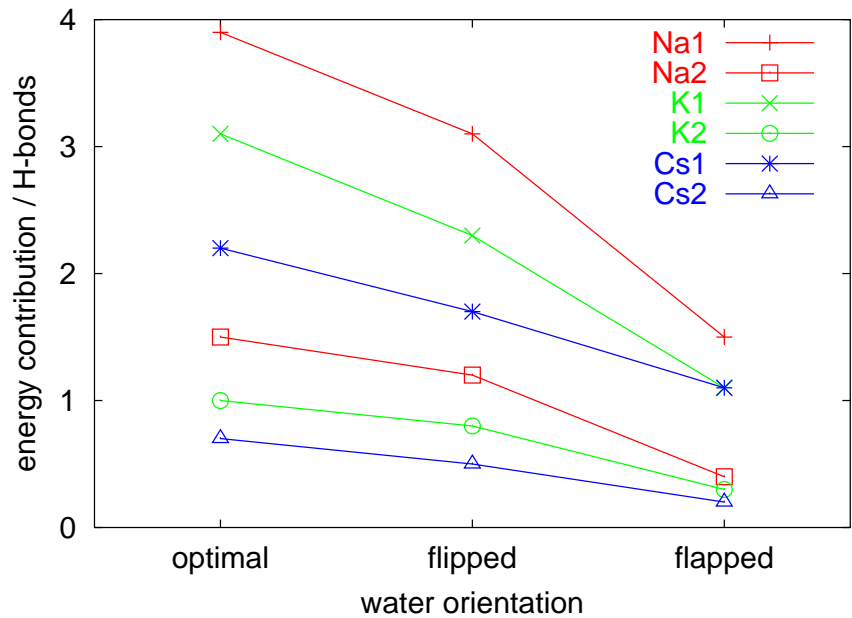
- for  $n \leq 9$  as  $\text{Na}^+(\text{H}_2\text{O})_n$
- for  $n \geq 10$  as  $\text{Cs}^+(\text{H}_2\text{O})_n$



- no coordination polyhedra
- instead: water rings
- closing to cages at  $n = 18$

<sup>38</sup> F. Schulz and B. Hartke, Chem. Phys. Chem. 3 (2002) 98.

# Rationalization of different structural preferences:

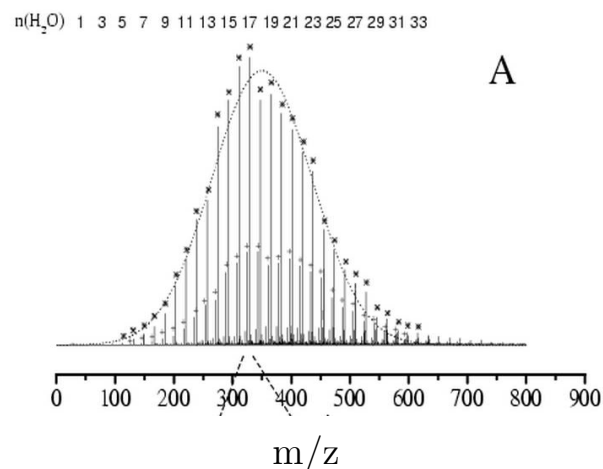


TIP4P/OPLS energy contributions in kJ/mol and in comparison to an optimal water-dimer H-bond (column “H”), for different ion-water distances

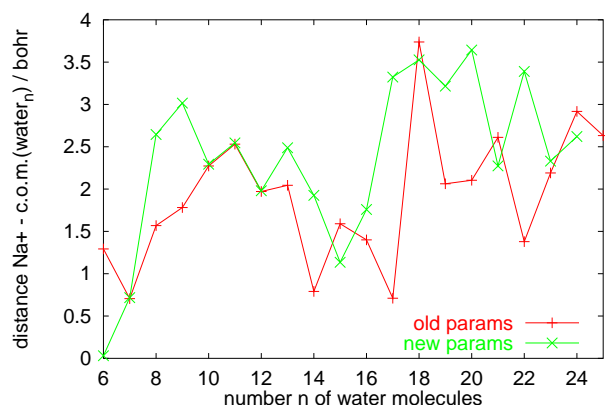
ion-water distance [Å]	optimal orientation	H	flipped	H	flapped	H
Na						
2.32	102.64	3.9	80	3.1	40	1.5
4.0	40	1.5	30	1.2	10	0.4
K						
2.68	79.59	3.1	60	2.3	30	1.1
5.0	25	1.0	20	0.8	8	0.3
Cs						
3.20	58.08	2.2	45	1.7	28	1.1
6.0	17	0.7	14	0.5	5	0.2

Na<sup>+</sup> is strongly and far-reachingly orienting, hard to beat by a good water network. For Cs<sup>+</sup>, the opposite applies.

# Can this be checked experimentally?



“jump” in the mass spectrum<sup>41</sup> of  $\text{Na}^+(\text{H}_2\text{O})_n$  at  $n = 17$  agrees with our structural observations.



