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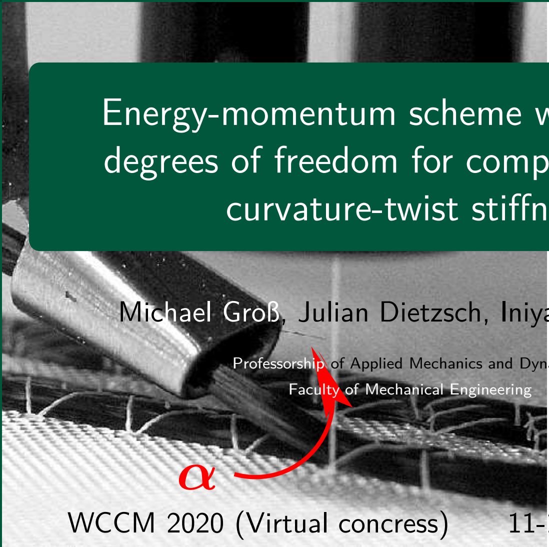
Energy-momentum scheme with drilling degrees of freedom for composites with curvature-twist stiffness



Michael Groß, Julian Dietzsch, Iniyan Kalaimani

Professorship of Applied Mechanics and Dynamics
Faculty of Mechanical Engineering

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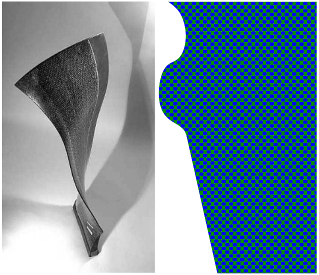
Motivation: lightweight rotating system design

Background: increasing application of fiber roving composites

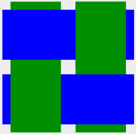
1. Rotor shafts
2. Turbine/pump rotors
3. Disc brake rotors

Macroscopic scale: turbine blade preform

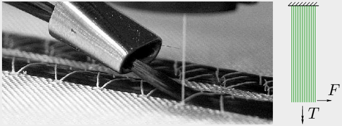


De Luycker, Morestin, Boisse, Marsal [2009]

Mesoscopic scale: Volume Element (RVE)



Micro scale: Roving with curvature stiffness



Kai Uhlig [2017] (IPF TU Dresden), recommended by IST TU Chemnitz

Goals: FE simulations taking into account the fiber twisting/bending (curvature) stiffness

1. We design dynamic variational principles for continua with multi-scale effects
2. We derive an energy-momentum time integration for a consistent simulation

Strategy (I): Extended continuum mechanics

(see e.g. Boisse et al. [2018], Asmanoglo & Menzel [2017], Madeo et al. [2015], Feretti et al. [2014], Spencer & Soldatos [2007])

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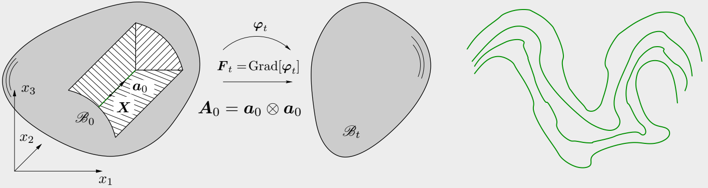
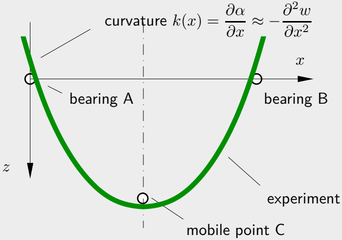
Formulation of anisotropic Cauchy continua with structural tensors

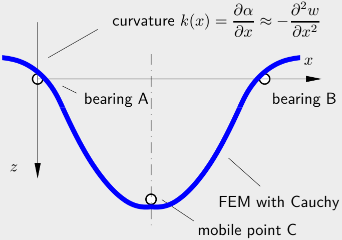
(see e.g. Reese, Raible & Wriggers [2001], Klinkel, Sansour & Wagner [2005], Schröder, Neff & Balzani [2005])



Three point bending test: experiment vs. Cauchy theory $\rightsquigarrow$ 'the need to a second-gradient theory'

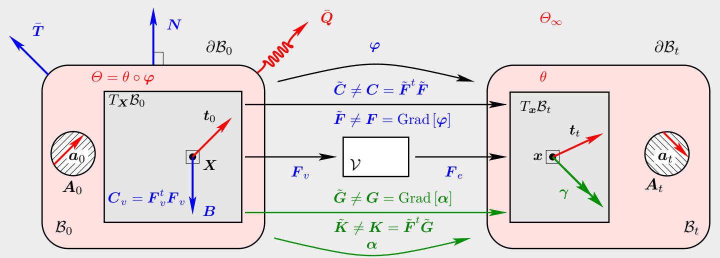
(see Charmetant et al. [2012], Madeo et al. [2015])





