Optimal control of solar energy systems

Viorel Badescu
Candida Oancea Institute
Polytechnic University of Bucharest
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3. Optimal operation - maximum exergy extraction

- The best operation strategies for open loop flat-plate solar collector systems are considered.
- A direct optimal control method (the TOMP algorithm) is implemented.
- A detailed collector model and realistic meteorological data from both cold and warm seasons are used.
Preliminaries

- Fully concentrated direct solar radiation is very rich in exergy (more than 90%).
- The exergy content of fully concentrated diffuse solar radiation is smaller but still high, ranging from 72.6% for single scattering to 9.6% in case of four scatterings.
- Therefore, solar energy collection systems may be used for power generation.
Preliminaries

- Part of the incident exergy flux is lost inside the solar energy conversion equipment due to various irreversible processes.
- Maximizing the exergy gain finally means minimizing the effects of these irreversible processes.
- It is known that thermal energy storage is associated to exergy destruction.
  - Therefore, the energy storage should normally be avoided in solar thermal systems designed for power generation.
- Open loop should be preferred to closed loop configurations in this case.
Preliminaries

- Solar energy conversion strategies are different from the point of view of their costs and feasibility.

- Optimization of these conversion processes can yield a variety of answers depending on the objective of the optimization and on the constraints that define the problem.

- The optimal paths are different for maximization of exergy gain and for maximization of energy gain.
Different objective functions related to the energy gain are considered:
- the cost per unit of energy transferred was minimized
- the amount of collected energy was maximized.

Different optimal strategies were found when the exergy gain are analyzed.
A realistic solar collector model is used.
Numerical optimization techniques are used.
Realistic mathematical model with time-dependent coefficients.
The model is implemented by using a large meteorological database.
Flow rate control can increase the performance of solar thermal systems.

The controller must be able to vary the flow rate in accordance with two types of fluctuations:

- One is attributed to disturbances
- Another one is caused by overshoots and undershoots in the manipulated variables that are caused by a lack of knowledge of future events.

Controllers for objective functions other than the solar energy gain are less studied.

Controller design in case the objective function is the exergy gain are presented here.
Model

- The energy balance at the level of the absorber plate:

\[ M' A_c c_m \frac{dT}{dt} = (\tau \alpha) G A_c - U_L A_c (T - T_{amb}) - \dot{m}' A_c c_p (T_{f,\text{out}} - T_{f,i}) \]

- The steady-state energy balance equation:

\[ h_f A'_t A_c (T - T_{f,m}) = \dot{m}' A_c c_p (T_{f,\text{out}} - T_{f,i}) \]
Optimum operation

- The gained exergy flux

\[ \dot{E}_x = \dot{m} c_p T_{amb} \left( \frac{T_{f,\text{out}} - T_{f,i}}{T_{amb}} - \ln \frac{T_{f,\text{out}}}{T_{f,i}} \right) \]

- The exergy collected in a time interval is:

\[ E_x = A_c \int_{t_1}^{t_2} \dot{m} c_p T_{amb} \left( \frac{T_{f,\text{out}} - T_{f,i}}{T_{amb}} - \ln \frac{T_{f,\text{out}}}{T_{f,i}} \right) dt \]

- The optimization problem consists of finding the optimum function \( \dot{m}_{opt}(t) \)

- that makes \( E_x \) a maximum, taking account of the constraint (energy balance equation).
Optimum operation

- The objective function is

\[
\tilde{E}_x \equiv \frac{E_x}{M' A_c c_m T_{ref}} = \int_{\tau_1}^{\tau_2} \theta_{amb} \tilde{h} \left( \frac{1 - \frac{1}{2}}{\mu} \right) \left\{ \mu \left( \frac{\theta - \theta_i}{\theta_{amb}} \right) - \ln \left[ 1 + \mu \left( \frac{\theta}{\theta_i} - 1 \right) \right] \right\} d\tau
\]

- The constraint (sometimes referred to as the state equation) is:

\[
\frac{d\theta}{d\tau} = (\tilde{\tau} \alpha) g - \tilde{U}(\theta - \theta_{amb}) - \tilde{h} \left( 1 - \frac{\mu}{2} \right) (\theta - \theta_i)
\]
Optimum operation

- The objective function may be seen as a function of 
  \[ \theta, \frac{d\theta}{d\tau}, \mu \]

- Procedures of variational calculus may be used to find the optimum function 
  \[ \mu_{opt}(\tau) \]

- The variational approach has no solution in the general case

- A direct optimal control technique is used to solve the problem
Variational simple case

