Optimal control of solar energy systems

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0. Introduction

- Thermodynamics (classical theory)
- Potentials formulation (maximum/minimum)
- Irreversible thermodynamics (Variational approaches; kinetic potentials)
- Finite time thermodynamics (optimisation techniques)
- Further optimisation techniques? Optimal control
0. Introduction

- This talk shows how the classical methods of optimal control can be used by the solar energy engineer.
- Four applications will give a broad idea about the usefulness of these optimization procedures.
Contents

1. Optimal operation - systems with water storage tanks
2. Sizing solar collectors
3. Optimal operation - maximum exergy extraction
4. Sizing solar collection area
5. Conclusions
Solar collector model

- “Absorbed” heat flux
- Lost heat flux
- Useful heat flux = “absorbed” - lost
Solar collector model

Hottel-Whillier-Bliss eq. (W):

\[ q_u^* = G'' F_R^* \eta_0^* - U_L^* F_R^* \left( T_{fi}^* - T_a^* \right) \]
Here we show the details of the flat-plate solar collector model used in calculations. The reference is [14].

1. Optical efficiency

A transparent cover consisting of $N$ identical layers is considered. Radiation is incident on the transparent cover at incidence angle $\theta_i$. The relative refractive indexes of transparent layer material and of the medium from where radiation is coming are denoted $n_2$ and $n_1 \,(= 1)$, respectively. Then, the refraction angle $\theta_2$ of the radiation inside the transparent layer may be computed by using the refraction law:

$$\frac{\sin \theta_2}{\sin \theta_i} = \frac{n_1}{n_2}. \quad (A1)$$

The reflectance $\rho$ of the transparent layer is given (for unpolarized radiation) by the following Fresnel formula:

$$\rho = \frac{1}{2} \left[ \frac{\sin^2 (\theta_2 - \theta_i)}{\sin^2 (\theta_2 + \theta_i)} + \frac{\tan^2 (\theta_2 - \theta_i)}{\tan^2 (\theta_2 + \theta_i)} \right]. \quad (A2)$$

The transparent cover transmittance due to reflection, $\tau_{r,N}$, is computed by

$$\tau_{r,N} = \frac{1 - \rho}{1 + (2N-1)\rho}. \quad (A3)$$

Let us denote $k_{abs}$ and $\alpha$ the absorption factor and the thickness of one transparent layer, respectively.

The actual path of radiation $L_1$ through a single transparent layer is given by

$$L_1 = \frac{\alpha}{\cos \theta_2}. \quad (A4)$$

The transparent cover transmittance due to absorption, $\tau_{a,N}$, is computed by Beer-Bouguer-Lambert law:

$$\tau_{a,N} = \exp(-k_{abs}NL_1). \quad (A5)$$

and the total cover transmittance $\tau$ is given by

$$\tau = \tau_{r,N}\tau_{a,N}. \quad (A6)$$
Let $\alpha$ be the absorptance of the absorber plate. The transmittance-absorptance product $(\tau\alpha)_N$ of the collector takes account of multiple scattering of radiation between the transparent layers and the absorber plate:

$$
(\tau\alpha)_N = \frac{\alpha}{1 - (1 - \alpha)\rho_{d,N}},
$$

where $\rho_{d,N}$ is the diffuse reflectance taking the values 0.16, 0.24, 0.29 and 0.32 for 1, 2, 3 and 4 transparent layers, respectively.

All of the solar radiation that is absorbed by a cover system is not lost, since this absorbed energy tends to increase layers temperature and consequently reduce the losses from the plate. Let $\varepsilon_p$ be the emittance of the absorber plate. A general analysis for a cover system yields the following expression for the optical efficiency $\eta_0$ of the collector

$$
\eta_0(N) = (\tau\alpha)_N + \left[1 - \tau_{d,N} \right] \sum_{i=1}^{N} \varepsilon_i \left( N, \varepsilon_p \right) \varepsilon_i^{-1}.
$$

Here $\alpha_i$ is the ratio of the overall loss coefficient to the loss coefficient from the $i$ layer to the surroundings, tabulated in [14, p. 156, Table 7.9.1]. The optical efficiency is sometime referred to as the effective transmittance-absorptance product.
1. Overall heat loss coefficient

The overall heat loss coefficient $U_L$ is given by

$$U_L = U_t + U_b,$$

(A9)

where $U_b$ is the bottom heat loss coefficient, given by

$$U_b = \frac{k_b}{L_b},$$

(A10)

where $k_b$ and $L_b$ are the thermal conductivity and the thickness of the bottom insulation, respectively.