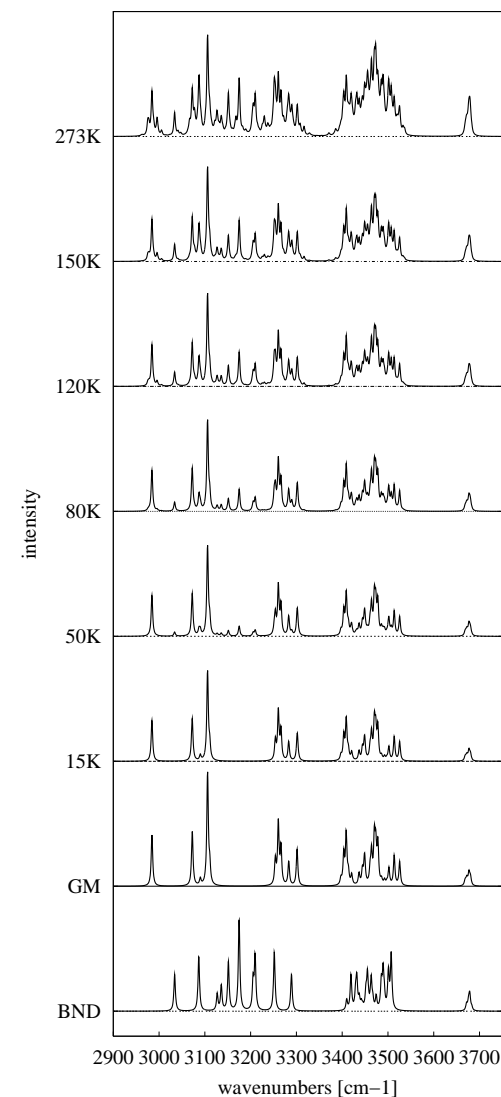
T-dependence<sup>42</sup> of IR spectra in OH-stretch region (anharmonic empirical model of Victoria Buch):

$\text{Cs}^+(\text{H}_2\text{O})_{20}$

GM: global minimum,  $T=0\text{K}$

BND: best non-dodecahedron,  $T=0\text{K}$

structural differences detectable



<sup>41</sup> B. Hartke, A. Charvat, M. Reich and B. Abel, J. Chem. Phys. 116 (2002) 3588.

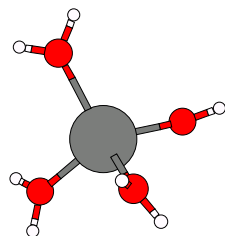
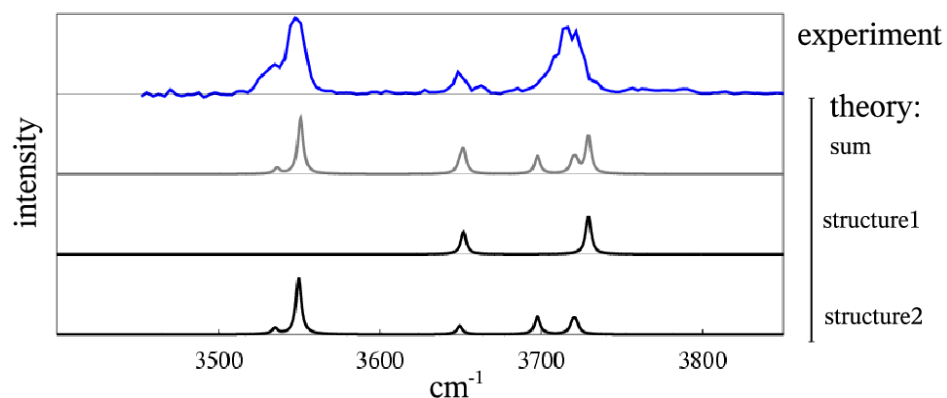
<sup>42</sup> F. Schulz and B. Hartke, Phys. Chem. Chem. Phys. 5 (2003) 5021.

## Direct structural comparison with experiment: IR spectra

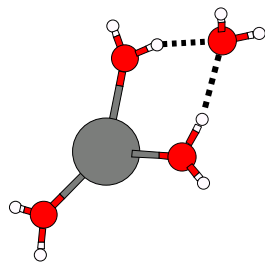
*Experiment:* James M. Lisy, University of Illinois at Urbana–Champaign

*Theory:* anharmonic empirical model by Victoria Buch<sup>43</sup>, for our global and best local minima structures

$\text{Na}^+(\text{H}_2\text{O})_4$ :

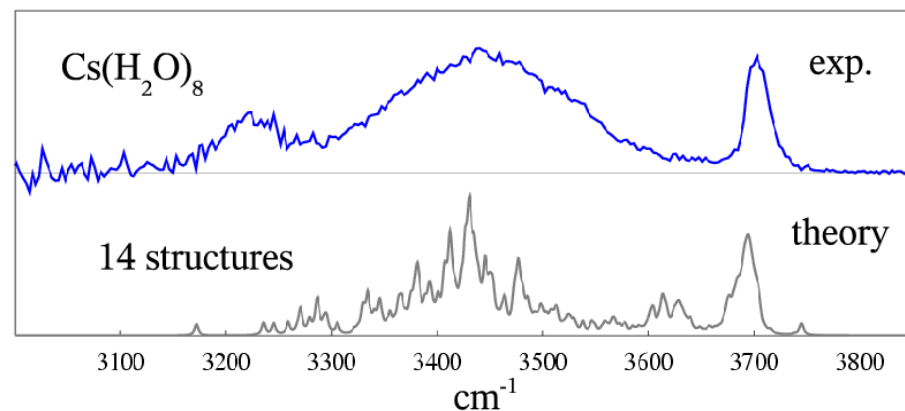
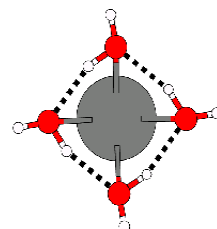
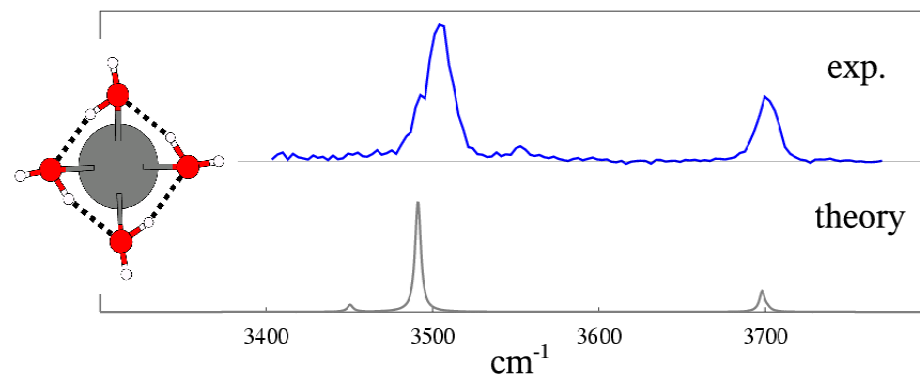