The following simplifying assumptions are adopted:

(i) \( T_{f,\text{out}} = T \)

(ii) \( T_{f,i} = T_{\text{amb}} \)

(iii) the characteristics of the solar collector do not depend on time

(iv) the ambient temperature is constant in time and equals the reference temperature
Variational simple case

\[ \hat{E}_x = \int_0^1 \mathcal{L}(\theta, d\theta) \, d\tau = \int_0^1 \theta amb \left\{ (\tau \alpha) g - \tilde{U}(\theta - \theta amb) - \frac{d\theta}{d\tau} \right\} \left[ \frac{(\theta - \theta_i)}{\theta amb} - \ln \left( \frac{\theta}{\theta_i} \right) \right] d\tau \]

- The objective function may be maximized by using the variational approach.
- The Euler-Lagrange equation:

\[ \frac{\partial F}{\partial \theta} - \frac{d}{d\tau} \left[ \frac{\partial F}{\partial (d\theta / d\tau)} \right] = 0 \]
Variational simple case

\[
\frac{(\tau \tilde{\alpha})_g}{\tilde{U} \theta_{amb}} = \frac{(\theta_{opt} - 1)^3}{\theta_{opt} \ln \theta_{opt} - \theta_{opt} + 1}
\]

- This relationship may be used in principle to build a flow rate “instantaneous” controller.
- (1) Measure solar irradiance
- (2) Find the optimum temperature
- (3) Optimum mass flow rate parameter from

\[
\tilde{h} \left( 1 - \frac{\mu}{2} \right) = \frac{(\tau \tilde{\alpha})_g - \tilde{U} (\theta - \theta_{amb}) - \frac{d\theta}{d\tau}}{\theta - \theta_{amb}}
\]
Variational general case

The objective function

\[
\tilde{E}_x = \int_{\tau_1}^{\tau_2} F\left(\theta, \frac{d\theta}{d\tau}, \mu\right) d\tau \equiv \int_{\tau_1}^{\tau_2} \theta_{amb} \left\{ \left(\tau\alpha\right) g - \tilde{U}(\theta - \theta_{amb}) - \frac{d\theta}{d\tau} \right\} \left\{ \frac{\theta - \theta_i}{\theta_{amb}} - \frac{1}{\mu} \ln \left[ 1 + \mu \left( \frac{\theta}{\theta_i} - 1 \right) \right] \right\} d\tau
\]

the Euler-Lagrange equations:

\[
\frac{dF'}{d\theta} - \frac{d}{d\tau} \left[ \frac{\partial F'}{\partial (d\theta / d\tau)} \right] = 0 \quad \frac{dF'}{d\mu} = 0
\]
Variational general case

\[
\frac{(\tilde{\tau}\alpha) \left( g - \tilde{U}(\theta - \theta_{\text{amb}}) \right)}{\theta_i - \theta_{\text{amb}} + \mu(\theta - \theta_i)} \frac{\mu}{\theta_{\text{amb}}} = \frac{\mu}{\theta_{\text{amb}}} - \ln \left[ 1 + \mu \left( \frac{\theta}{\theta_i} - 1 \right) \right] \frac{1}{\theta - \theta_i}
\]

\[
\left[ 1 + \mu \left( \frac{\theta}{\theta_i} - 1 \right) \right] \ln \left[ 1 + \mu \left( \frac{\theta}{\theta_i} - 1 \right) \right] - \mu \left( \frac{\theta}{\theta_i} - 1 \right) = 0
\]
Variational general case

\[ x \equiv \mu(\theta / \theta_i - 1) \]

- is generally smaller than 0.35
- the Euler-Lagrange equation has no solution.
- The variational approach yields no useful result in the general case,
  - at least when the method of frozen parameters is adopted.
Direct optimal control approach

- The optimization problem is solved by using optimal control techniques.
- One may choose between
  - indirect methods (Pontryagin principle) and
  - direct methods.
- Indirect methods need preparing an adjoined (or co-state) differential equation to the state equation.
- This task is difficult to implement in the present case
  - because of the implicit dependence of the Hamiltonian on the state variable (the overall heat loss coefficient depends on the plate temperature).
- Accurate modeling should take account of this dependence
  - which creates difficulties in computing the derivatives of the Hamiltonian.
Direct method

- Direct shooting approach,
  - i.e. Trajectory Optimization by Mathematical Programming (TOMP).