Strategy (II): Mixed drilling degrees of freedom

Anisotropic constrained micropolar continuum (cf. Steinmann & Stein [1997])



Deformation of fibers and matrix (cf. Klinkel, Sansour & Wagner [2005])

1. Fiber stretch per unit length with right Cauchy-Green tensor

$$C_F := \boldsymbol{a}_t \cdot \boldsymbol{a}_t \equiv \boldsymbol{a}_0 \cdot \boldsymbol{C} \cdot \boldsymbol{a}_0 \equiv \mathrm{tr}\,[\boldsymbol{A}_0\boldsymbol{C}] =: I_1^{\boldsymbol{C}_F} \qquad \boldsymbol{C} := \boldsymbol{F}^t\boldsymbol{F} \qquad \boldsymbol{C}_F := \boldsymbol{A}_0\,\boldsymbol{C}$$

2. Fiber twist per unit length with curvature-twist tensor

$$T_F := \boldsymbol{a}_t \cdot \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{x}} \cdot \boldsymbol{a}_t \equiv \boldsymbol{a}_0 \cdot \boldsymbol{K} \cdot \boldsymbol{a}_0 \equiv \mathrm{tr}\,[\boldsymbol{A}_0\boldsymbol{K}] =: I_1^{\boldsymbol{K}_F} \qquad \boldsymbol{K} := \boldsymbol{F}^t\boldsymbol{G} \qquad \boldsymbol{K}_F := \boldsymbol{A}_0\boldsymbol{K}$$

3. Fiber bending per unit length with curvature-twist tensor

$$-2\,I_2^{\boldsymbol{K}_F} := B_F - T_F^2 \qquad B_F := \boldsymbol{a}_0\boldsymbol{K} \cdot \boldsymbol{a}_0\boldsymbol{K} \equiv \boldsymbol{a}_0 \cdot \boldsymbol{K}\boldsymbol{K}^t \cdot \boldsymbol{a}_0 \equiv \boldsymbol{K}_F : \boldsymbol{K}_F$$

Strategy (III): Principle of virtual power

Principle of virtual work / least action (cf. Dimarogonas et al. [2013])

1. Simo-Taylor-Pister functional (Simo, Taylor, Pister [1985])

$$\Pi_{\mathrm{STP}}(\boldsymbol{\varphi}, \tilde{J}, p) := \int_{\mathscr{B}_0} \Psi^{\mathrm{iso}}(\boldsymbol{\varphi})\,\mathrm{d}V + \int_{\mathscr{B}_0} \Psi^{\mathrm{vol}}(\tilde{J})\,\mathrm{d}V - \int_{\mathscr{B}_0} p\,[\tilde{J} - \det(\mathrm{Grad}[\boldsymbol{\varphi}])]\,\mathrm{d}V$$

2. Spatial weak form for $\tilde{J}$

$$\int_{\mathscr{B}_0} \delta p\, G(\boldsymbol{\varphi}, \tilde{J})\,\mathrm{d}V = 0 \quad \text{with} \quad G(\boldsymbol{\varphi}, \tilde{J}) := \tilde{J} - \det(\mathrm{Grad}[\boldsymbol{\varphi}])$$

Energy-momentum time integration (motivated by Betsch & Steinmann [2002], Armero [2008])

1. Preservation of balance law of total energy $H := T(\dot{\boldsymbol{\varphi}}) + \Psi^{\mathrm{iso}}(\boldsymbol{\varphi}) + \Psi^{\mathrm{vol}}(\tilde{J})$

$$\mathcal{H}_{t_{n+1}} - \mathcal{H}_{t_n} = \int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \left[\frac{\partial H}{\partial \dot{\boldsymbol{\varphi}}} \cdot \ddot{\boldsymbol{\varphi}} + \frac{\partial H}{\partial \boldsymbol{\varphi}} \cdot \dot{\boldsymbol{\varphi}} + \frac{\partial H}{\partial \tilde{J}}\, \dot{\tilde{J}} \right] \mathrm{d}V\,\mathrm{d}t$$

2. Time evolution of volume dilatation $\tilde{J}$

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta p\, \dot{G}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}})\,\mathrm{d}V\,\mathrm{d}t = 0 \quad \text{with} \quad \dot{G}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}}) := \dot{\tilde{J}} - \overline{\det(\mathrm{Grad}[\boldsymbol{\varphi}])}^{\boldsymbol{\cdot}}$$

Variational principle of choice (cf. Schröder & Kuhl [2015])

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}}, p)\,\mathrm{d}t = 0 \quad \text{with} \quad \mathcal{H} := \frac{1}{2} \int_{\mathscr{B}_0} \rho_0\, \dot{\boldsymbol{\varphi}} \cdot \dot{\boldsymbol{\varphi}}\,\mathrm{d}V + \Pi_{\mathrm{STP}}(\boldsymbol{\varphi}, \tilde{J}, p)$$

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Fiber twist kinematics and strain energy