structure 1



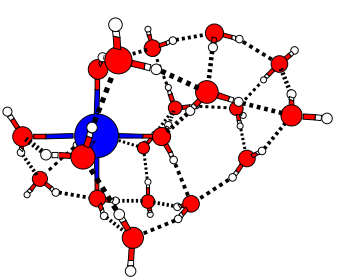
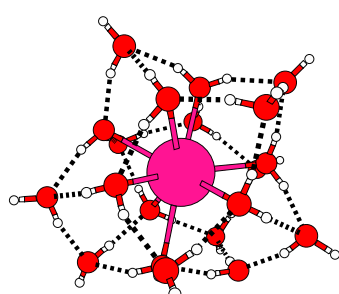
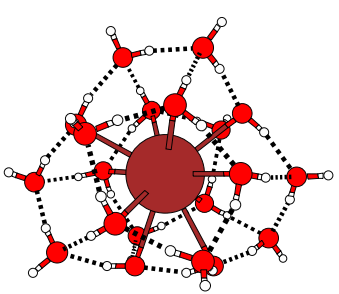
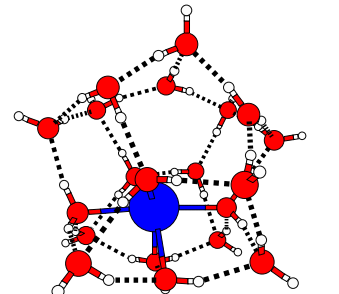
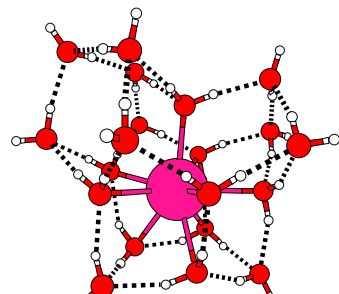
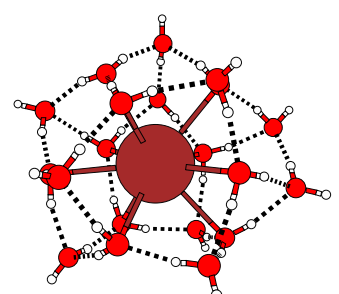
structure 2

$\text{Cs}^+(\text{H}_2\text{O})_4$ :



<sup>43</sup> J. Sadlej, V. Buch, J. K. Kazimirski and U. Buck, J. Phys. Chem. A, 1999, **103**, 4933.

# Are dodecahedra important? <sup>44</sup>

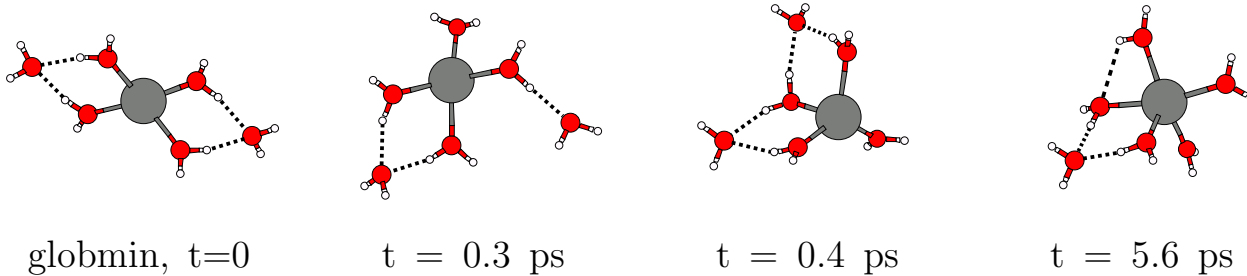
	$\text{Na}^+(\text{H}_2\text{O})_{20}$	$\text{K}^+(\text{H}_2\text{O})_{20}$	$\text{Cs}^+(\text{H}_2\text{O})_{20}$
global minimum	 <p>irregular, off-center</p>	 <p>distorted dodecahedron</p>	 <p>distorted dodecahedron</p>
local minimum	 <p>dodecahedron: +13 kJ/mol</p>	 <p>no cage: +3.3 kJ/mol</p>	 <p>4-/5-/6-rings: +0.88 kJ/mol</p>

<sup>44</sup> F. Schulz and B. Hartke, Chem. Phys. Chem. 3 (2002) 98.

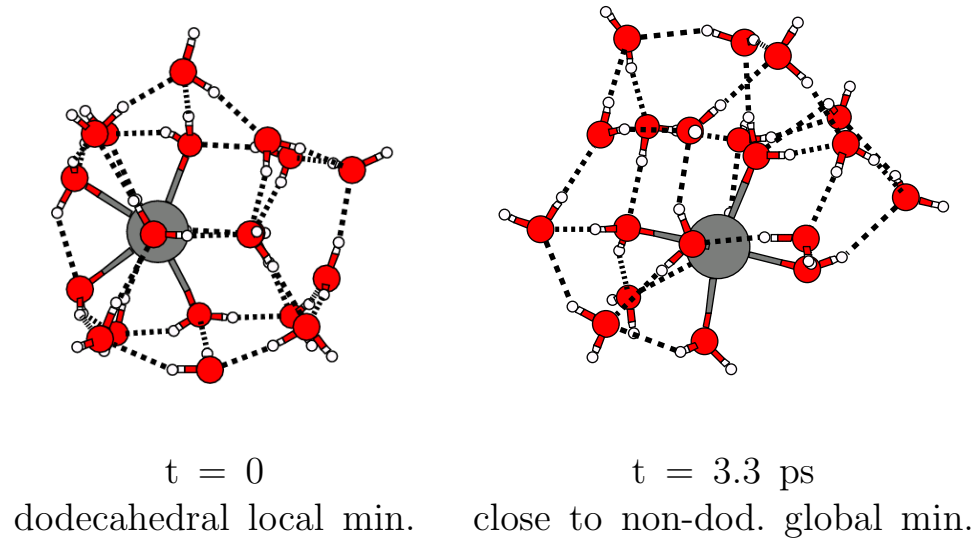
## Stability and dynamics: free dynamcis

molecular dynamics (canonical ensemble,  $40 \text{ K} \leq T \leq 150 \text{ K}$ ); input: global & low-energy local minima