For a glazed solar collector, the top heat loss coefficient $U_t$ in Eq. (A.9) is given by

$$U_t = \hat{U}_t \left[1 - (s - 45)(0.00259 - 0.00144\varepsilon_g)\right],$$

(A11)

where $\hat{U}_t$ is the top heat loss coefficient for a collector tilted $45^\circ$ while $s$ is collector actual tilt in degrees. The empirical relation proposed in [14] is used here for $\hat{U}_t$:

$$\hat{U}_t = \left[\frac{N}{244 \left(\frac{2}{T_p - T_a} + \frac{1}{h_w}\right)^{0.82}} + \frac{\sigma (T_p + T_a) (T_p^2 + T_a^2)}{\left[\varepsilon_g + 0.0425N(1-\varepsilon_g)\right]^{1.1} + \frac{2N + f - 1}{\varepsilon_g} - n}\right].$$

(A12)

Here $T_p$ and $T_a$ are the space averaged absorber temperature and ambient temperature, respectively, $\varepsilon_g$ is glass emittance, $h_w$ [W/m$^2$/K] is the convection heat loss coefficient due to the wind speed $w_{wind}$ [m/s]. In practice we used $h_w = 5.7 + 3.8w_{wind}$. Also, $\sigma$ is Stefan-Boltzmann constant and $f = \left(1 - 0.04h_w + 5 \times 10^{-4}h_w^2\right)(1 + 0.058N).

Note that in case of collectors with straight fins with rectangular profile $U_L$ and $T_p$ are computed together by using an iterative procedure shown later in section 4 of this Appendix A. When fins of variable thickness are considered a simpler iterative procedure was used (see Section 5 of the paper). This is possible because the heat removal factor does not enter the calculations in this second case.
3. Collector heat removal factor

A register-type collector is considered here. Then, \( d \) is tube external diameter and \( W \) is the distance between the centers of two neighbor tubes. Let \( \delta_p \) and \( k_p \) be plate thickness and its material thermal conductivity, respectively. The standard fin efficiency \( F \) for straight fins with rectangular profile is given by:

\[
F = \left[ \frac{m(W-d)}{2} \right]^{-1} \tan \left[ \frac{m(W-d)}{2} \right], \quad \left( m = \sqrt{ \frac{U_L}{k_p \delta_p} } \right) \quad \text{(A13,14)}
\]

The collector efficiency factor \( F' \) is given by:

\[
F' = \left( \frac{1}{WU_L} \right) \left( \frac{1}{U_L [d + (W-d)F]} + \frac{1}{C_b} + \frac{1}{\pi d h_f} \right)^{-1/2} \quad \text{(A15)}
\]

Here \( C_b \) is bond conductance, \( d_i \) is the inside tube diameter and \( h_f \) is the heat transfer coefficient between the working fluid and the tube wall. Here we used \( d_i = d - 2\delta_p \). The working fluid is formally equivalent to water and the following empirical formula was used to evaluate \( h_f \) [W/m²/K] [16, p 54]

\[
h_f = (1430 + 23.3t - 0.048t^2) \cdot \frac{0.8}{w_\text{water}} \cdot \delta_i^{-0.2} \quad \text{(A16)}
\]

where \( t = T_{f,m} - 273.15 \) (with \( T_{f,m} \) [K] - the average working fluid temperature inside the tube) and \( w_\text{water} \) [m/s] is water speed in the tube. In Eq. (A16) the unit for \( \delta_i \) is [m]. The following common value was adopted during calculations

\[
w_\text{water} = 0.1 \text{ m/s} \quad \text{(A17)}
\]
In Eq. (A16) $T_{f,m}$ was evaluated as a function of the working fluid temperatures at collector inlet and outlet, $T_{f,i}$ and $T_{f,out}$, respectively, by:

$$T_{f,m} = \left( T_{f,i} + T_{f,out} \right)/2.$$  \hspace{1cm} (A18)

The energy balance of the fluid of mass flow rate $\dot{m}$ yields:

$$T_{f,\text{out}} - T_{f,i} = \frac{Q_u}{\dot{m} c_p} = \frac{Q_u}{(\dot{m}/A) c_p} = \frac{q_u}{\dot{m} c_p},$$  \hspace{1cm} (A19)

Here $Q_u$ is the useful heat provided by the collection area $A_c$ and $q_u \left(= Q_u / A \right)$ is the useful heat per unit area given by the following formula (A21).