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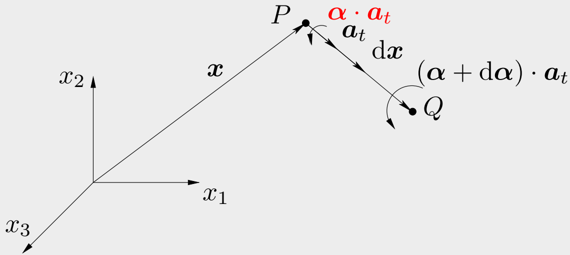
Twist of a fiber based on drilling degrees of freedom (cf. Stokes [2012])



St. Venant-Kirchhoff strain energy of fiber curvature-twist

1. Twist of a fiber bundle relative to point P

$$\mathrm{d}\boldsymbol{\alpha}\cdot\boldsymbol{a}_t=\left[\frac{\partial\boldsymbol{\alpha}}{\partial\boldsymbol{x}}\cdot\mathrm{d}\boldsymbol{x}\right]\cdot\boldsymbol{a}_t=[\boldsymbol{g}\cdot\boldsymbol{a}_t\,\mathrm{d}\xi]\cdot\boldsymbol{a}_t \qquad \boldsymbol{g}:=\frac{\partial\boldsymbol{\alpha}}{\partial\boldsymbol{x}} \qquad \boldsymbol{a}_t:=\boldsymbol{F}\boldsymbol{a}_0$$

2. Fiber twist per unit length $\mathrm{d}\xi$ with curvature-twist tensor

$$T_F:=\boldsymbol{a}_t\cdot\boldsymbol{g}\cdot\boldsymbol{a}_t=\boldsymbol{K}:\boldsymbol{A}_0=\boldsymbol{K}_F:\boldsymbol{I} \qquad \boldsymbol{K}_F:=\boldsymbol{A}_0\boldsymbol{K} \qquad \boldsymbol{K}:=\boldsymbol{F}^t\boldsymbol{G}$$

3. Quadratic fiber curvature-twist strain energy

$$\Psi^{\mathrm{ctw}}_{\mathrm{SVK}}:=\Psi^{\mathrm{twi}}_F+\Psi^{\mathrm{cur}}_F=\frac{1}{2}\,K_{\mathrm{twi}}\,(T_F)^2+\frac{1}{2}\,\mu_{\mathrm{cur}}\,(b_F)^2$$

Concept of St. Venant-Kirchhoff strain energy (I)

St. Venant-Kirchhoff (matrix) strain energy $\quad \boldsymbol{E} := [\boldsymbol{C} - \boldsymbol{I}]/2$

1. Spherical and deviatoric part

$$\Psi^{\mathrm{ela}}_{\mathrm{SVK}} := \Psi^{\mathrm{vol}}_{\mathrm{SVK}} + \Psi^{\mathrm{dev}}_{\mathrm{SVK}} = \frac{K}{2}\,(E^{\mathrm{vol}})^2 + \frac{\mu}{2}\,(E^{\mathrm{dev}})^2 = \frac{\lambda}{2}\,[\boldsymbol{E} : \boldsymbol{I}]^2 + \mu\,\boldsymbol{E} : \boldsymbol{E}$$

2. Volumetric strain (spherical part)

$$E^{\mathrm{vol}} := \boldsymbol{E} : \boldsymbol{I} = \sqrt{n_{\mathrm{dim}}\,\mathrm{sph}_{\boldsymbol{I}}[\boldsymbol{E}] : \mathrm{sph}_{\boldsymbol{I}}[\boldsymbol{E}]} \qquad \lambda := K - \frac{2\,\mu}{n_{\mathrm{dim}}}$$

3. Deviatoric strain

$$E^{\mathrm{dev}} := 2\sqrt{\frac{1}{2}\,\mathrm{dev}_{\boldsymbol{I}}[\boldsymbol{E}] : \mathrm{dev}_{\boldsymbol{I}}[\boldsymbol{E}]} = \sqrt{2\,\mathrm{dev}_{\boldsymbol{I}}[\boldsymbol{E}] : \mathrm{dev}_{\boldsymbol{I}}[\boldsymbol{E}]}$$

St. Venant-Kirchhoff fiber strain energy

1. Spherical and deviatoric split

$$\Psi^{\mathrm{ela}}_{\mathrm{F_{SVK}}} := \Psi^{\mathrm{str}}_{F} + \Psi^{\mathrm{dis}}_{F} = \frac{K_F}{2}\,(E^{\mathrm{str}}_F)^2 + \frac{\mu_F}{2}\,(E^{\mathrm{dis}}_F)^2 = \frac{\lambda_F}{2}\,[\boldsymbol{E}_F : \boldsymbol{I}]^2 + \mu_F\,\boldsymbol{E}_F : \boldsymbol{E}_F$$