- prejudice: very rapid fluctuations, “structure” fluxional: true only for  $\text{Na}(\text{H}_2\text{O})_n$ ,  $n \leq 10$ ,  $T = 120\text{--}150\text{K}$ : visits various coordination polyhedra:

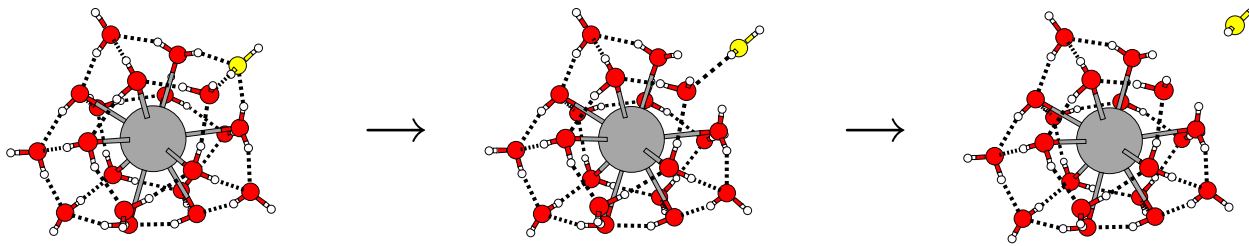


- for larger cases of all ions, *no qualitative structural changes* even at 150 K
- big exception:  $\text{Na}(\text{H}_2\text{O})_{20}$  at  $T \geq 120 \text{ K}$ :



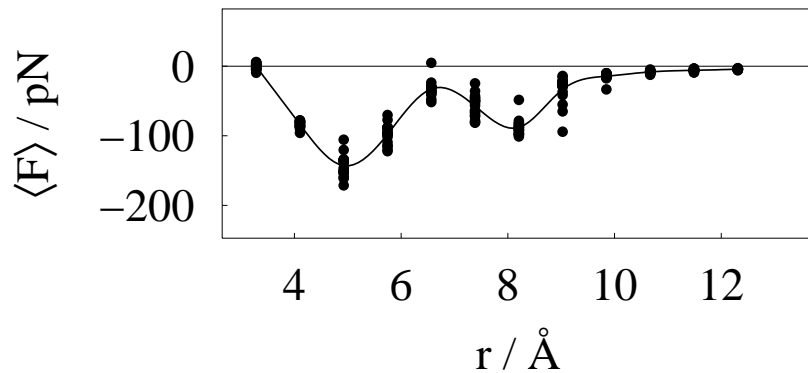
## Stability and dynamics: constrained dynamics

- dissociation of single water molecule enforced by fixing its distance to the central ion, for increasing values:

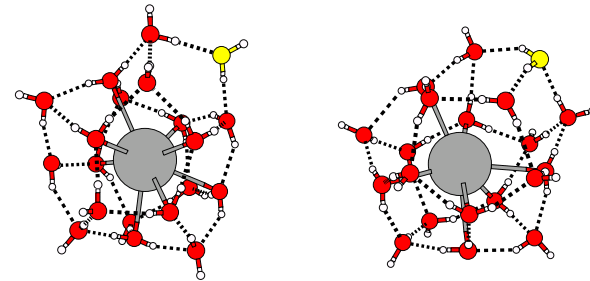


- calculation of the average force  $\langle F \rangle$  at each distance
- integration of  $\langle F \rangle \rightarrow$  free energy  $\Delta G$  of dissociation

Example:  $\text{Cs}(\text{H}_2\text{O})_{20}$ ,  $T = 120 \text{ K}$   
 $\Delta G = 28.4 \text{ kJ/mol}$