The collector removal factor $F_R$ is given by

$$F_R = \frac{\dot{m} c_p}{U_L} \left[ 1 - \exp \left( - \frac{U_L F'}{\dot{m} c_p} \right) \right].$$  \hspace{1cm} (A20)

One reminds that $U_L$ entering Eq. (A20) is a function of the unknown space averaged collector temperature $T_p$ that may be evaluated from two equivalent expressions of collector energy balance:

$$q_u = \left( \eta_0 - U_L (T_p - T_a) \right) = F_R \left( \eta_0 - U_L (T_{f,i} - T_a) \right),$$  \hspace{1cm} (A21)

One easily finds:

$$T_p = T_a + \frac{\eta_0 (1 - F_R)}{U_L} + F_R (T_{f,i} - T_a).$$  \hspace{1cm} (A22)
4. Iterative procedure

The quantities $U_L, F, F', F_R$ and the temperature $T_p$ are evaluated all together through the following iterative procedure with $T_a$ and $T_{f,i}$ as input (given) parameters. A guessed value for $T_p$ is first adopted. Next, $U_L, F, F'$ and $F_R$ are evaluated from Eqs. (A12), (A13), (A15) and (A20), respectively. Finally, a new value for $T_p$ is obtained from Eq. (A22). It is compared with the guessed $T_p$ value and if they differ significantly the procedure is repeated by using the new $T_p$ value as entry.

Note that $T_a$ and $T_{f,i}$ in Appendix A correspond to $T_a^*$ and $T_{f,i}^*$ respectively, in the paper.
# Collector

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transparent cover</strong></td>
<td></td>
</tr>
<tr>
<td>Number of transparent layers $N$</td>
<td></td>
</tr>
<tr>
<td>Collectors I and III</td>
<td>1</td>
</tr>
<tr>
<td>Collectors II and IV</td>
<td>2</td>
</tr>
<tr>
<td>Thickness of one transparent layer</td>
<td>0.004 (m)</td>
</tr>
<tr>
<td>Relative refraction index</td>
<td>1.526</td>
</tr>
<tr>
<td>Absorption coefficient (water white glass)</td>
<td>4 (m⁻¹)</td>
</tr>
<tr>
<td>Emittance</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Absorber plate</strong></td>
<td></td>
</tr>
<tr>
<td>Thickness</td>
<td>0.0015 (m)</td>
</tr>
<tr>
<td>Absorptance</td>
<td>0.9</td>
</tr>
<tr>
<td>Emittance</td>
<td>0.1</td>
</tr>
<tr>
<td>Thermal conductivity (aluminum)</td>
<td>211 (Wm⁻¹K⁻¹)</td>
</tr>
<tr>
<td>Distance between tubes</td>
<td>0.1 (m)</td>
</tr>
<tr>
<td>Tube external diameter</td>
<td>0.013 (m)</td>
</tr>
<tr>
<td>Tube internal diameter</td>
<td>0.01 (m)</td>
</tr>
<tr>
<td>Bond conductance</td>
<td>0.03 (mKW⁻¹)</td>
</tr>
<tr>
<td><strong>Bottom thermal insulation</strong></td>
<td></td>
</tr>
<tr>
<td>Thickness $L_b$</td>
<td></td>
</tr>
<tr>
<td>Collectors I and II</td>
<td>0.05 (m)</td>
</tr>
<tr>
<td>Collectors III and IV</td>
<td>0.1 (m)</td>
</tr>
<tr>
<td>Thermal conductivity (polyurethane)</td>
<td>0.034 (Wm⁻¹K⁻¹)</td>
</tr>
</tbody>
</table>
1. Optimal operation - systems with water storage tanks

- Closed loop flat-plate solar collector systems are considered.
- The water storage tank operates in fully mixed regime.
- Two design configurations are considered:
  - (A) one serpentine in the tank (for the secondary circuit) and
  - (B) two serpentines in the tank (for both primary and secondary circuits).
CLOSED LOOP SYSTEM

- Solar energy collection area
- Primary circuit
- Water storage tank (fully mixed regime)
- Secondary circuit
CLOSED LOOP SYSTEM
OPERATION MODEL

\[ C_s \frac{dT_s}{dt} = Q_{in} - Q_{loss} - Q_{out} \]

\[ Q_{in} = \begin{cases} 
0 & \quad \text{(pump P does not operate)} \\
\dot{m}_1 c_1 (T_{f,out} - T_{f,in}) & \quad \text{(pump P operates)}
\end{cases} \]

\[ Q_{loss} = U_s A_s \left( T_s - T_{int} \right) \]
\[ Q_{out} = \dot{m}_0 c_1 \left[ 1 - \exp \left( - \frac{h_0 S_0}{\eta_0 \dot{m}_0 c_0} \right) \right] (T_s - T'_0) \]

\[ T''_0 = T_s + (T'_0 - T_s) \exp \left( - \frac{h_0 S_0}{\eta_0 \dot{m}_0 c_0} \right) \]
OPERATION MODEL

\[ Q_u = AF_R \left[ (\tau \alpha)G - U_L (T_{f,\text{in}} - T_a) \right] = \dot{m}_1 c_1 (T_{f,\text{out}} - T_{f,\text{in}}) \]

\[ Q_{in} = \begin{cases} 
0 & \text{(pump P does not operate)} \\
AF_R \left[ (\tau \alpha)G - U_L (T_s - T_a) \right] & \text{(pump P operates)}
\end{cases} \]
OPERATION MODEL

\[
\frac{dT_s}{dt} = \begin{cases} 
- \Gamma^2(T_s - T_{\text{int}}) - \Gamma^3(T_s - T'_0) & \text{(pump P does not operate)} \\
\Gamma^0 - \Gamma^1(T_s - T_a) - \Gamma^2(T_s - T_{\text{int}}) - \Gamma^3(T_s - T'_0) & \text{(pump P operates)}
\end{cases}
\]

\[
\frac{dT_s}{dt} = \begin{cases} 
- \Gamma^2(T_s - T_{\text{int}}) - \Gamma^3(T_s - T'_0) & \text{(pump P does not operate)} \\
\Gamma^6 + \Gamma^7 T_s - \Gamma^2(T_s - T_{\text{int}}) - \Gamma^3(T_s - T'_0) & \text{(pump P operates)}
\end{cases}
\]
OPTIMAL CONTROL

\[ J[\dot{m}_1(t)] = -\int_{t_1}^{t_2} \left\{ Q_{in}[\dot{m}_1(t)] - E_{pump}[\dot{m}_1(t)] \right\} dt \equiv -\int_{t_1}^{t_2} f_J \, dt \]

\[ E_{pump} = K_{pump} \dot{m}_1^3 \]
OPTIMAL CONTROL

\[ H \equiv -f_J + \psi f_T \]

\[ \frac{d\psi}{dt} = -\frac{\partial H}{\partial T_s} \]

\[ \frac{\partial H}{\partial m_1} = 0 \]
OPTIMAL CONTROL

\[ H = \left\{ C_s \left[ \Gamma^0 - \Gamma^1 (T_s - T_a) \right] - K_{pump} \dot{m}_1^3 \right\} \\
+ \psi \left[ \Gamma^0 - \Gamma^1 (T_s - T_a) - \Gamma^2 (T_s - T_{int}) - \Gamma^3 (T_s - T'_0) \right] \]

\[ \frac{d\psi}{dt} = C_s \Gamma^1 + \psi \left( \Gamma^1 + \Gamma^2 + \Gamma^3 \right) \]

\[ T_s(t_1) = T_{s, ini} \quad \psi(t_2) = 0 \]
Application

Here we used the daily time-schedule of mass flow rate for a family.
Computational procedures

- The time period was divided into “day-time” and “night-time” sub-intervals.
- During the “night-time” one ordinary differential Eq. (IVP) was solved in the unknown T_s.
- During the “day-time” the optimal control problem was solved as follows.
  - Two boundary value problems (BVPs) were associated to the optimal control problem, depending on system configuration.
  - In both cases the same boundary conditions were used.
Application

- Three days with different radiative characteristics:
  - a day with overcast sky
  - a day with cloudy sky and
  - a day with clear sky

- Two values of the maximum fluid mass flow rate were considered,
  - 0.01 kg/s and
  - 0.2 kg/s.
The optimal control strategy is almost similar to the common bang-bang strategy.

The optimal strategy involves two-step up and down jumps.

This is different from the one step jump of the bang-bang strategy.
The maximum inlet value may be as high as 10 kWh/day.

The outlet value is higher during the day-time because the warm water demand by the user is higher during that period of time.
Results
Conclusions

- The optimal control strategy is simple:
  - most of the time the pump is stopped or works at maximum speed.
- The present optimal strategy involves two-step up and down jumps.
- During days with overcast sky the pump operates almost continuously.
- During days with cloudy or clear sky the pump often stops.
Conclusions

- When a constant flow rate strategy is adopted,
  - there is an optimum ratio between the volume of the storage tank and the area of the solar energy collection surface: $V/A = 33.3 \text{ L/m}^2$.

- The optimal control strategy does not exhibit such an optimum:
  - the thermal energy supply to the user (slightly) decreases by increasing the ratio $V/A$. 
End of part ¼

Thank you!