2. Fiber strain (spherical part)

$$E^{\mathrm{str}}_F := \boldsymbol{E} : \boldsymbol{A}_0 = \sqrt{\mathrm{sph}_{\boldsymbol{A}_0}[\boldsymbol{E}_F] : \mathrm{sph}_{\boldsymbol{A}_0}[\boldsymbol{E}_F]} \qquad \boldsymbol{E}_F := \boldsymbol{A}_0\boldsymbol{E} \qquad \lambda_F := K_F - 2\,\mu_F$$

3. Deviatoric fiber-matrix strain

$$E^{\mathrm{dis}}_F := \sqrt{2\,\mathrm{dev}_{\boldsymbol{A}_0}[\boldsymbol{E}_F] : \mathrm{dev}_{\boldsymbol{A}_0}[\boldsymbol{E}_F]} = \sqrt{2\,[J^{\boldsymbol{E}}_5 - (E^{\mathrm{str}}_F)^2]} \qquad J^{\boldsymbol{E}}_5 := \boldsymbol{a}_0 \cdot \boldsymbol{E}\boldsymbol{E} \cdot \boldsymbol{a}_0$$

Concept of St. Venant-Kirchh. strain energy (II)

St. Venant-Kirchhoff fiber curvature-twist strain energy

1. Spherical and deviatoric split

$$\Psi^{\text{ctw}}_{\text{SVK}} := \Psi^{\text{twi}}_{F} + \Psi^{\text{cur}}_{F} = \frac{K_{\text{twi}}}{2}\,(T_F)^2 + \frac{\mu_{\text{cur}}}{2}\,(b_F)^2 = \frac{\lambda_{\text{ctw}}}{2}\,[\boldsymbol{K}_F : \boldsymbol{I}]^2 + \mu_{\text{cur}}\,\boldsymbol{K}_F : \boldsymbol{K}_F$$

2. Twisting strain (spherical part)

$$T_F := \boldsymbol{K}_F : \boldsymbol{I} = \sqrt{\text{sph}_{\boldsymbol{A}_0}[\boldsymbol{K}_F] : \text{sph}_{\boldsymbol{A}_0}[\boldsymbol{K}_F]} \qquad \lambda_{\text{ctw}} := K_{\text{twi}} - 2\,\mu_{\text{cur}}$$

3. Bending strain (deviatoric part)

$$b_F := \sqrt{2\,\text{dev}_{\boldsymbol{A}_0}[\boldsymbol{K}_F] : \text{dev}_{\boldsymbol{A}_0}[\boldsymbol{K}_F]}$$

Physical meaning of the (matrix) strains

(cf. Marsden & Hughes [1983])

1. First Piola-Kirchhoff stress tensor

$$\boldsymbol{P}^{\text{ela}}_{\text{SVK}}(\boldsymbol{F}) := \boldsymbol{F}\,\boldsymbol{S}^{\text{ela}}_{\text{SVK}}(\boldsymbol{E}(\boldsymbol{F})) = \boldsymbol{F}\,\frac{\partial \Psi^{\text{ela}}_{\text{SVK}}(\boldsymbol{E}(\boldsymbol{F}))}{\partial \boldsymbol{E}} = \boldsymbol{F}\,\{\lambda\,[\boldsymbol{E}(\boldsymbol{F}) : \boldsymbol{I}]\,\boldsymbol{I} + 2\,\mu\,\boldsymbol{E}(\boldsymbol{F})\}$$

2. Linearization

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[\boldsymbol{P}^{\text{ela}}_{\text{SVK}}(\boldsymbol{I} + s\,\Delta\boldsymbol{H})\right]_{s=0} = \lambda\,[\boldsymbol{\varepsilon} : \boldsymbol{I}]\,\boldsymbol{I} + 2\,\mu\,\boldsymbol{\varepsilon} =: \boldsymbol{\sigma}^{\text{ela}}_{\text{H}} \qquad \boldsymbol{\varepsilon} := \frac{1}{2}\,[\Delta\boldsymbol{H} + \Delta\boldsymbol{H}^t]$$

3. Hooke's (matrix) strain energy function

$$\Psi^{\text{ela}}_{\text{H}} = \frac{\lambda}{2}\,[\boldsymbol{\varepsilon} : \boldsymbol{I}]^2 + \mu\,\boldsymbol{\varepsilon} : \boldsymbol{\varepsilon} = \frac{K}{2}\,(e^{\text{vol}})^2 + \frac{\mu}{2}\,(e^{\text{dev}})^2 \qquad \boldsymbol{\varepsilon} : \boldsymbol{I} = \text{div}[\boldsymbol{u}] = \frac{\Delta V}{V_0}$$

Concept of St. Venant-Kirchh. strain energy (III)

