Optimal control of solar energy systems

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2. Sizing solar collectors

- The optimum fin geometry in flat-plate solar collector systems.
- The width and thickness of fins is optimized by minimizing the cost per unit useful heat flux.
- The proposed procedure allows computation of the necessary collection surface area.
- A rather involved but still simple flat-plate solar collector model is used

Preliminaries

- The main objective:
  - reduce the parameters of the design process which are influenced by the changing cost conditions to the smaller possible number

- Fins with
  - constant and
  - variable thickness are studied
Preliminaries

- Two different objective functions are considered.
- In case of fins with constant thickness
  - the cost per unit useful heat is minimized
- when fins of variable thickness were considered
  - the material cost per unit fin length was minimized
    - to find the optimum fin thickness variation.
  - Subsequently, the cost per unit useful heat allowed to obtain the optimum fin width.
Preliminaries

- Computations are performed on a hourly basis
- The temperature is interpolated linearly between neighboring measured data
Uniform fin thickness

- standard Hottel-Whillier-Bliss Eq.

\[ q_u^* = G'' F_R^* \eta_0^* - U_L^* F_R^* \left( T_{fi}^* - T_a^* \right) \]

- the time averaged useful heat flux provided per unit collection area:

\[ q_u = G \eta'_0 - U'T' \]

- the time averaged solar energy conversion efficiency

\[ \eta \equiv \frac{q_u}{G} = \eta'_0 - \frac{U'T'}{G} \]
Uniform fin thickness
Uniform fin thickness

- The collector cost per unit length for a single tube is given by:
  \[ C \equiv c_F + c_A W + (W - d) \delta c_v \]

- The cost per unit useful heat flux is given by
  \[ J \equiv \frac{C}{Wq_u} \]

- Reduced cost parameters are defined by
  \[ a_1 \equiv \frac{c_A}{c_v}, \quad a_2 \equiv \frac{c_F}{c_v} \]

- A new cost function may be defined as
  \[ J' \equiv \frac{J}{c_v} = \frac{a_1 W + a_2 + (W - d) \delta}{Wq_u} \]
the minimum value of $J'$ may be found by solving the equations

$$\frac{\partial J'}{\partial W} = \frac{\partial J'}{\partial \delta} = 0$$
Results depend on

- the meteorological features of the site,
- the solar collector design,
- the solar collector economics (through the reduced cost parameters)
- the operation regime (through the average temperature difference).

Among these factors

- the average temperature difference only may be at user’s choice.
Results

- The optimum distance between tube centers decreases by increasing the difference of temperature (Fig. a).
- This is reasonable since higher operating temperatures, associated to a rather constant heat transfer rate in the tubes, require a smaller collection surface per tube.
- The optimum fin thickness is relatively the same, whatever the operation temperature and meteorological factors (Fig. b).
Important result

- A remarkable early result states that
  \[ 4W\delta = a_2 \]

- for an optimum fin.
- In fact, the optimum product
  \[ W\delta \]

- is rather constant, especially at higher operation temperatures.
Optimized shape of the collection surface area
Optimized shape of the collection surface area

- Time averaged energy balance for the collection area

\[ \dot{m} c_p (T_{f, \text{out}} - T_{f, i}) = A_c q_u \]

\[ \dot{m}' \equiv \frac{A_c}{\dot{m}} \]

\[ n_{\text{tube}} = \frac{\dot{m}}{\dot{m}_{\text{tube}}} \]

- The width of the collection surface area is given by

\[ l = n_{\text{tube}} W \]

- The tube length is

\[ L = A_c / l = \dot{m} \dot{m}' / l \]
Results

(a) Width of the collection surface area as a function of the distance $x$ from fluid inlet and

(b) the distance necessary to increase by one degree Celsius the working fluid temperature as a function of the temperature difference

![Graph showing width and distance as functions](image-url)
Variable fin thickness

The useful heat flux provided by an elemental collector area of unit length and width is given by:

\[ dq^*(x') = - \left( G'' \eta_0^* - U_L^* \left[ T^*(x') - T_a^* \right] \right) dx' \]
Variable fin thickness

The useful heat flux per unit collection surface area

\[ q_u^* = \left| \int dq^* \right| = \frac{2}{W} \int_0^{w/2} \left\{ G\eta_0^* - U_L^*[T^*(x') - T^*_a] \right\} dx' \]

After integration over time

\[ dq = -\left\{ G\eta_0 - U[T(x') - T_a] \right\} dx' \]

\[ q_u = \frac{2}{W} \int_0^{w/2} \left\{ G\eta_0 - U[T(x') - T_a] \right\} dx' \]
Variable fin thickness

The cost per unit collection area is given by

\[ C = \frac{1}{W} \left( c_F + c_A W + 2 c_v \int_{d/2}^{W/2} \delta(x') dx' \right) \]

