Modeling of fiber-bending stiffness in fiber-reinforced composites with a dynamic mixed-finite element method based on the principle of virtual power

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Fiber-reinforced composites(FRC)

- Previous works Madeo et al. [2015]
  - On mesoscopic level - Fibers induce deformation patterns
  - On macroscopic level - Influence the bending stiffness of the material

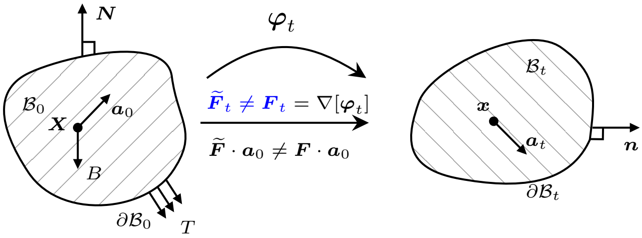
- We formulate a constitutive equation to account for out-of-plane bending

- Our approach for dynamical simulation:

Strategy	Goal	Property
Momentum consistent time integrator	to improve	Stability & accuracy
Higher-order gradients	to capture	Fiber-curvature
Mixed finite elements	to reduce	Locking effect

- We model numericaly based on variational-mixed principle of virtual power Gross et al. [2018]

New extended Cauchy-Boltzmann continuum model



$||\boldsymbol{a}_0|| = 1$ $\quad$ $\boldsymbol{a}_t = \tilde{\lambda}_{\widetilde{\boldsymbol{F}}} \bar{\boldsymbol{a}}_t = \widetilde{\boldsymbol{F}} \cdot \boldsymbol{a}_0$ $\quad$ $||\boldsymbol{a}_t|| = 1$

Variable	Asmanoglo et al. [2017]	This talk
System	Static	Dynamic
$\widetilde{\boldsymbol{F}}$	global	element wise
$\nabla\widetilde{\boldsymbol{F}}$	dependent	Independent
$\boldsymbol{G} = \nabla_X[\boldsymbol{a}_t]$	dependent	dependent
$\boldsymbol{\Lambda} = \widetilde{\boldsymbol{F}}^t \cdot \boldsymbol{G}$	dependent	dependent

Constitutive Model — Asmanoglo et al. [2017]

- General Strain energy function

$$\Psi^{total}(I_i) = \Psi^{iso}(I_1, I_2, I_3) + \Psi^{aniso}(I_4, I_5) + \Psi^{hg}(I_6)$$

$$I_1 = \boldsymbol{C} : \boldsymbol{I}$$
$$I_2 = cof\,(\boldsymbol{C}) : \boldsymbol{I}$$
$$I_3 = det\,(\boldsymbol{C})$$

$$I_4 = \boldsymbol{a}_0 \cdot \boldsymbol{C} \cdot \boldsymbol{a}_0$$
$$I_5 = \boldsymbol{a}_0 \cdot \boldsymbol{C}^2 \cdot \boldsymbol{a}_0$$

$$I_6 = (\boldsymbol{\Lambda} \cdot \boldsymbol{a}_0) \cdot (\boldsymbol{\Lambda} \cdot \boldsymbol{a}_0)$$

with $\boldsymbol{C} = \widetilde{\boldsymbol{F}}^t \cdot \widetilde{\boldsymbol{F}}$

$$\boldsymbol{\Lambda} = \widetilde{\boldsymbol{F}}^t \cdot \boldsymbol{G}$$

- Example: Polyconvex material formulation for the hyperelastic parts

$$\Psi^{total}(I_1, I_3, I_6; \lambda, \mu, c) = \underbrace{\lambda \frac{I_3 - 1}{4} - \left[\frac{\lambda}{2} + \mu\right] \ln\left(\sqrt{I_3}\right) + \frac{\mu}{2}\left[I_1 - 3\right]}_{\text{Simple Neo-Hookean isotropic part}} + c\, I_6$$