Physical meaning of the fiber strains T_F and b_F

1. Piola-Kirchhoff couple stress tensor

$$\boldsymbol{P}_K^{\mathrm{SVK}}(\boldsymbol{F},\boldsymbol{G}) := \boldsymbol{F}\,\boldsymbol{S}_K^{\mathrm{SVK}}(\boldsymbol{F},\boldsymbol{G}) = \boldsymbol{F}\,\frac{\partial \Psi_{\mathrm{SVK}}^{\mathrm{ctw}}(\boldsymbol{K}(\boldsymbol{F},\boldsymbol{G}))}{\partial \boldsymbol{K}} = \boldsymbol{F}\left\{\lambda_{\mathrm{ctw}}\,[\boldsymbol{K}_F(\boldsymbol{F},\boldsymbol{G}):\boldsymbol{I}]\,\boldsymbol{A}_0 + 2\,\mu_{\mathrm{cur}}\,\boldsymbol{K}_F(\boldsymbol{F},\boldsymbol{G})\right\}$$

2. Linearization

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[\boldsymbol{P}_K^{\mathrm{SVK}}(\boldsymbol{I}+s\,\Delta\boldsymbol{H},\boldsymbol{O}+s\,\Delta\boldsymbol{G})\right]_{s=0} = \lambda_{\mathrm{ctw}}\,[\boldsymbol{\kappa}_F:\boldsymbol{I}]\,\boldsymbol{A}_0 + 2\,\mu_{\mathrm{cur}}\,\boldsymbol{\kappa}_F =: \boldsymbol{\mu}_K^{\mathrm{H}}$$

$$\frac{\mathrm{d}}{\mathrm{d}s}\left[\boldsymbol{K}_F(\boldsymbol{I}+s\,\Delta\boldsymbol{H},\boldsymbol{O}+s\,\Delta\boldsymbol{G})\right]_{s=0} = \boldsymbol{A}_0\,\Delta\boldsymbol{G} \qquad \boldsymbol{\kappa}_F := \boldsymbol{A}_0\,\Delta\boldsymbol{G}$$

3. Hooke's fiber curvature-twist strain energy function

$$\Psi_K^H := \frac{\lambda_{\mathrm{ctw}}}{2}\,[\boldsymbol{\kappa}_F:\boldsymbol{I}]^2 + \mu_{\mathrm{cur}}\,\boldsymbol{\kappa}_F:\boldsymbol{\kappa}_F \qquad \boldsymbol{\mu}_K^{\mathrm{H}} := \frac{\partial \Psi_K^H}{\partial \boldsymbol{\kappa}_F}\,\frac{\partial \boldsymbol{\kappa}_F}{\partial \Delta\boldsymbol{G}}$$

$$= \frac{K_{\mathrm{twi}}}{2}\,[\boldsymbol{\kappa}_F:\boldsymbol{I}]^2 + \mu_{\mathrm{cur}}\left[\boldsymbol{\kappa}_F:\boldsymbol{\kappa}_F - (\boldsymbol{\kappa}_F:\boldsymbol{I})^2\right]$$

$$\Psi_K^H = \frac{K_{\mathrm{twi}}}{2}\,[\boldsymbol{a}_0\cdot\Delta\boldsymbol{G}\cdot\boldsymbol{a}_0]^2 + \mu_{\mathrm{cur}}\left[\boldsymbol{a}_0\cdot\Delta\boldsymbol{G}\,\Delta\boldsymbol{G}^t\cdot\boldsymbol{a}_0 - (\boldsymbol{a}_0\cdot\Delta\boldsymbol{G}\cdot\boldsymbol{a}_0)^2\right]$$

4. Twisting and bending modes in direction of $\boldsymbol{a}_0 := \boldsymbol{e}_i,\ i = 1,2,3$

$$\Psi_K^H\big|_{\boldsymbol{a}_0:=\boldsymbol{e}_i} = \frac{K_{\mathrm{twi}}}{2}\left[\frac{\partial\Delta\alpha_i}{\partial X_i}\right]^2 + \mu_{\mathrm{cur}}\sum_{\substack{j=1\\ i\neq j}}^{n_{\mathrm{dim}}}\left[\frac{\partial\Delta\alpha_i}{\partial X_j}\right]^2 \qquad \Delta\boldsymbol{G} := \frac{\partial\Delta\alpha_i}{\partial X_j}\,\boldsymbol{e}_i\otimes\boldsymbol{e}_j$$

Anisotropic Cauchy continuum

Decoupled thermo-viscoelastic free energy for matrix and fibers

1. Hyperelastic strain energy

$$\Psi^{\text{ela}}(\tilde{\boldsymbol{C}}, \tilde{C}_V, \tilde{C}_F; \boldsymbol{A}_0) = \hat{\Psi}^{\text{iso}}_M(I_1^{\tilde{C}}, I_2^{\tilde{C}}, I_3^{\tilde{C}}, I_4) + \hat{\Psi}^{\text{vol}}_M(\tilde{C}_V) + \hat{\Psi}^{\text{ela}}_F(\tilde{C}_F)$$