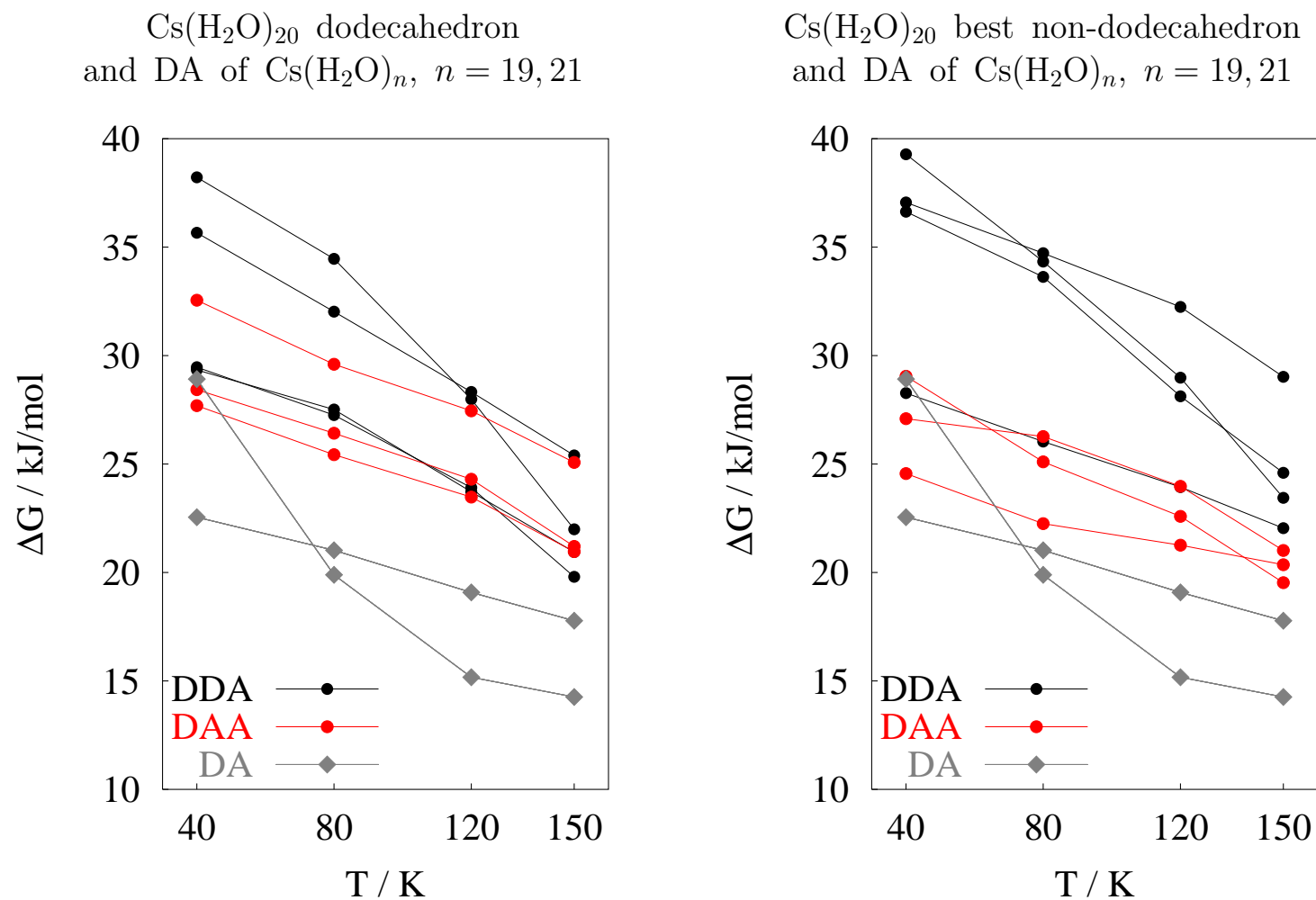


scatter at each distance due to different reaction pathways:





representative free energy values, as a function of temperature, H-bonding environment, and cluster structure:



- no significant differences between dodecahedron and non-dodecahedron
- resistance to dissociation:  $\text{DA} < \text{DAA} \leq \text{DDA}$

⇒ new hypothesis:<sup>45</sup> magic-number status depends on presence/absence of DA molecules

(note: this is a combination of structural and dynamical propensities)

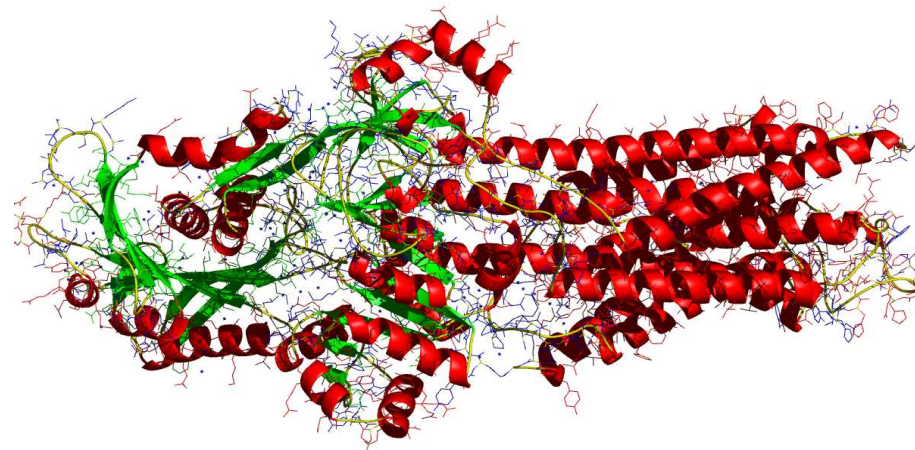
	single	A	D	DA	AA	DD	DDA	DAA	DDAA		single	A	D	DA	AA	DD	DDA	DAA	DDAA
$\text{Na}^+(\text{H}_2\text{O})_4$	4	—	—	—	—	—	—	—	—	$\text{Cs}^+(\text{H}_2\text{O})_4$	—	—	—	4	—	—	—	—	—
$\text{Na}^+(\text{H}_2\text{O})_5$	5	—	—	—	—	—	—	—	—	$\text{Cs}^+(\text{H}_2\text{O})_5$	—	1	1	3	—	—	—	—	—
$\text{Na}^+(\text{H}_2\text{O})_6$	—	—	4	—	2	—	—	—	—	$\text{Cs}^+(\text{H}_2\text{O})_6$	2	—	—	4	—	—	—	—	—
$\text{Na}^+(\text{H}_2\text{O})_7$	1	—	4	—	1	—	—	—	—	$\text{Cs}^+(\text{H}_2\text{O})_7$	—	—	—	5	—	—	1	1	—
$\text{Na}^+(\text{H}_2\text{O})_8$	1	—	3	1	—	—	—	3	—	$\text{Cs}^+(\text{H}_2\text{O})_8$	—	—	1	5	—	—	1	1	—
$\text{Na}^+(\text{H}_2\text{O})_9$	1	—	2	1	—	—	1	4	—	$\text{Cs}^+(\text{H}_2\text{O})_9$	—	—	—	5	—	—	2	2	—
$\text{Na}^+(\text{H}_2\text{O})_{10}$	—	—	2	2	—	—	2	4	—	$\text{Cs}^+(\text{H}_2\text{O})_{10}$	—	—	—	6	—	—	2	2	—
$\text{Na}^+(\text{H}_2\text{O})_{11}$	1	—	2	1	—	1	1	5	—	$\text{Cs}^+(\text{H}_2\text{O})_{11}$	—	—	—	5	—	—	3	3	—
$\text{Na}^+(\text{H}_2\text{O})_{12}$	—	—	2	1	—	1	2	6	—	$\text{Cs}^+(\text{H}_2\text{O})_{12}$	—	—	—	4	—	—	4	4	—
$\text{Na}^+(\text{H}_2\text{O})_{13}$	—	—	—	1	—	2	3	7	—	$\text{Cs}^+(\text{H}_2\text{O})_{13}$	—	—	—	5	—	—	4	4	—
$\text{Na}^+(\text{H}_2\text{O})_{14}$	—	—	—	—	—	—	7	7	—	$\text{Cs}^+(\text{H}_2\text{O})_{14}$	—	—	—	4	—	—	5	5	—
$\text{Na}^+(\text{H}_2\text{O})_{15}$	—	—	—	1	—	2	4	8	—	$\text{Cs}^+(\text{H}_2\text{O})_{15}$	—	—	—	3	—	—	6	6	—
$\text{Na}^+(\text{H}_2\text{O})_{16}$	—	—	—	1	—	1	6	8	—	$\text{Cs}^+(\text{H}_2\text{O})_{16}$	—	—	—	2	—	—	7	7	—
$\text{Na}^+(\text{H}_2\text{O})_{17}$	—	—	—	1	—	2	5	9	—	$\text{Cs}^+(\text{H}_2\text{O})_{17}$	—	—	—	3	—	—	7	7	—
$\text{Na}^+(\text{H}_2\text{O})_{18}$	—	—	—	1	—	2	4	8	3	$\text{Cs}^+(\text{H}_2\text{O})_{18}$	—	—	—	—	—	—	9	9	—
$\text{Na}^+(\text{H}_2\text{O})_{19}$	—	—	—	2	—	2	4	8	3	$\text{Cs}^+(\text{H}_2\text{O})_{19}$	—	—	—	1	—	—	9	9	—
$\text{Na}^+(\text{H}_2\text{O})_{20}$	—	—	—	1	—	2	5	9	3	$\text{Cs}^+(\text{H}_2\text{O})_{20}$	—	—	—	—	—	—	10	10	—
$\text{Na}^+(\text{H}_2\text{O})_{21}$	—	—	2	—	—	1	4	9	4	$\text{Cs}^+(\text{H}_2\text{O})_{21}$	—	—	—	1	—	—	10	10	—
										$\text{Cs}^+(\text{H}_2\text{O})_{22}$	—	—	—	—	—	—	11	11	—
										$\text{Cs}^+(\text{H}_2\text{O})_{23}$	—	—	—	1	—	—	11	11	—
										$\text{Cs}^+(\text{H}_2\text{O})_{24}$	—	—	—	—	—	—	11	11	2