Hu-Washizu based functionals

$\odot_3$ – triple contraction

Energy functionals

$$\Pi_{int}(\boldsymbol{\varphi}, \widetilde{\boldsymbol{F}}, \widetilde{\boldsymbol{P}}, \widetilde{\boldsymbol{\Gamma}}, \widetilde{\mathfrak{B}}) := \int_{\mathcal{B}_0} \Psi^{hg}(\widetilde{\boldsymbol{F}}, \widetilde{\boldsymbol{\Gamma}})\, dV - \int_{\mathcal{B}_0} \widetilde{\boldsymbol{P}} : \left[\widetilde{\boldsymbol{F}} - \nabla\boldsymbol{\varphi}\right] dV - \int_{\mathcal{B}_0} \widetilde{\mathfrak{B}} \odot_3 \left[\widetilde{\boldsymbol{\Gamma}} - \nabla\widetilde{\boldsymbol{F}}\right] dV$$

$$\Pi_{ext}(\boldsymbol{\varphi}) := -\int_{\mathcal{B}_0} \rho_0 \boldsymbol{B}\cdot\boldsymbol{\varphi}\, dV - \int_{\partial\mathcal{B}_0} \boldsymbol{T}\cdot\boldsymbol{\varphi}\, dA - \int_{\partial\mathcal{B}_0} \boldsymbol{\lambda}_{\varphi}\cdot(\boldsymbol{\varphi} - \boldsymbol{\varphi}^{ref})\, dA$$

$$\mathcal{T}_{kin}(\dot{\boldsymbol{\varphi}}, \boldsymbol{v}, \boldsymbol{p}) := \int_{\mathcal{B}_0} \frac{1}{2}\rho_0 \boldsymbol{v}^2\, dV - \int_{\mathcal{B}_0} \boldsymbol{p}\cdot(\boldsymbol{v} - \dot{\boldsymbol{\varphi}})\, dV$$

Power functionals

$$\dot{\Pi}_{int}(\dot{\boldsymbol{\varphi}}, \dot{\widetilde{\boldsymbol{F}}}, \widetilde{\boldsymbol{P}}, \dot{\widetilde{\boldsymbol{\Gamma}}}, \widetilde{\mathfrak{B}}) := \int_{\mathcal{B}_0} \left[\frac{\partial\Psi^{hg}(\widetilde{\boldsymbol{F}}, \widetilde{\boldsymbol{\Gamma}})}{\partial\widetilde{\boldsymbol{F}}} : \dot{\widetilde{\boldsymbol{F}}} + \frac{\partial\Psi^{hg}(\widetilde{\boldsymbol{F}}, \widetilde{\boldsymbol{\Gamma}})}{\partial\widetilde{\boldsymbol{\Gamma}}} \odot_3 \dot{\widetilde{\boldsymbol{\Gamma}}}\right] dV - \int_{\mathcal{B}_0} \widetilde{\boldsymbol{P}} : \left[\dot{\widetilde{\boldsymbol{F}}} - \nabla\dot{\boldsymbol{\varphi}}\right] dV$$
$$- \int_{\mathcal{B}_0} \widetilde{\mathfrak{B}} \odot_3 \left[\dot{\widetilde{\boldsymbol{\Gamma}}} - \nabla\dot{\widetilde{\boldsymbol{F}}}\right] dV$$

$$\dot{\Pi}_{ext}(\dot{\boldsymbol{\varphi}}, \boldsymbol{\lambda}_{\varphi}) := -\int_{\mathcal{B}_0} \rho_0 \boldsymbol{B}\cdot\dot{\boldsymbol{\varphi}}\, dV - \int_{\partial\mathcal{B}_0} \boldsymbol{T}\cdot\dot{\boldsymbol{\varphi}}\, dA - \int_{\partial\mathcal{B}_0} \boldsymbol{\lambda}_{\varphi}\cdot(\dot{\boldsymbol{\varphi}} - \dot{\boldsymbol{\varphi}}^{ref})\, dA$$

$$\dot{\mathcal{T}}_{kin}(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) := \int_{\mathcal{B}_0} \rho_0 \boldsymbol{v}\cdot\dot{\boldsymbol{v}}\, dV - \int_{\mathcal{B}_0} \boldsymbol{p}\cdot(\dot{\boldsymbol{v}} - \ddot{\boldsymbol{\varphi}})\, dV - \int_{\mathcal{B}_0} \dot{\boldsymbol{p}}\cdot(\boldsymbol{v} - \dot{\boldsymbol{\varphi}})\, dV$$