2. Thermoelastic free energy

$$\Psi^{\text{the}}(\tilde{C}_V, \tilde{C}_F, \Theta) = \hat{\Psi}^{\text{cap}}(\Theta) - 2\sqrt{\tilde{C}_V^{\,2-n_{\text{dim}}}}\,\beta_M\,[\Theta - \Theta_\infty]\,\mathrm{D}\hat{\Psi}^{\text{vol}}_M(\tilde{C}_V) - 2\sqrt{\tilde{C}_F^{2-1}}\,\beta_F\,[\Theta - \Theta_\infty]\,\mathrm{D}\hat{\Psi}^{\text{ela}}_F(\tilde{C}_F)$$

3. Viscoelastic free energy

$$\Psi^{\text{vis}}_M(\tilde{\boldsymbol{C}}, \boldsymbol{C}_v) = \hat{\Psi}^{\text{ela}}_M(I_1^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}, I_2^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}, I_3^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}) \qquad \Psi^{\text{vis}}_F(\tilde{C}_F, \tilde{C}_F^v) = \Psi^{\text{ela}}_F(\tilde{C}_F[\tilde{C}_F^v]^{-1})$$

Viscous evolution equations

(cf. Reese [2001], Nedjar [2007], Krüger et. al. [2011])

1. Viscous matrix evolution equation and non-equilibrium stress tensor

$$\boxed{\boldsymbol{Y} = \boldsymbol{\Sigma}_v} \qquad \boldsymbol{Y} := -\frac{\partial \Psi^{\text{vis}}_M}{\partial \boldsymbol{C}_v} \qquad D^{\text{int}}_M := \dot{\boldsymbol{C}}_v : \boldsymbol{\Sigma}_v = \dot{\boldsymbol{C}}_v : \mathbb{V}(\boldsymbol{C}_v) : \dot{\boldsymbol{C}}_v \geq 0$$

$$\mathbb{V}(\boldsymbol{C}_v) = \frac{1}{4}\left[V_{\text{vol}} - \frac{V_{\text{dev}}}{n_{\text{dim}}}\right]\boldsymbol{C}_v^{-1} \otimes \boldsymbol{C}_v^{-1} + \frac{V_{\text{dev}}}{4}\,\boldsymbol{C}_v^{-1} \odot \boldsymbol{C}_v^{-1}$$

2. Viscous fiber evolution equation and non-equilibrium stress

$$\boxed{Y_F = \Sigma_F^v} \qquad Y_F := -\frac{\partial \Psi^{\text{vis}}_F}{\partial C_F^v} \qquad D^{\text{int}}_F := \dot{C}_F^v\,\Sigma_F^v = \dot{C}_F^v\,\frac{V_F}{4\,(C_F^v)^2}\,\dot{C}_F^v \geq 0$$

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Principle of virtual power (I)

Total energy balance law

$$\dot{\mathcal{H}} = \dot{\mathcal{T}}^{\mathrm{tra}} + \dot{\mathcal{T}}^{\mathrm{rot}} + \dot{\Pi}^{\mathrm{ext}} + \dot{\Pi}^{\mathrm{int}}$$

$$\dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\boldsymbol{C}}_v, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F, \dot{\tilde{C}}_F^v, \dot{\Theta}, \dot{\eta}}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \tilde{\boldsymbol{P}}, \tilde{\boldsymbol{S}}, \tilde{\boldsymbol{P}}_K, \tilde{\boldsymbol{S}}_K, \tilde{\boldsymbol{\tau}}^t_{\mathrm{skw}}, \tilde{S}_V, \tilde{S}_F, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Kinetic power functionals (motivated by Altenbach et al. [2003], Askes & Aifantis [2011])

$$\dot{\mathcal{T}}^{\mathrm{tra}}(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) := \int_{\mathscr{B}_0} [\rho_0 \boldsymbol{I}\,\boldsymbol{v} - \boldsymbol{p}] \cdot \dot{\boldsymbol{v}}\,\mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{p}} \cdot [\boldsymbol{v} - \dot{\boldsymbol{\varphi}}]\mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{p} \cdot \ddot{\boldsymbol{\varphi}}\,\mathrm{d}V$$

$$\dot{\mathcal{T}}^{\mathrm{rot}}(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}) := \int_{\mathscr{B}_0} \left[\rho_0 \left[(l_F^2 - l_0^2)\,\boldsymbol{A}_0 + l_0^2 \boldsymbol{I}\right] \boldsymbol{\omega} - \boldsymbol{\pi}\right] \cdot \dot{\boldsymbol{\omega}}\,\mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{\pi}} \cdot [\boldsymbol{\omega} - \dot{\boldsymbol{\alpha}}]\mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\pi} \cdot \ddot{\boldsymbol{\alpha}}\,\mathrm{d}V$$

External power functional

$$D^{\mathrm{cdu}} = -\mathrm{Grad}[\ln \Theta] \cdot \boldsymbol{Q}$$

$$\begin{aligned}
\dot{\Pi}^{\mathrm{ext}} := & -\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \cdot \dot{\boldsymbol{\varphi}}\,\mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}}\,\mathrm{d}A + \int_{\partial_Q \mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta} \bar{Q}\,\mathrm{d}A \\
& + \int_{\mathscr{B}_0} \frac{1}{\Theta} \mathrm{Grad}[\tilde{\Theta}] \cdot \boldsymbol{Q}\,\mathrm{d}V + \int_{\mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta} \left(D^{\mathrm{cdu}} + D^{\mathrm{int}}\right) \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\Sigma}_v : \dot{\boldsymbol{C}}_v\,\mathrm{d}V \\
& + \int_{\partial_\Theta \mathscr{B}_0} \lambda \left[\tilde{\Theta} - \Theta_\infty\right] \mathrm{d}A - \int_{\partial_{\dot{\Theta}} \mathscr{B}_0} h \left[\dot{\Theta} - \dot{\bar{\Theta}}\right] \mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \cdot \left[\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}\right] \mathrm{d}A \\
& - \int_{\partial_\alpha \mathscr{B}_0} \boldsymbol{Z} \cdot \left[\dot{\boldsymbol{\alpha}} - \dot{\bar{\boldsymbol{\alpha}}}\right] \mathrm{d}A - \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \left[\boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\mathrm{skw}}\right] \mathrm{d}A - \int_{\partial_W \mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\alpha}}\,\mathrm{d}A \\
& + \int_{\mathscr{B}_0} \frac{1}{2} \bar{\boldsymbol{S}} : \dot{\tilde{\boldsymbol{C}}}\,\mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2} \bar{S}_V\,\dot{\tilde{C}}_V\,\mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2} \bar{S}_F\,\dot{\tilde{C}}_F\,\mathrm{d}V + \int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\,\mathrm{d}V \\
& + \int_{\mathscr{B}_0} \dot{\tilde{C}}_F^v\,\Sigma_F^v\,\mathrm{d}V + \int_{\mathscr{B}_0} \bar{M}_F^v \left[L_F(\dot{\tilde{C}}_F) - L_F(\dot{\tilde{C}}_F^v)\right] \mathrm{d}V \quad \text{with} \quad L_F(\dot{\epsilon}) = \frac{\dot{\overline{\ln(\epsilon)}}}{2}
\end{aligned}$$

Principle of virtual power (II)

Internal power functional

(motivated by Steinmann & Stein [1997])

$$\dot{\Pi}^{\mathrm{int}} := \int_{\mathscr{B}_0} \left\{ \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{\boldsymbol{C}}} + S_V\,\boldsymbol{A}^{\mathrm{vol}} + S_F\,\boldsymbol{A}_0 - \boldsymbol{S} \right] : \frac{1}{2}\dot{\tilde{\boldsymbol{C}}} + \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{C}_V} - S_V \right] \frac{\dot{\tilde{C}}_V}{2} + \left[2\,\frac{\partial \hat{\Psi}_F}{\partial \tilde{C}_F} - S_F \right] \frac{\dot{\tilde{C}}_F}{2} \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ [\Theta - \tilde{\Theta}]\,\dot{\eta} + \left[\frac{\partial \Psi}{\partial \Theta} + \eta \right] \dot{\Theta} + \frac{\partial \hat{\Psi}_M}{\partial \boldsymbol{C}_v} : \dot{\boldsymbol{C}}_v + \frac{\partial \Psi_F}{\partial \tilde{C}_F^v}\,\dot{\tilde{C}}_F^v + \left[\tilde{\boldsymbol{F}}\boldsymbol{S} + \tilde{\boldsymbol{G}}(\boldsymbol{S}_K)^t - \boldsymbol{P} \right] : \dot{\tilde{\boldsymbol{F}}} + \boldsymbol{P} : \mathrm{Grad}\,[\dot{\boldsymbol{\varphi}}] \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ \boldsymbol{P}_K : \mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] + \boldsymbol{\tau}^t_{\mathrm{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] + \left[\tilde{\boldsymbol{F}}\boldsymbol{S}_K - \boldsymbol{P}_K \right] : \dot{\tilde{\boldsymbol{G}}} + \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} - \boldsymbol{S}_K \right] : \dot{\tilde{\boldsymbol{K}}} \right\} \mathrm{d}V$$

Principle of virtual power

$$\delta_* \dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\boldsymbol{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \boldsymbol{P}, \boldsymbol{P}_K, \boldsymbol{S}, S_V, S_F, \boldsymbol{S}_K, \boldsymbol{\tau}^t_{\mathrm{skw}}, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Rotational weak forms