<sup>45</sup> F. Schulz and B. Hartke, Theor. Chem. Acc. 114 (2005) 357.

# Protein folding

The next grand challenge after the human genome project.  
NP-hard. Currently favored strategies:

- **databank-based:** homology modeling, threading, ...
  - *advantage:* sometimes very successful
  - *disadvantages:*
    - \* premise “similar primary sequences fold similarly” may be wrong;
    - \* we know 3D structures for some but not all primary sequences.
- **ab-initio:** (without using databank information)
  - *advantage:* should work for unknown sequences
  - *disadvantages:*
    - \* so far limited to short oligopeptides;
    - \* performance hard to judge independent of force field;
    - \* accuracy much worse than for good cases of threading.



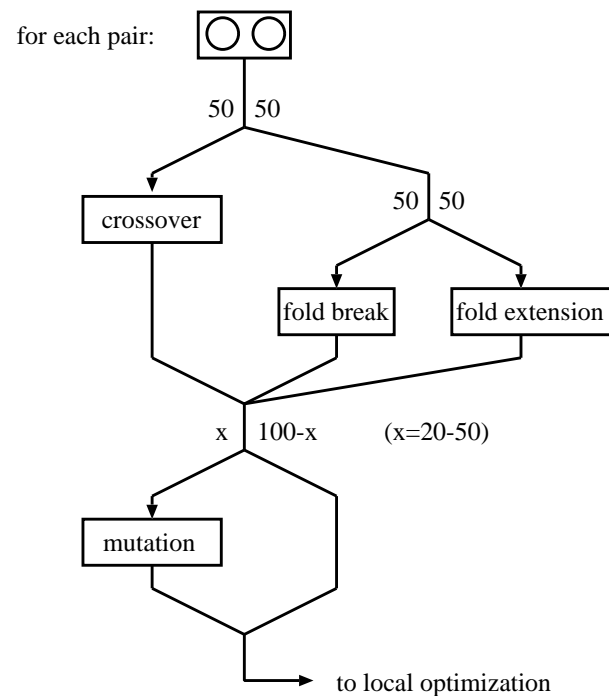
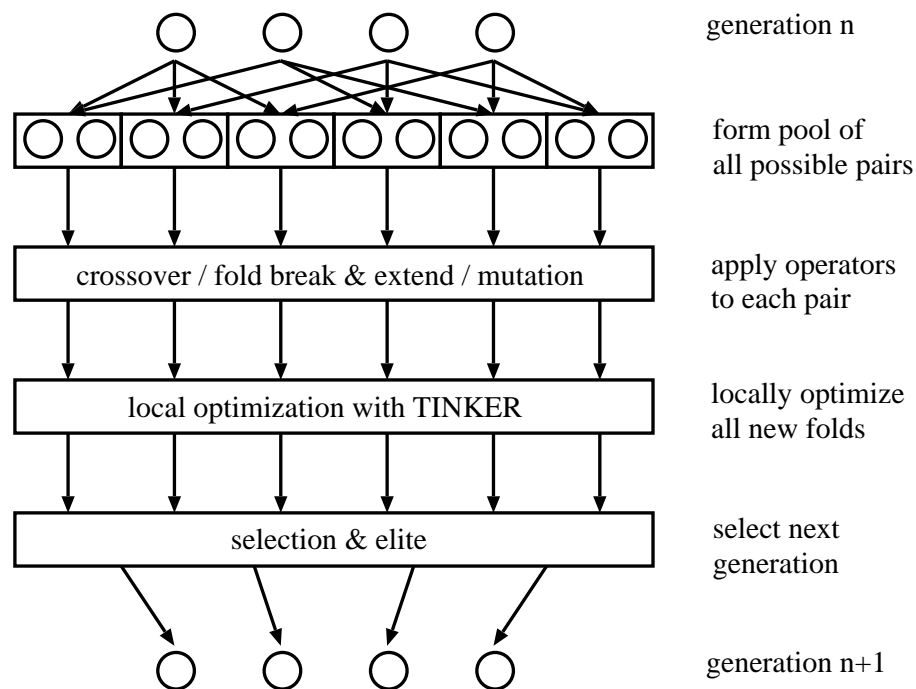
New algorithm PROFET<sup>46</sup>, based on our cluster structure evolutionary algorithm:

- It does not operate on backbone dihedral angles exclusively
- instead, secondary structure elements are defined as correlated dihedral angle sets  $(\phi, \psi, \omega)$
- operations of the algorithm introduce, extend, split, and destroy these secondary structure elements
- in all operations, clashes are avoided by construction.

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<sup>45</sup> F. Koskowski and B. Hartke, J. Comput. Chem. 26 (2005) 1169.

# PROtein Folding with Evolutionary Techniques: PROFET<sup>47</sup>

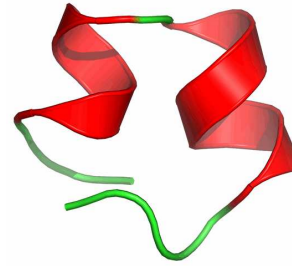


- v1.0: good secondary structure elements (but not their location) as a-priori input information  
v2.0: during the folding, algorithm learns which dihedral angle pairs are good
- side chains explicitly present only during local optimization
- clash avoidance: (1) finite thickness of backbone in global moves,  
(2) no dihedral angle value is enforced, but only optimally approached
- default folding moves not random, but towards maximum compactness

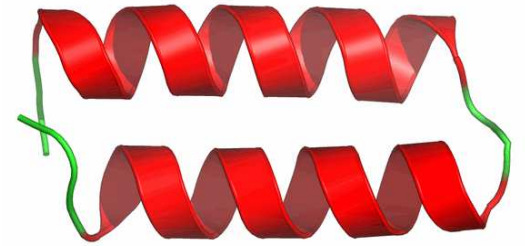
<sup>47</sup> F. Koskowski and B. Hartke, J. Comput. Chem. 26 (2005) 1169.

## Test application: Polyalanines

- OPLSAA and AMBER94/96 force fields, as implemented in TINKER
- in vacuo, and with implicit solvent model (distance dependent dielectric)
- $\text{ala}_n$ ,  $n = 20 - 60$



$\text{ala}_{20}$



$\text{ala}_{40}$

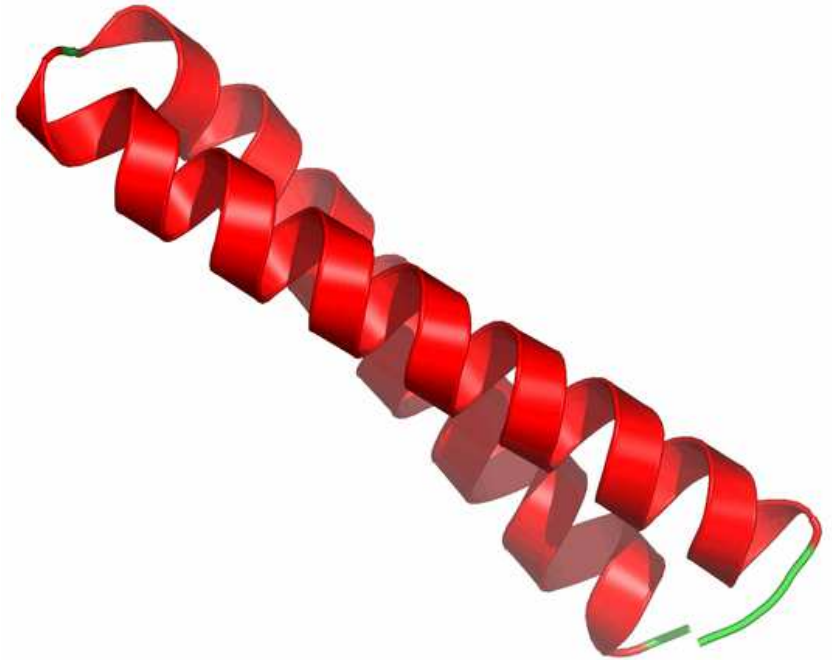
Result:

overwhelming preference for bundles of 2 alpha-helices.