Principle of Virtual power

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}\, dt \equiv \int_{t_n}^{t_{n+1}} \left[\delta_* \dot{\mathcal{T}}_{kin} \left(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}} \right) + \delta_* \dot{\Pi}_{int} \left(\dot{\boldsymbol{\varphi}}, \dot{\widetilde{\boldsymbol{F}}}, \widetilde{\boldsymbol{P}}, \dot{\widetilde{\boldsymbol{\Gamma}}}, \widetilde{\mathfrak{B}} \right) + \delta_* \dot{\Pi}_{ext} \left(\dot{\boldsymbol{\varphi}}, \boldsymbol{\lambda}_\varphi \right) \right] dt = 0$$

Variation with respect to dependencies

Virtual kinetic power

$$\delta_* \dot{\mathcal{T}}_{kin} := \int_{\mathcal{B}_0} [\rho_0 \boldsymbol{v} - \boldsymbol{p}] \cdot \delta_* \dot{\boldsymbol{v}}\, dV - \int_{\mathcal{B}_0} [\boldsymbol{v} - \dot{\boldsymbol{\varphi}}] \cdot \delta_* \dot{\boldsymbol{p}}\, dV + \int_{\mathcal{B}_0} \dot{\boldsymbol{p}} \cdot \delta_* \dot{\boldsymbol{\varphi}}\, dV$$

Virtual external power

$$\begin{aligned} \delta_* \dot{\Pi}_{ext} := & - \int_{\mathcal{B}_0} \rho_0 \boldsymbol{B} \cdot \delta_* \dot{\boldsymbol{\varphi}}\, dV - \int_{\partial \mathcal{B}_0} \boldsymbol{T} \cdot \delta_* \dot{\boldsymbol{\varphi}}\, dA - \int_{\partial \mathcal{B}_0} \boldsymbol{\lambda}_\varphi \cdot \delta_* \dot{\boldsymbol{\varphi}}\, dA \\ & - \int_{\partial \mathcal{B}_0} (\dot{\boldsymbol{\varphi}} - \dot{\boldsymbol{\varphi}}^{ref}) \cdot \delta_* \boldsymbol{\lambda}_\varphi\, dA \end{aligned}$$

Virtual internal power

$$\begin{aligned} \delta_* \dot{\Pi}_{int} := & \int_{\mathcal{B}_0} \left[\frac{\partial \Psi}{\partial \widetilde{\boldsymbol{F}}} + \left(\widetilde{\mathfrak{B}} \odot_3 \frac{\partial (\nabla \dot{\widetilde{\boldsymbol{F}}})}{\partial \dot{\widetilde{\boldsymbol{F}}}} \right) - \widetilde{\boldsymbol{P}} \right] : \delta_* \dot{\widetilde{\boldsymbol{F}}}\, dV - \int_{\mathcal{B}_0} \left[\dot{\widetilde{\boldsymbol{F}}} - \nabla \dot{\boldsymbol{\varphi}} \right] : \delta_* \widetilde{\boldsymbol{P}}\, dV \\ & + \int_{\mathcal{B}_0} \left[\frac{\partial \Psi}{\partial \widetilde{\boldsymbol{\Gamma}}} - \widetilde{\mathfrak{B}} \right] \odot_3 \delta_* \dot{\widetilde{\boldsymbol{\Gamma}}}\, dV - \int_{\mathcal{B}_0} \left[\dot{\widetilde{\boldsymbol{\Gamma}}} - \nabla \dot{\widetilde{\boldsymbol{F}}} \right] \odot_3 \delta_* \widetilde{\mathfrak{B}}\, dV + \int_{\mathcal{B}_0} \widetilde{\boldsymbol{P}} : \nabla [\delta_* \dot{\boldsymbol{\varphi}}]\, dV \end{aligned}$$

Weak forms

Weak deformation gradient equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\dot{\widetilde{\boldsymbol{F}}} - \nabla \dot{\boldsymbol{\varphi}} \right] : \delta_* \widetilde{\boldsymbol{P}} \, dV \, dt = 0$$

Weak curvature equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\dot{\widetilde{\boldsymbol{\Gamma}}} - \nabla \dot{\widetilde{\boldsymbol{F}}} \right] \odot_3 \delta_* \widetilde{\mathfrak{B}} \, dV \, dt = 0$$

Weak first Piola-Kirchoff stress equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\frac{\partial \Psi}{\partial \widetilde{\boldsymbol{F}}} + \left(\widetilde{\mathfrak{B}} \odot_3 \frac{\partial (\nabla \dot{\widetilde{\boldsymbol{F}}})}{\partial \dot{\widetilde{\boldsymbol{F}}}} \right) - \widetilde{\boldsymbol{P}} \right] : \delta_* \dot{\widetilde{\boldsymbol{F}}} \, dV \, dt = 0$$

Weak curvature stress equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\frac{\partial \Psi}{\partial \widetilde{\boldsymbol{\Gamma}}} - \widetilde{\mathfrak{B}} \right] \odot_3 \delta_* \dot{\widetilde{\boldsymbol{\Gamma}}} \, dV \, dt = 0$$

Weak boundary displacement equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} (\dot{\boldsymbol{\varphi}} - \dot{\boldsymbol{\varphi}}^{ref}) \cdot \delta_* \boldsymbol{\lambda}_{\varphi} \, dA \, dt = 0$$

Weak momentum balance equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} [\dot{\boldsymbol{p}} - \rho_0 \boldsymbol{B}] \cdot \delta_* \dot{\boldsymbol{\varphi}} \, dV \, dt - \int_{t_n}^{t_{n+1}} \int_{\partial \mathcal{B}_0} [\boldsymbol{T} + \boldsymbol{\lambda}_{\varphi}] \cdot \delta_* \dot{\boldsymbol{\varphi}} \, dA \, dt + \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \widetilde{\boldsymbol{P}} : \nabla [\delta_* \dot{\boldsymbol{\varphi}}] \, dV \, dt = 0$$

Weak velocity equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} [\boldsymbol{v} - \dot{\boldsymbol{\varphi}}] \cdot \delta_* \dot{\boldsymbol{p}} \, dV \, dt = 0$$

Weak momentum equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} [\rho_0 \boldsymbol{v} - \boldsymbol{p}] \cdot \delta_* \dot{\boldsymbol{v}} \, dV \, dt = 0$$

Angular momentum balance law $\mathcal{J} := \int_{\mathcal{B}_0} \boldsymbol{\varphi} \times \boldsymbol{p} \, dV$

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \dot{\mathcal{J}} = \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \boldsymbol{\varphi} \times \dot{\boldsymbol{p}} \, dV \, dt + \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \underbrace{\dot{\boldsymbol{\varphi}} \times \boldsymbol{p}}_{=0} \, dV \, dt$$

Employing the test function $\delta_* \dot{\boldsymbol{\varphi}} = \boldsymbol{c} \times \boldsymbol{\varphi}$ in weak momentum balance equation

axial vector $\boldsymbol{c} =$ const.

$$\begin{aligned}
\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \boldsymbol{c} \cdot [\boldsymbol{\varphi} \times \dot{\boldsymbol{p}}] \, dV dt \quad &= \quad \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \boldsymbol{c} \cdot [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}] \, dV dt + \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \boldsymbol{c} \cdot \left[\widetilde{\boldsymbol{P}} \times \widetilde{\boldsymbol{F}} \right] \, dV \, dt \\
&+ \int_{t_n}^{t_{n+1}} \int_{\partial \mathcal{B}_0} \boldsymbol{c} \cdot [\boldsymbol{\varphi} \times (\boldsymbol{T} + \boldsymbol{\lambda}_\varphi)] \, dA \, dt
\end{aligned}$$

$$\begin{aligned}
\mathcal{J}_{n+1} - \mathcal{J}_n \quad &= \quad \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} [\boldsymbol{\varphi} \times \rho_0 \boldsymbol{B}] \, dV dt + \int_{t_n}^{t_{n+1}} \int_{\partial \mathcal{B}_0} [\boldsymbol{\varphi} \times (\boldsymbol{T} + \boldsymbol{\lambda}_\varphi)] \, dA \, dt \\
&+ \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \epsilon : \left[\frac{\partial \Psi}{\partial \widetilde{\boldsymbol{F}}} + \left(\widetilde{\mathfrak{B}} \odot_3 \frac{\partial (\nabla \dot{\widetilde{\boldsymbol{F}}})}{\partial \dot{\widetilde{\boldsymbol{F}}}} \right) \right] \widetilde{\boldsymbol{F}}^t \, dV \, dt
\end{aligned}$$

Lagrangian ansatz functions - Space (N) and Time (M, M', $\tilde{M}$) Gross et al. [2018]

Polynomial degree in time - k

- Time rate variables and mixed fields ($\boldsymbol{\varphi}$, $\boldsymbol{v}$, $\boldsymbol{p}$, $\widetilde{\boldsymbol{F}}$, $\widetilde{\boldsymbol{\Gamma}}$)

$$(\bullet)^{e,h} = \sum_{I=1}^{k+1}\sum_{A=1}^{n_{no}} M_I(\alpha)\mathrm{N}^A(\boldsymbol{\xi})(\bullet)_I^{eA}$$

$$(\dot{\bullet})^{e,h} = \frac{1}{h_n}\sum_{I=1}^{k+1}\sum_{A=1}^{n_{no}} M'_I(\alpha)\mathrm{N}^A(\boldsymbol{\xi})(\bullet)_I^{eA}$$

$$M_i(\alpha) = \prod_{\substack{j=1\\ j\neq i}}^{k+1} \frac{\alpha-\alpha_j}{\alpha_i-\alpha_j}, \; 1 \leq i \leq k+1$$

- Lagrange multiplier and variation fields ($\widetilde{\boldsymbol{P}}$, $\widetilde{\boldsymbol{\mathcal{B}}}$, $\delta_*\bullet$)

$$(\bullet)^{e,h} = \sum_{I=1}^{k}\sum_{A=1}^{n_{no}} \tilde{M}_I(\alpha)\mathrm{N}^A(\boldsymbol{\xi})(\bullet)_I^{eA}$$

$$\tilde{M}_i(\alpha) = \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\alpha-\alpha_j}{\alpha_i-\alpha_j}, 1 \leq i \leq k$$

- Lagrange multipliers are approximated with same shape functions corresponding to their mixed fields e.g. ($\widetilde{\boldsymbol{F}}$ & $\widetilde{\boldsymbol{P}}$), ($\widetilde{\boldsymbol{\Gamma}}$ & $\widetilde{\boldsymbol{\mathcal{B}}}$)

Discontinuous at the boundaries of spatial elements

- Solve for $\widetilde{\boldsymbol{F}}$ with its local weak form $\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0^e} \left[\dot{\widetilde{\boldsymbol{F}}}^{e,h} - \nabla \boldsymbol{\varphi}^{e,h} \right] : \delta_* \widetilde{\boldsymbol{P}}^{e,h} \, dV \, dt \quad = 0$

  Solve for $\widetilde{\boldsymbol{\Gamma}}$ with its local weak form $\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0^e} \left[\dot{\widetilde{\boldsymbol{\Gamma}}}^{e,h} - \nabla \dot{\widetilde{\boldsymbol{F}}}^{e,h} \right] \odot_3 \delta_* \widetilde{\mathfrak{B}}^{e,h} \, dV \, dt \; = 0$

- Condensate at element level to pure displacement form

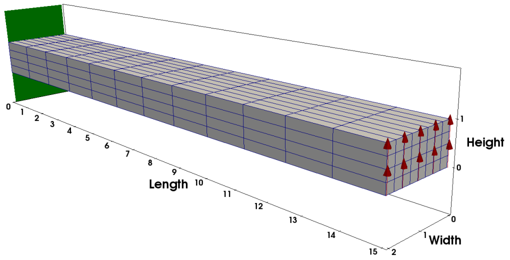
$$
\begin{bmatrix}
\boldsymbol{K}^e_{\varphi\varphi} & \mathbf{0} & \boldsymbol{\mathcal{K}}^e_{\varphi p} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbb{K}^e_{ff} & \mathbb{K}^e_{fp} & \overline{\boldsymbol{\mathcal{K}}}^e_{fg} & \overline{\boldsymbol{\mathcal{K}}}^e_{fb} \\
\boldsymbol{\mathcal{K}}^e_{p\varphi} & \mathbb{K}^e_{pf} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \overline{\boldsymbol{\mathcal{K}}}^e_{gf} & \mathbf{0} & \overline{\mathbb{K}}^e_{gg} & \overline{\mathbb{K}}^e_{gb} \\
\mathbf{0} & \overline{\boldsymbol{\mathcal{K}}}^e_{bf} & \mathbf{0} & \overline{\mathbb{K}}^e_{bg} & \mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\Delta \boldsymbol{\varphi} \\ \Delta \widetilde{\boldsymbol{F}} \\ \Delta \widetilde{\boldsymbol{P}} \\ \Delta \widetilde{\boldsymbol{\Gamma}} \\ \Delta \widetilde{\boldsymbol{\mathcal{B}}}
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{r}^e_{\varphi} \\ \mathbf{r}^e_{\widetilde{\boldsymbol{F}}} \\ \mathbf{r}^e_{\widetilde{\boldsymbol{P}}} \\ \mathbf{r}^e_{\widetilde{\boldsymbol{\Gamma}}} \\ \mathbf{r}^e_{\widetilde{\boldsymbol{\mathcal{B}}}}
\end{bmatrix}
$$

$$
\widetilde{\boldsymbol{K}}^e_{\varphi\varphi} \; \Delta\varphi \; = \boldsymbol{r}^e_{\varphi}
$$

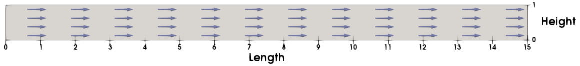
- Eliminating $\boldsymbol{p}$, e.g.

$$
\int_T \int_{\mathcal{B}_0} \left[\mathrm{Div}[\widetilde{\boldsymbol{P}}] - \dot{\boldsymbol{p}} \right] \cdot \delta\dot{\boldsymbol{\varphi}} \, dV \, dt = 0
$$

$$
\int_T \int_{\mathcal{B}_0} \left[\frac{1}{\rho_0} \boldsymbol{p} - \dot{\boldsymbol{\varphi}} \right] \cdot \delta\boldsymbol{\dot{v}} \, dV \, dt = 0
$$



Dirichlet: Fixed support   Neumann: In-plane load



Fiber orientation along X-axis

* Dal et al. [2017]

Initial conditions

$$\boldsymbol{\varphi}_0 = \boldsymbol{X}$$
$$\boldsymbol{v}_0 = \boldsymbol{0}$$
$$\boldsymbol{a}_0^t = [1 \;\; 0 \;\; 0]^T$$
$$p = 3.5\, e^6$$

Material parameters

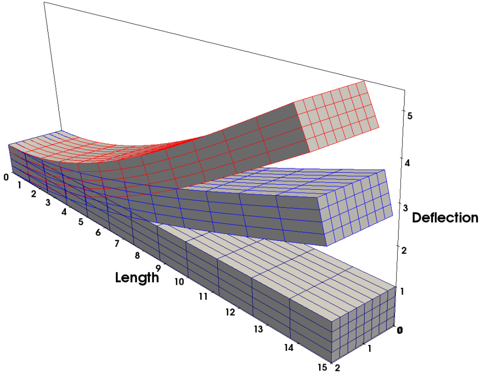
$$\mu^* = 4.04\, e^9$$
$$K^* = 5.2\, e^9$$
$$c = [0, e^{12}]$$

Temporal parameters

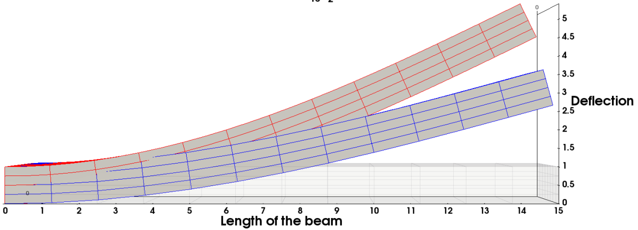
$$h_n = 0.005$$
$$T = 1$$
$$Tol = 1e^{-2}$$

Numerical studies

Maximum deflection vs. fiber-stiffness parameter c



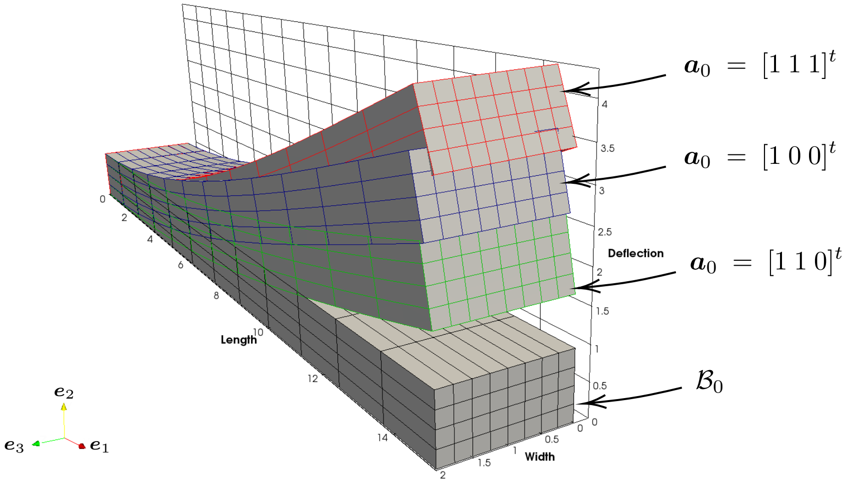
$$\boldsymbol{a}_0 = [1 \;\; 0 \;\; 0]^t$$



$c = 0$

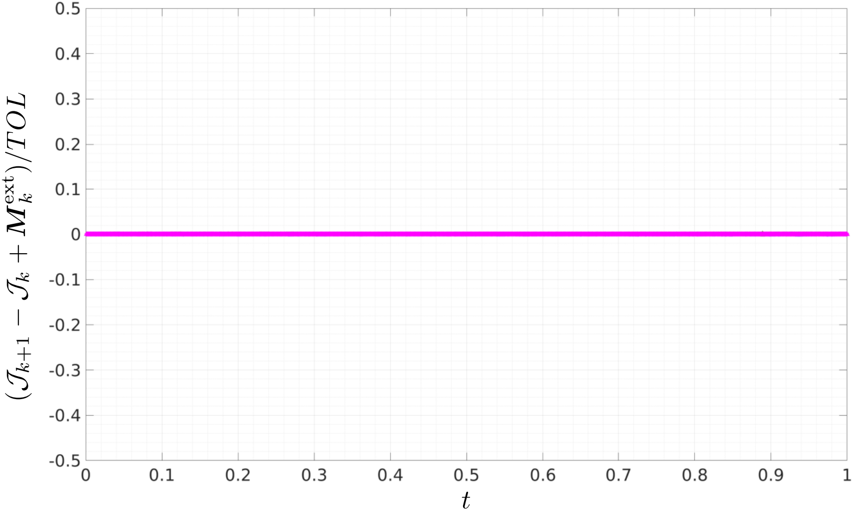
$c = e^{12}$

Numerical studies
Maximum deflection vs. fiber orientation $\boldsymbol{a}_0$



Fiber stiffness parameter $c = e^{12}$

Conserved total angular momentum



Summary

1. Motivation
   - Accurate dynamical FE simulations for fiber-reinforced composites
   - Capture fiber bending stiffness
2. Approach
   - Higher-order gradient based independent field for the fiber curvature
   - Variational-based mixed finite element formulation
3. Important results
   - Additional Invariant (I_6) captures the fiber curvature effect
   - Time integrator conserves angular momentum consistently
   - Gradient shape functions can be approximated with different polynomials
4. Further steps
   - Energy-momentum consistent time integrator
   - Different types of strain energy functions related to gradient