A constant temperature of the tube on direction \( x' \) is assumed here

\[ T(x') = \begin{cases} 
  T_b \ (= \text{constant}) & \text{for } 0 \leq x' \leq d/2 \\
  \text{variable} & \text{for } d/2 \leq x' \leq W/2
\end{cases} \]
Variable fin thickness

The objective (cost) function is defined

\[ J' = \frac{1}{c_v \, q_u} \frac{1}{W} \left[ \frac{2}{W} \int_0^{(w-d)/2} \left( \frac{a_2 + a_1 W}{W - d} + \delta(x) \right) dx \right] \]

the solution satisfies the heat flow equations

\[ \frac{dT}{dx} = \frac{q}{k_p \, \delta(x)} \quad \frac{dq}{dx} = -\{ G \eta_0 - [T(x) - T_a] \} \]

The solution satisfies the boundary conditions

\[ T(x = 0) = T_b \quad q \left( x = \frac{W - d}{2} \right) = 0 \]
Optimal control problem

- Pontryagin theory is used.
- The Hamiltonian is defined:

\[
H = -\frac{a_2 + a_1 W}{W - d} + \delta(x) + \frac{d}{W} [G \eta_0 - U(T_b - T_a)] + \frac{2}{W} \left( \int_0^{(w-d)/2} G \eta_0 - U[T(x) - T_a] \right) dx + \frac{q}{k_b} \delta(x)
\]

- The adjoint variables satisfy the equations

\[
\frac{d \psi_1}{dx} = -\frac{\partial H}{\partial T} = -\psi_2 U + \frac{a_2 + a_1 W}{W - d} \frac{U}{W} + \delta(x) \frac{1}{\left\{ \frac{d}{W} [G \eta_0 - U(T_b - T_a)] + \frac{2}{W} \left( \int_0^{(w-d)/2} G \eta_0 - U[T(x) - T_a] \right) dx \right\}^2}
\]

\[
\frac{d \psi_2}{dx} = -\frac{\partial H}{\partial q} = -\psi_1 \frac{q}{k_p} \delta(x)
\]
Optimal control problem

- The boundary conditions are:
  \[ \psi_1\left(x = \frac{W-d}{2}\right) = 0 \quad \psi_2(x = 0) = 0 \]

- The Hamiltonian is a maximum for
  \[ \frac{\partial H}{\partial \delta} = 0 \]

- Then, the optimum control is
  \[
  \delta_{opt}(x) = \left\{-\frac{Wq(x)\psi_1(x)}{2k_p} \left\{ \frac{d}{W} \left[ G\eta_0 - U(T_b - T_a) \right] + \frac{2}{W} \int_0^{(w-d)/2} \left[ G\eta_0 - U[T(x) - T_a] \right] dx \right\} \right\}^{1/2}
  \]
Optimal control problem

To find the optimum width value the following equation should be solved

\[ \frac{\partial J'}{\partial W} = 0 \]
Previous results

- Fins of variable thickness
  - the minimum amount of material as an objective function.
  - constant value for the heat loss coefficient and
  - operation temperature and meteorological factors neglected.

- Results
  - A circular shape requires the least material
    - but a triangular shape requires almost the same material.
  - In practice an absorber fin having a step-change in local thickness may be adopted
    - for compatibility with existing manufacturing methods.
Four different shapes of fins:
- (a) straight rectangular fins,
- (b) fins with a step-change in local thickness;
- (c) straight triangular fins and
- (d) straight fins of inverse parabolic profile.
  - the minimum amount of material was the objective function
  - The model does not take account of the thermal operation regime

The inverse parabolic profile save the larger amount of material
- when the ratio of material amount reduction to reduction in collector efficiency is calculated, design (b) has the highest value
Results

(a) \(\langle T \rangle - T_a = 20^\circ C\)

(b) \(\langle T \rangle - T_a = 40^\circ C\)

The optimum shape is very close to an isosceles triangle

The fin is longer for the cold season as compared to the warm season (Fig. a).

- This allows a larger quantity of solar energy to be collected and transferred when the insolation is smaller.

At larger operating temperatures the fin is much shorter and thicker for the warm season as compared to the cold season (Fig. b).

- This way the amount of solar energy collected and transformed in thermal energy is easier transferred to the fluid in the tube.
Results

- The optimum distance between the tubes increases by increasing the inlet fluid temperature.
- The distance is larger in the cold season than in the warm season.
Results

- The width of the collection area increases when the inlet fluid temperature increases.
- The width is larger for the cold season.
Conclusions

- The optimum fin cross-section is very close to an isosceles triangle.
- The fin width is shorter and the seasonal influence is weaker at lower operation temperatures.
- Fin width and thickness at base depend on season.
- The optimum distance between the tubes increases by increasing the inlet fluid temperature, and it is larger in the cold season than in the warm season.
End of part 2/4
Thank you!