This agrees with theoretical expectations and with experimental observations (mobility experiments<sup>a</sup>)

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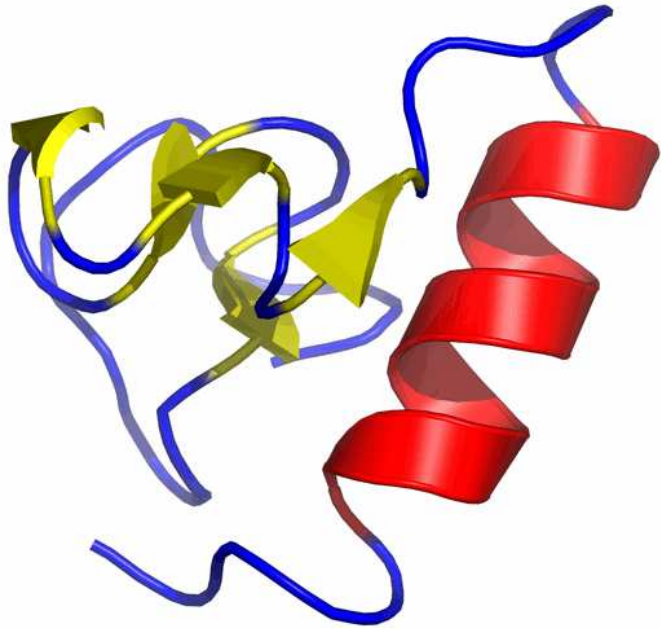
<sup>a</sup> R. R. Hudgins and M. F. J. Jarrold, J. Am. Chem. Soc. 121 (1999) 3494; A. E. Counterman and D. E. Clemmer, J. Phys. Chem. B 107 (2003) 2111.



$\text{ala}_{60}$

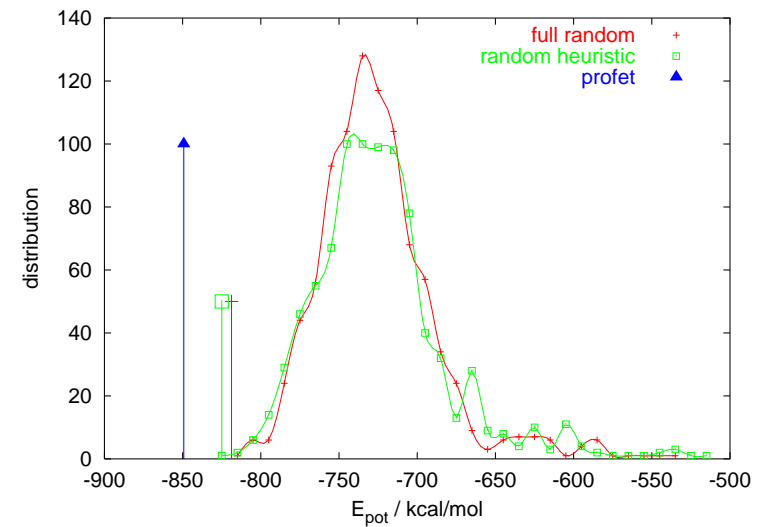
## Test application: random sequences

totally different results: globular random coils, sometimes with short secondary structure elements:



⇒ PROFET does *not* enforce the given secondary structure elements!

PROFET clearly beats multiple local optimizations, both without and with random secondary structure elements initialization:



⇒ not the given secondary structure elements are important, but the global optimization algorithm.

## Real life tests: PDB sequences

Connection of PROFET with the SMMP suite<sup>48</sup> as force-field engine for ECEPP and FLEX.

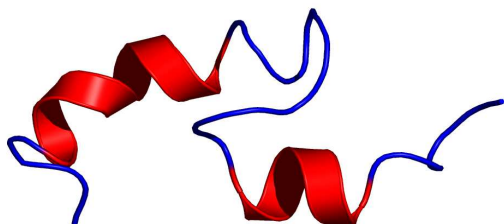
### benchmark case 1:

biologically active PTH fragment (residues 1–34) of the human parathyroid hormone (84 residues)

X-ray structure (PDB code: 1ET1):



NMR solution structure (PDB code: 1ZWA):



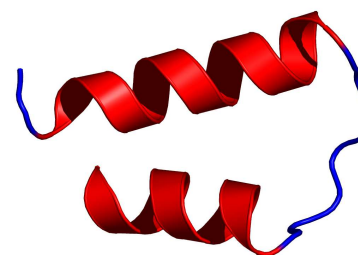
large-scale simulated-annealing/multicanonical sampling study by Hansmann<sup>49</sup> results in RMS=0.8Å from the crystal structure:



We find RMS=1.2Å with FLEX and  $\epsilon(r)$ :



and differing structures with ECEPP:



<sup>48</sup> F. Eisenmenger, U. H. E. Hansmann, Sh. Hayryan and C.-K. Hu, Comput. Phys. Commun. 138 (2001) 192.

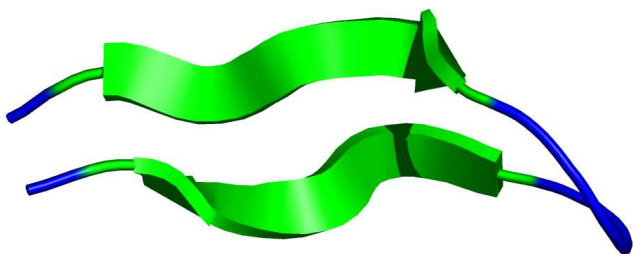
<sup>49</sup> U. H. E. Hansmann, J. Chem. Phys. 120 (2004) 417.



**benchmark case 2:** cis proline turn linking two beta-hairpin strands in HIV-1IIIB V3, 18 residues

NMR solution structure (PDB entry 1B03)

top view:

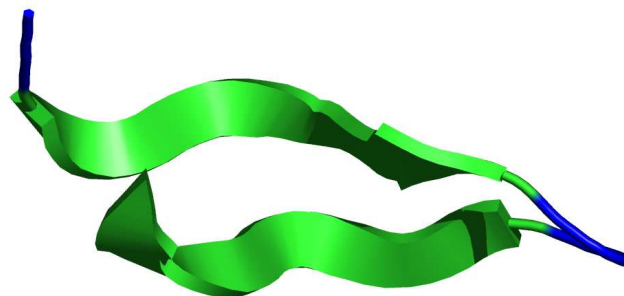


side view:



PROFET result with FLEX/ $\epsilon(r)$ : RMS=4.4Å

top view:



side view:

