The Role of Entropy Generation in Momentum and Heat Transfer

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Overview
Instead of an Overview
Instead of an Overview

I hope you enjoy the talk
Instead of an Overview

I hope you enjoy the talk about Entropy & what it is Good for
\[ dQ = T \, dS \]
\[ dQ = T \, dS \]

- HEAT transferred
- ENTROPY transferred
Introduction
Introduction

1024 pages

1148 pages
Introduction

- Index
  - Energy storage, 71
  - English system of units, 35–38
  - Enhancement surfaces, 632
  - Enhancement devices, 521–524
  - Entry region, 512–513
  - Environmental radiation, 770–776
  - Equilibrium states, 12–13
  - Error function, 961
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Two options

HEAT TRANSFER  

ENTROPY
Two options

irrelevant $\rightarrow$ negligible

HEAT TRANSFER $\rightarrow$ ENTROPY
Introduction

Two options

irrelevant $\rightarrow$ negligible

HEAT TRANSFER

relevant $\rightarrow$ important

ENTROPY
Introduction

Two options

- **irrelevant → negligible**
- **relevant → important**

**HEAT TRANSFER**

**ENTROPY**

H. Herwig (TUHH)
Two crucial questions (often asked by students)
Introduction

Two crucial questions (often asked by students)

1. What is entropy?
Two crucial questions (often asked by students)

1. What is entropy?
2. What is entropy good for?
1\textsuperscript{st} Question: What is entropy?
1st Question: What is entropy?

Expected answer
Entropy is . . .
1st Question: What is entropy?

**Expected answer**

Entropy is . . .

**But**

If entropy were . . . it would be only another word for something else.
1st Question: What is entropy?

**Expected answer**

Entropy is . . .

**But**

If entropy were . . . it would be only another word for something else.

**Instead**

Entropy is a concept waiting to be learned by those who ask.
2\textsuperscript{nd} Question: What is entropy good for?  
(In engineering heat transfer problems)
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(In engineering heat transfer problems)

Note: Entropy $\rightarrow$ entropy change
2nd Question: What is entropy good for? (In engineering heat transfer problems)

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- Transfer (reversible)
2\textsuperscript{nd} Question: What is entropy good for?
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- Transfer (reversible)
- Generation (irreversible)
2nd Question: What is entropy good for? (In engineering heat transfer problems)

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Answer: Entropy is good for
2nd Question: What is entropy good for?
(In engineering heat transfer problems)

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Answer: Entropy is good for
1. Conceptual considerations
2nd Question: What is entropy good for?
(In engineering heat transfer problems)

**Note:** Entropy $\rightarrow$ entropy change

- Transfer (reversible)
- Generation (irreversible)

**Answer:** Entropy is good for

I. Conceptual considerations
II. Determination of losses
2nd Question: What is entropy good for?
(In engineering heat transfer problems)

Note: Entropy $\rightarrow$ entropy change
- Transfer (reversible)
- Generation (irreversible)

Answer: Entropy is good for
- I. Conceptual considerations
- II. Determination of losses
- III. Assessment of processes
## I. General Conceptual Considerations

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<th>Technical aspects:</th>
<th>real ↔ ideal processes</th>
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I. General Conceptual Considerations

<table>
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<tr>
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| • Which is the ideal process (no entropy generation)?
I. General Conceptual Considerations

Technical aspects: real ↔ ideal processes

- Which is the ideal process (no entropy generation)?
- Where does entropy generation occur?
I. General Conceptual Considerations

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- Which is the ideal process (no entropy generation)?
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I. General Conceptual Considerations

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- Why does entropy generation occur?
- How can entropy generation be reduced?
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| Philosophical aspects: |
| Creation, existence & destruction of order |
I. General Conceptual Considerations

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- Mahulikar, Herwig (2009), Chaos, Solitons and Fractals, 41, 1939–1948
I. Special Conceptual Considerations

The “quality” of heat transfer
I. Special Conceptual Considerations

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\[ h \equiv \frac{\dot{q}_w}{\Delta T} \] (heat transfer coefficient)
I. Special Conceptual Considerations

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- \( h \equiv \frac{\dot{q}_w}{\Delta T} \) (heat transfer coefficient)
- \( \text{Nu} \equiv h \frac{L}{k} \) (Nußelt number)
I. Special Conceptual Considerations

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Crucial questions
I. Special Conceptual Considerations

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ideal: \( h = \infty, \text{Nu} = \infty \)
real: \( h \neq \infty, \text{Nu} \neq \infty \) (due to “losses”)

Crucial questions

- What is lost?
- Does \( \text{Nu} \) measure these “losses”?
### I. Special Conceptual Considerations

#### The “quality” of heat transfer

- \( h \equiv \frac{\dot{q}_w}{\Delta T} \) (heat transfer coefficient)
- \( \text{Nu} \equiv h \frac{L}{k} \) (Nußelt number)

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- **ideal**: \( h = \infty, \text{Nu} = \infty \)
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#### Crucial questions

- What is lost?
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---

Thermodynamics
“Quality” and “losses” from a thermodynamics point of view

\[ \dot{Q}_w = \dot{q}_w A \]
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

\[ \dot{Q}_w = \dot{q}_w A \]

\( T_\infty \): environmental temperature

\( T_0 \): environmental temperature
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

\[ \dot{Q}_w = \dot{q}_w A \]

\( T_\infty \): environmental temperature

\( T_0 \): environmental temperature

\[ \dot{S}_Q = \frac{\dot{Q}_w}{T_s} \]

Entropy

Transport \( \rightarrow \) reversible
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

\[ \dot{Q}_w = \dot{q}_w A \]

\( T_0 \): environmental temperature

Entropy

\[ \dot{S}_Q = \frac{\dot{Q}_w}{T_s} \]

\[ \dot{S}_C = \dot{Q}_w \left( \frac{1}{T_\infty} - \frac{1}{T_s} \right) \]

Transport \( \rightarrow \) reversible

Generation \( \rightarrow \) irreversible
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

\[ T_\infty \]

\[ T_0: \text{environmental temperature} \]

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**Entropy**

**Transport** \( \rightarrow \) reversible

\[ \dot{S}_Q = \frac{\dot{Q}_w}{T_s} \]

**Generation** \( \rightarrow \) irreversible

\[ \dot{S}_C = \dot{Q}_w \left( \frac{1}{T_\infty} - \frac{1}{T_s} \right) \]

**Nu** \( \equiv \)

\[ \frac{\dot{Q}_w L}{A k \Delta T} \]

(Nußelt number)
“Quality” and “losses” from a thermodynamics point of view

\[ T_0: \text{environmental temperature} \]

\[ T_s \]

\[ \dot{Q}_w = \dot{q}_w A \]

\[ \frac{1}{T_\infty} - \frac{1}{T_s} \]

\[ \Delta T \rightarrow \dot{Q}_w \]
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

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(Nußelt number)

\[ \text{Devaluation number} \]

\[ \Delta T \rightarrow \dot{Q}_w \]
I. Special Conceptual Considerations

“Quality” and “losses” from a thermodynamics point of view

\[ T_\infty \]

\[ T_s \]

\[ \dot{Q}_w = \dot{q}_w A \]

\[ T_0: \text{ environmental temperature} \]

\[ S_Q = \frac{\dot{Q}_w}{T_s} \]

\[ S_C = \dot{Q}_w \left( \frac{1}{T_\infty} - \frac{1}{T_s} \right) \]

\[ Nu \equiv \frac{\dot{Q}_w L}{A k \Delta T} \]

(Nußelt number)

\[ N_{dev} \equiv \frac{T_0 S_C}{\dot{Q}_w} \]

(Devaluation number)

\[ \Delta T \rightarrow \dot{Q}_w \]

\[ \Delta T \rightarrow T_0 S_C \]
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

\[ \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{Q}_{\text{out}} - \dot{Q}_{\text{in}} \]
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

\[ \eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}} \]
Example: Heat transfer in a power cycle

Heat transfer with \( \text{Nu} = 100 \)

\[
\dot{Q}_{\text{in}}
\]

\[
\dot{Q}_{\text{out}}
\]

\[
\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}
\]
Example: Heat transfer in a power cycle

Heat transfer with $\text{Nu} = 100$

\[
\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - P \\
\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}
\]
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

Heat transfer with $\text{Nu} = 100$

$\dot{Q}_{\text{in}}$  $\dot{Q}_{\text{out}}$

$\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}$

Steam power cycle  Organic Rankine cycle
- $\text{H}_2\text{O}$  - $\text{NH}_3$

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I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

Heat transfer with \( \text{Nu} = 100 \)

\[
\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \eta_{\text{cycle}} P
\]

Steam power cycle
- \( \text{H}_2\text{O} \)
- \( T_s = 600 \, ^\circ\text{C} \)

Organic Rankine cycle
- \( \text{NH}_3 \)
- \( T_s = 100 \, ^\circ\text{C} \)
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

Heat transfer with $\text{Nu} = 100$

Steam power cycle
- $\text{H}_2\text{O}$
- $T_s = 600\, ^\circ\text{C}$
- $\text{Nu} = 100$

Organic Rankine cycle
- $\text{NH}_3$
- $T_s = 100\, ^\circ\text{C}$
- $\text{Nu} = 100$

$\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}$
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

Heat transfer with $\text{Nu} = 100$

Steam power cycle
- $\text{H}_2\text{O}$
- $T_s = 600 \degree \text{C}$
- $\text{Nu} = 100$
- $N_{\text{dev}} = 0.4\%$

Organic Rankine cycle
- $\text{NH}_3$
- $T_s = 100 \degree \text{C}$
- $\text{Nu} = 100$
- $N_{\text{dev}} = 5\%$

$\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}$
I. Special Conceptual Considerations

Example: Heat transfer in a power cycle

Heat transfer with $\text{Nu} = 100$

Steam power cycle
- $\text{H}_2\text{O}$
- $T_s = 600 \degree \text{C}$
- $\text{Nu} = 100$
- $N_{\text{dev}} = 0.4 \%$
- lost exergy: 0.7 \% (at turbine)

Organic Rankine cycle
- $\text{NH}_3$
- $T_s = 100 \degree \text{C}$
- $\text{Nu} = 100$
- $N_{\text{dev}} = 5 \%$
- lost exergy: 24 \% (at turbine)

$\eta_{\text{cycle}} = \frac{P}{\dot{Q}_{\text{in}}}$
II. Determination of Losses

### Losses

- **Flow field** → velocity gradients → entropy generation, $\dot{S}_D$
- **Temperature field** → temperature gradients → entropy generation, $\dot{S}_C$
### II. Determination of Losses

#### Losses

| Flow field | velocity gradients | entropy generation, $\dot{S}_D'''$ |
| Temperature field | temperature gradients | entropy generation, $\dot{S}_C'''$ |

key quantity: $\dot{S}''' = \dot{S}_D''' + \dot{S}_C'''$
II. Determination of Losses

**Losses**

Flow field → velocity gradients → entropy generation, $\dot{S}_{D}^{\prime\prime\prime}$

Temperature field → temperature gradients → entropy generation, $\dot{S}_{C}^{\prime\prime\prime}$

Key quantity: $\dot{S}^{\prime\prime\prime} = \dot{S}_{D}^{\prime\prime\prime} + \dot{S}_{C}^{\prime\prime\prime}$

**Entropy balance equation**

\[
q \left( \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} \right) = \text{div} \left( \frac{\dot{q}}{T} \right) + \dot{S}_{D}^{\prime\prime\prime} + \dot{S}_{C}^{\prime\prime\prime}
\]

- Convection
- Transport
- Generation
II. Determination of Losses

**Losses**

Flow field → velocity gradients → entropy generation, $\dot{S}'''
o
Temperature field → temperature gradients → entropy generation, $\dot{S}'''

key quantity: $\dot{S}''' = \dot{S}_{D}''' + \dot{S}_{C}'''$

**Entropy balance equation**

$$q \left( \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} \right) = \text{div} \left( \frac{\bar{q}}{T} \right) + \dot{S}_{D}''' + \dot{S}_{C}'''$$

convection
transport

generation

- No need to solve: $s = s(p, T)$ is a postprocessing quantity
II. Determination of Losses

### Losses

<table>
<thead>
<tr>
<th>Flow field</th>
<th>velocity gradients</th>
<th>entropy generation, $\dot{S}_D^{'''}$</th>
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<td>Temperature field</td>
<td>temperature gradients</td>
<td>entropy generation, $\dot{S}_C^{'''}$</td>
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key quantity: $\dot{S}^{''''} = \dot{S}_D^{''''} + \dot{S}_C^{''''}$

### Entropy balance equation

$$\rho \left( \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} \right) = \text{div} \left( \frac{\vec{q}}{T} \right) + \dot{S}_D^{''''} + \dot{S}_C^{''''}$$

- convection
- transport
- generation

- No need to solve: $s = s(p, T)$ is a postprocessing quantity
- Identify $\dot{S}_D^{''''} = \frac{\mu}{T} \left( \left( \frac{\partial u}{\partial x} \right)^2 + \ldots \right)$; dissipation in the flow field
II. Determination of Losses

**Losses**

Flow field $\rightarrow$ velocity gradients $\rightarrow$ entropy generation, $\dot{S}_D'''

Temperature field $\rightarrow$ temperature gradients $\rightarrow$ entropy generation, $\dot{S}_C'''

key quantity: $\dot{S}''' = \dot{S}_D'''+ \dot{S}_C'''

**Entropy balance equation**

$$ \rho \left( \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} \right) = \text{div} \left( \frac{\vec{q}}{T} \right) + \dot{S}_D'''+ \dot{S}_C'''

- No need to solve: $s = s(p, T)$ is a postprocessing quantity
- Identify $\dot{S}_D''' = \frac{\mu}{T} \left( \left( \frac{\partial u}{\partial x} \right)^2 + \ldots \right)$; dissipation in the flow field
- Identify $\dot{S}_C''' = \frac{k}{T^2} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \ldots \right)$; conduction in the temperature field
II. Determination of Losses

Strategy

1. Determine $\dot{S}''', \dot{S}'''_C$ (local entropy generation)
II. Determination of Losses

### Strategy

1. Determine $\dot{S}_{D}'''$, $\dot{S}_{C}'''$ (local entropy generation)
2. Determine $\dot{S}_{D} = \int \dot{S}_{D}''' dV$, $\dot{S}_{C} = \int \dot{S}_{C}''' dV$ (overall entropy generation)
Strategy

1. Determine $\dot{S}_D'''$, $\dot{S}_C'''$ (local entropy generation)
2. Determine $\dot{S}_D = \int \dot{S}_D''' \, dV$, $\dot{S}_C = \int \dot{S}_C''' \, dV$ (overall entropy generation)
3. Link them to loss coefficients of flow and heat transfer
II. Determination of Losses

Strategy

1. Determine $\dot{S}'''_D$, $\dot{S}'''_C$ (local entropy generation)
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Straight forward strategy with some “potholes”
II. Determination of Losses

Strategy

1. Determine $\dot{S}^{\prime\prime\prime}_D$, $\dot{S}^{\prime\prime\prime}_C$ (local entropy generation)
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Straight forward strategy with some "potholes"

- Turbulent flow
II. Determination of Losses

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3. Link them to loss coefficients of flow and heat transfer

Straight forward strategy with some “potholes”

- Turbulent flow
- Definition of loss coefficients
II. Determination of Losses

1st “pothole”: Turbulence

\[ \dot{S}_D'' = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] \right. \]
\[ + \left. \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right) \]

\[ \dot{S}_D''' = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right] \right. \]
\[ + \left. \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 \right) \]
II. Determination of Losses

1st “pothole”: Turbulence

\[ \dot{S}_{D}'' = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right) \]

\[ \dot{S}_{D}''' = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right] + \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 \right) \]

\[ \dot{S}_{D}''' = \frac{\rho \varepsilon}{T} \rightarrow \varepsilon \text{ (turbulent dissipation rate)} \]
## II. Determination of Losses

1st “pothole”: Turbulence

<table>
<thead>
<tr>
<th>Known</th>
<th>To be modeled</th>
</tr>
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<tbody>
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<td>( \dot{S}_{D}^{\prime\prime\prime} ):</td>
<td>( \dot{S}_{D}^{\prime\prime\prime} ):</td>
</tr>
<tr>
<td>( \dot{S}_{D}^{\prime\prime\prime} = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right) )</td>
<td>( \dot{S}_{D}^{\prime\prime\prime} = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right] + \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 \right) )</td>
</tr>
<tr>
<td>( \dot{S}_{C}^{\prime\prime\prime} = \frac{k}{T^2} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right) )</td>
<td>( \dot{S}_{C}^{\prime\prime\prime} = \frac{k}{T^2} \left( \left( \frac{\partial T'}{\partial x} \right)^2 + \left( \frac{\partial T'}{\partial y} \right)^2 + \left( \frac{\partial T'}{\partial z} \right)^2 \right) )</td>
</tr>
</tbody>
</table>

\( \dot{S}_{D}^{\prime\prime\prime} = \frac{Q \varepsilon}{T} \rightarrow \varepsilon \) (turbulent dissipation rate)
II. Determination of Losses

1st “pothole”: Turbulence

\[ \dot{S}_{D}^{III} = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] 
+ \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right) \]

\[ \dot{S}_{D}'^{III} = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial y} \right)^2 + \left( \frac{\partial w'}{\partial z} \right)^2 \right] 
+ \left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2 + \left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2 + \left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2 \right) \]

\[ \dot{S}_{C}^{III} = \frac{k}{T^2} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right) \]

\[ \dot{S}_{C}'^{III} = \frac{\alpha}{\alpha_t} \dot{S}_{C}^{III} \rightarrow \alpha_t \] (turbulent diffusivity)
II. Determination of Losses

2nd “pothole”: Definition of loss coefficients

Flow-losses: dissipation

\[ K = \frac{\varphi}{u_m^2/2} \]
II. Determination of Losses

2\textsuperscript{nd} “pothole”: Definition of loss coefficients

**Flow-losses: dissipation**

\[ \varphi \rightarrow K = \frac{\varphi}{u_m^2/2} \]

\[ \varphi = T_\infty \dot{S}_D / \dot{m} \]
II. Determination of Losses

2\textsuperscript{nd} “pothole”: Definition of loss coefficients

Flow-losses: dissipation

\[ K = \frac{\phi}{u_m^2 / 2} \]

\[ \phi = T_\infty \dot{S}_D / \dot{m} \]  
(lost exergy: \( T_0 \dot{S}_D / \dot{m} \))
II. Determination of Losses

2\textsuperscript{nd} “pothole”: Definition of loss coefficients

Flow-losses: dissipation

\[ \varphi \rightarrow K = \frac{\varphi}{u_m^2/2} \]

\( \varphi = T_\infty \dot{S}_D/\dot{m} \)  

(lost exergy: \( T_0 \dot{S}_D/\dot{m} \))

\[ \Rightarrow \]

 entropy generation

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} u_m^2/2} \]  

(exergy loss coefficient)
II. Determination of Losses

2nd “pothole”: Definition of loss coefficients

**Flow-losses: dissipation**

\[ \varphi \rightarrow K = \frac{\varphi}{u_m^2/2} \]

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(lost exergy: \( T_0 \dot{S}_D / \dot{m} \))

**Heat transfer losses: energy devaluation**

**entropy generation**

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} u_m^2/2} \]

(exergy loss coefficient)
II. Determination of Losses

2nd “pothole”: Definition of loss coefficients

Flow-losses: dissipation

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(lost exergy: \( T_0 \dot{S}_D / \dot{m} \))

⇒ entropy generation

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} \, u_m^2/2} \]

(exergy loss coefficient)

Heat transfer losses: energy devaluation

⇒ entropy generation

\[ N_{\text{dev}} = \frac{T_0 \dot{S}_C}{Q_w} \]

(energy devaluation number)
II. Determination of Losses

2nd “pothole”: Definition of loss coefficients

Flow-losses: dissipation

\[ \varphi \rightarrow K = \frac{\varphi}{u_m^2/2} \]

\[ \varphi = T_\infty \dot{S}_D/m \]

(lost exergy: \( T_0 \dot{S}_D/m \))

⇒ entropy generation

\[ K^E = \frac{T_0 \dot{S}_D}{m u_m^2/2} \]

(exergy loss coefficient)

Heat transfer losses: energy devaluation

<table>
<thead>
<tr>
<th>flow field</th>
<th>K</th>
<th>( K^E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature field</td>
<td>Nu</td>
<td>( N_{\text{dev}} )</td>
</tr>
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⇒ entropy generation

\[ N_{\text{dev}} = \frac{T_0 \dot{S}_C}{Q_w} \]

(energy devaluation number)
II. Determination of Losses

Example: Rough wall pipe flow (→ Moody chart)
II. Determination of Losses

Example: Rough wall pipe flow (→ Moody chart)

\[
\begin{align*}
\frac{k}{D} & \quad f \\
10^3 & \quad 10^4 & \quad 10^5 \\
0.1 & \quad 0.06 & \quad 0.03 \\
\end{align*}
\]

Löwenherz thread

[Schiller, 1923]
II. Determination of Losses

Example: Rough wall pipe flow (→ Moody chart)

\[
f, \quad k/D, \quad \text{Loewenherz thread}
\]

Schiller, 1923
II. Determination of Losses

Example: Rough wall pipe flow (→ Moody chart)

\[ f = \frac{k}{2\pi R} \]

\[ \frac{k}{D} \]

\[ f \]

\[ \text{Re} \]

\[ \text{SLA} \]

\[ \text{exp.} \]

\[ \text{Herwig, Gloss, Wenterodt, 2008: JFM} \]

\[ \text{Schiller, 1923} \]

Löwenherz thread

\[ k/D \quad \text{exp.} \quad \text{SLA} \]

<table>
<thead>
<tr>
<th>k/D</th>
<th>exp.</th>
<th>SLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0%</td>
<td>+</td>
<td>+ + +</td>
</tr>
<tr>
<td>3.6%</td>
<td>•</td>
<td>• •</td>
</tr>
<tr>
<td>1.8%</td>
<td>×</td>
<td>× × ×</td>
</tr>
</tbody>
</table>
II. Determination of Losses

Example: Differentially heated channel flow (benchmark solution)
II. Determination of Losses

Example: Differentially heated channel flow (benchmark solution)
II. Determination of Losses

Example: Differentially heated channel flow (benchmark solution)
II. Determination of Losses

\[ \dot{S}_C^{***} \]

RANS
II. Determination of Losses

\[ \dot{S}_C \]

RANS

DNS
II. Determination of Losses

RANS

\[ \dot{S}_C \]

DNS

\[ \text{Entropy Generation} \]
II. Determination of Losses

RANS

\[ \dot{\mathcal{S}}_{C} \]

DNS
II. Determination of Losses

\[ \dot{S}_C^{'''} \]

RANS

DNS

H. Herwig (TUHH)  Entropy Generation
II. Determination of Losses

RANS

\[ \dot{S}_C^{''''} \]

DNS

H. Herwig (TUHH)
II. Determination of Losses

RANS

\[ \dot{S}_C \]

DNS

Entropy Generation
II. Determination of Losses

RANS

\[ \dot{S}_C \]

DNS

[Graph and 3D visualization]
II. Determination of Losses

\[ \dot{\mathcal{S}}_c^{III} \]

**RANS**

**DNS**
II. Determination of Losses

\[ \dot{S}_C \]

**RANS**

**DNS**
II. Determination of Losses

\[ \dot{S}_C \]

RANS

DNS
### III. Assessment of Processes

**Convective heat transfer**

<table>
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### III. Assessment of Processes

**Convective heat transfer**

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\[
\eta = \frac{St}{St_0} \left( \frac{f}{f_0} \right)^{-\frac{1}{3}}
\]

(thermo hydraulic performance parameter)
### III. Assessment of Processes

#### Convective heat transfer

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$$\eta = \frac{St}{St_0} \left(\frac{f}{f_0}\right)^{-\frac{1}{3}}$$  
(thermo hydraulic performance parameter)

$$\eta = \text{apple} / \frac{1}{3}$$
### III. Assessment of Processes

#### Convective heat transfer

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\[
\eta = \frac{St}{St_0} \left( \frac{f}{f_0} \right)^{-\frac{1}{3}}
\]

(thermo hydraulic performance parameter)

\[
\eta = 1 - \frac{1}{3} (f_0, St_0) \rightarrow (f, St) \rightarrow \eta \geq 1 ; \quad \eta: ???
\]
III. Assessment of Processes

Convective heat transfer

Flow

\[ \dot{S}_D \]

Heat conduction

\[ \dot{S}_C \]
### Convective heat transfer

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<td>$\dot{S}_C$</td>
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\[ \dot{S} = \dot{S}_D + \dot{S}_C \]
III. Assessment of Processes

Convective heat transfer

\[ \dot{S} = \dot{S_D} + \dot{S_C} \]
III. Assessment of Processes

Convective heat transfer

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$\dot{S} = \dot{S}_D + \dot{S}_C$

$(\dot{S}_{D,0}, \dot{S}_{C,0}) \rightarrow (\dot{S}_D, \dot{S}_C) \rightarrow \dot{S} \geq \dot{S}_0$; $\dot{S}$: entropy generation

H. Herwig (TUHH) Entropy Generation
Example: Bejan’s problem

\[ \dot{m} = \text{const} \; ; \; \frac{dT_m}{dx} = \text{const} \]

[Bejan, 1996]
Example: Bejan’s problem

\[ \dot{m} = \text{const} \quad ; \quad \frac{d T_m}{dx} = \text{const} \]

[Bejan, 1996]
III. Assessment of Processes

Example: Bejan’s problem

\[ \dot{m} = \text{const} ; \quad \frac{d T_m}{dx} = \text{const} \]

\[ \text{Re} = \frac{4 \dot{m}}{\pi \eta D} \]

[Bejan, 1996]

\[ \dot{S}_D / \dot{S}_o \]

\[ \dot{S}_C / \dot{S}_o' \]
III. Assessment of Processes

Example: Bejan’s problem

\[ \dot{m} = \text{const} ; \quad \frac{d T_m}{d x} = \text{const} \]

\[ \text{Re} = \frac{4 \dot{m}}{\pi \eta D} \]

\[ \dot{q}_w' \]

\[ \dot{m} \quad D \]

\[ \dot{S}/\dot{S}_o' \]

\[ \dot{S}_D/\dot{S}_o' \]

\[ \dot{S}_C/\dot{S}_o' \]

[Bejan, 1996]
Example: Bejan’s problem

\[ \dot{m} = \text{const} \quad ; \quad \frac{d T_m}{d x} = \text{const} \]

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[Bejan, 1996]
Example: Bejan’s problem extended

\[ \dot{m} = \text{const} \; ; \; \frac{dT_m}{dx} = \text{const} \]

\[ \text{Re} = \frac{4\dot{m}}{\pi \eta D} \]
III. Assessment of Processes

Example: Bejan’s problem extended

\[ \dot{m} = \text{const} ; \quad \frac{dT_m}{dx} = \text{const} \]

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III. Assessment of Processes

Example: Bejan’s problem extended

\[ \dot{m} = \text{const} \quad ; \quad \frac{dT_m}{dx} = \text{const} \]
III. Assessment of Processes

Example: Bejan’s problem extended

\[ \dot{m} = \text{const} \; ; \; \frac{dT_m}{dx} = \text{const} \]

\[
\dot{q}'_w \quad \dot{m} \quad \text{Re} = \frac{4\dot{m}}{\pi \eta D}
\]

\[
\dot{S}'/\dot{S}'_o \\
\dot{S}_D/\dot{S}'_o \\
\dot{S}_C/\dot{S}'_o
\]

\begin{align*}
10^{-0.2} & \quad 10^{0.2} & \quad 10^{0.4} & \quad 10^{0.6} \\
10^4 & \quad 10^{4.1} & \quad 10^{4.2} & \quad 10^{4.3} & \quad 10^{4.5} & \quad \text{Re}
\end{align*}
Example: Bejan’s problem extended

\[ \dot{m} = \text{const} ; \quad \frac{d T_m}{d x} = \text{const} \]

\[ \text{Re} = \frac{4 \dot{m}}{\pi \eta D} \]

Decreasing entropy generation for increasing roughness
III. Assessment of Processes

Assessment by $\eta$ (ائها)
III. Assessment of Processes

Assessment by $\eta$ (🍎 & 🍊)

Assessment by $\eta$ (🍎 & 🍊)

$\text{Re} \quad \text{opt}(\eta)$

0 0.1% 0.3% $K_s$ 0.5%
III. Assessment of Processes

Assessment by $\eta$ (🍎&🍊)

![Graph showing assessment by $\eta$ and $S'$ with Re on the y-axis and $K_s$ on the x-axis. The graph has two curves: one for $\text{opt}(\eta)$ and another for $\text{opt}(S')$. The y-axis ranges from 10000 to 30000, and the x-axis ranges from 0 to 0.5%.]

H. Herwig (TUHH)
III. Assessment of Processes

Assessment by $\eta (\bigcirc \& \bigcirc)$

![Graph 1](image1.png)

![Graph 2](image2.png)
III. Assessment of Processes

Assessment by $\dot{S} (\text{🍎&🍎})$

![Graph showing assessment of processes]

(previous diagram)
III. Assessment of Processes

Assessment by $\dot{S}$ ($\bullet$ & $\odot$)

$\frac{\dot{S}'}{\dot{S}_{o}'}$

Previous diagram

5% roughness $\rightarrow$ 30% less entropy generation
Towards the end
Towards the end

I. conceptual considerations

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} u_m^2 / 2} \]

\[ N_{\text{dev}} = \frac{T_0 \dot{S}_C}{\dot{Q}_w} \]
Towards the end

I. Conceptual considerations

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} u_m^2 / 2} \]

\[ N_{\text{dev}} = \frac{T_0 \dot{S}_C}{Q_w} \]

II. Determination of losses

\[ \dot{S}_D = \int \dot{S}'_D \, dV \]

\[ \dot{S}_C = \int \dot{S}'_C \, dV \]
Towards the end

I. conceptual considerations

\[ K^E = \frac{T_0 \dot{S}_D}{\dot{m} u_m^2 / 2} \]

\[ N_{dev} = \frac{T_0 \dot{S}_C}{Q_w} \]

II. determination of losses

\[ \dot{S}_D = \int \dot{S}_D''' dV \]

\[ \dot{S}_C = \int \dot{S}_C''' dV \]

III. assessment of processes

\[ \dot{S} = \dot{S}_D + \dot{S}_C \geq \dot{S}_0 \]
Conclusion
Instead of a Conclusion
INSTEAD OF A CONCLUSION

I HOPE YOU ENJOYED THE TALK
Instead of a Conclusion

I hope you enjoyed the talk about Entropy & what it is Good for