- This avoids the need for the co-state equation
  - by transforming the original optimal control problem into a nonlinear programming problem (NPP).
Basic ideas of TOMP algorithm

- The state equation is an initial value problem (IVP) as a sub-problem.
- The integration time interval is divided into sub-intervals separated by nodes. The values of the control parameter in these nodes constitute the so called parameter vector.
- Initially, this parameter vector is unknown and a guess is necessary.
- The IVP is solved on the above integration interval by using common Runge-Kutta techniques.
Basic ideas of TOMP algorithm

- The resulted values of the state variable in the nodes of the integration interval depend on the parameter vector.
- Consequently, the objective function is dependent on this parameter vector.
- The NPP consists in maximizing the objective function in terms of the parameter vector.
- The resulted optimized parameter vector is returned as an entry to the IVP and a new set of values of the state variables in the nodes of the integration interval is obtained.
- Then, the objective function is maximized again and a new optimized parameter vector is obtained.
- This process continues until a given convergence condition for the parameter vector is satisfied.
Indicators of performance

\[ \eta_{en} \equiv \frac{\dot{m}' c_p (T_{f, out} - T_{f, i})}{G} \]

\[ \bar{\eta}_{en} \equiv \frac{\int_{t_1}^{t_2} \dot{m}' c_p (T_{f, out} - T_{f, i}) dt}{G(t_2 - t_1)} \]

\[ \eta_{ex} \equiv \frac{\dot{E}_x}{A_c G} \]

\[ \bar{\eta}_{ex} \equiv \frac{E_x}{A_c G(t_2 - t_1)} \]
Results

- In January the exergetic efficiency is low (less than 3%).
- The time variation of eta_ex is rather well correlated to the time variation of solar global irradiance.
- There is no obvious correlation between the time evolution of eta_ex and ambient temperature.
Aspects of controller design

- Controllers in solar energy collection systems are differentiated upon objective, complexity and way of operation.
- In case of closed loop solar thermal systems the typical control system has:
  - one sensor mounted on the collector absorber plate near the fluid outlet and
  - another mounted in the bottom of the storage tank.
- With no flow through the collector,
  - the collector sensor essentially measures the mean plate temperature.
- With flow,
  - the collector sensor measures the outlet fluid temperature.
- The optimal condition for the controller is simply
  - to turn on the pumps when the value of the solar energy that is delivered to the load just exceeds the value of the energy needed to operate the pump.
Controllers

- In case of solar space heating applications the usual classification of controllers is.
- Controllers of first kind (also called distribution controllers)
  - allow optimal heat distribution in a building.
  - This means that a certain objective function related to the thermal energy provided or living discomfort is minimized.
- Controllers of second kind (collection controllers)
  - maximize the difference between the useful collected energy and the energy required to transport the working fluid.
- Controllers of third kind combine collection and distribution functions.
Controllers

- The second kind controllers are responsible for the optimum operation of the pumps.
- Two sorts of second kind controllers are often used in applications.
  - One is the bang-bang controller
    - the mass flow rate has two allowable values: maximum and zero.
  - The other is the proportional controller
    - the mass flow rate is a linear function of the difference between the outlet working fluid temperature and the temperature inside the storage tank.
- Variants of proportional controllers exist such as
  - PID (proportional integral plus derivative mode) and
  - PSD (proportional sum derivative) controllers.
Controllers

- In case of systems for work generation a different control strategy is usually adopted.
- Published studies concerning solar thermal power plant operation consider that the purpose of the control is
  - to regulate the outlet temperature of the collector field
  - by suitable adjusting the working fluid flow.
Controllers

- Designing a mass flow rate controller based on optimal control theory encounters a major difficulty:
  - one needs a priori knowledge of meteorological data time series.
- An “instantaneous” controller,
  - able to optimally adjust the mass flow rate
  - by using as input just the last (in time) measured value of the meteorological parameters, would be highly desirable.
- This would avoid modeling the future history of irradiance and ambient temperature.
- We showed that the variational methods allow such an “instantaneous” controller to be build.
- However, the case studied
  - is very simple and
  - the additional simplifications make the results of little practical interest.
Constant mass flow strategy

- Results suggest that a constant mass flow rate may be a good strategy during the warm season.
- The strategy of maximum exergy collection is different from that of maximum energy collection.
Conclusions

- The maximum exergetic efficiency is low
  - usually less than 3 %
  - in good agreement with experimental measurements
- The optimum mass flow rate increases
  - near sunrise
  - and sunset
  - and by increasing the fluid inlet temperature.
- The optimum mass flow rate is well correlated with global solar irradiance during the warm season
- Operation at a properly defined constant mass flow rate may be close to the optimal operation
End of part \( \frac{3}{4} \)

Thank you!