(here only the volume weak forms)

$$\int_{\mathscr{B}_0} \delta_* \boldsymbol{\tau}^t_{\mathrm{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] \mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \delta_* \boldsymbol{\tau}^t_{\mathrm{skw}}\,\mathrm{d}A \qquad \int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{K}}} : \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} + \bar{\boldsymbol{S}}_K - \boldsymbol{S}_K \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{G}}} : \left[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}}_K - \boldsymbol{P}_K \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_* \boldsymbol{P}_K : \left[\mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] - \dot{\tilde{\boldsymbol{G}}} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\pi}} \cdot [\dot{\boldsymbol{\alpha}} - \boldsymbol{\omega}]\,\mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\omega}} \cdot \left[\rho_0 \left[(l_F^2 - l_0^2)\,\boldsymbol{A}_0 + l_0^2 \boldsymbol{I} \right] \boldsymbol{\omega} - \boldsymbol{\pi} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_* \boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} + \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}} - \dot{\tilde{\boldsymbol{K}}} \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \left[\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\mathrm{skw}} \right] \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{P}_K : \mathrm{Grad}\,[\delta_* \dot{\boldsymbol{\alpha}}]\,\mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A$$

Potential energy balance and algorithmic stresses

Balance law of potential energy Π^{int}

$$\Pi_{n+1}^{\text{int}} - \Pi_n^{\text{int}} = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left[\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \dot{\hat{\Psi}}_K\right] \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left[\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} : \dot{\tilde{\boldsymbol{K}}}\right] \mathrm{d}V\,\mathrm{d}t$$

Setting $\delta_* \dot{\tilde{\boldsymbol{K}}} = \dot{\tilde{\boldsymbol{K}}}$ and $\delta_* \boldsymbol{S}_K = \boldsymbol{S}_K$, leading to

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \dot{\tilde{\boldsymbol{K}}} : \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} + \bar{\boldsymbol{S}}_K - \boldsymbol{S}_K\right] \mathrm{d}V\,\mathrm{d}t = 0 \quad \text{and} \quad \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{K}}} - \dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} - \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}}\right] \mathrm{d}V\,\mathrm{d}t = 0$$

we arrive at

$$\Pi_{n+1}^{\text{int}} - \Pi_n^{\text{int}} - \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left\{\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \boldsymbol{S}_K : \dot{\tilde{\boldsymbol{K}}}\right\} \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\,\mathrm{d}V\,\mathrm{d}t$$

$$\Pi_{n+1}^{\text{int}} - \Pi_n^{\text{int}} - \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left\{\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} + \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}}\right]\right\} \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\,\mathrm{d}V\,\mathrm{d}t$$

Algorithmic curvature-twist stress $\bar{\boldsymbol{S}}_K$ (cp. Simo & Tarnow [1992], Betsch & Steinmann [2002])

Local algorithmic constraint:

$$\hat{\Psi}_{K_{n+1}} - \hat{\Psi}_{K_n} - \int_0^1 \boldsymbol{S}_K : \frac{\partial \tilde{\boldsymbol{K}}}{\partial \alpha}\,\mathrm{d}\alpha = \int_0^1 \bar{\boldsymbol{S}}_K : \frac{\partial \tilde{\boldsymbol{K}}}{\partial \alpha}\,\mathrm{d}\alpha \qquad \alpha(t) := \frac{t - t_n}{t_{n+1} - t_n}$$

$\bar{\boldsymbol{S}}_K$ vanishes with

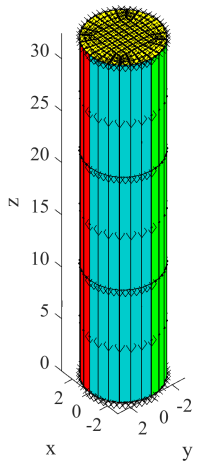
$$\tilde{\boldsymbol{K}} := \sum_{I=1}^{k+1} M_I(\alpha)\,\tilde{\boldsymbol{K}}_I \qquad M_I(\alpha) := \prod_{\substack{J=1 \\ J\neq I}}^{k+1} \frac{\alpha - \alpha_J}{\alpha_I - \alpha_J} \qquad I_{\text{Gauss}}\{f\} = \sum_{l=1}^{k} f(\xi_l)\,w_l$$

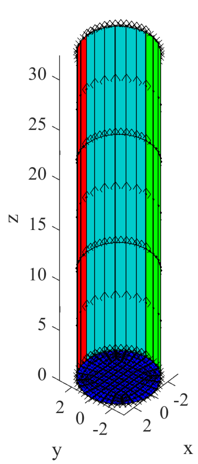
for the Saint Venant-Kirchhoff curvature-twist model

$$\Psi^{\text{cur}}(\tilde{\boldsymbol{K}}; \boldsymbol{A}_0) = \frac{\mu_{\text{twi}}}{2}\left[\boldsymbol{A}_0 \tilde{\boldsymbol{K}} : \boldsymbol{I}\right]^2 + \mu_{\text{ben}}\,\boldsymbol{A}_0 \tilde{\boldsymbol{K}} : \boldsymbol{A}_0 \tilde{\boldsymbol{K}}$$

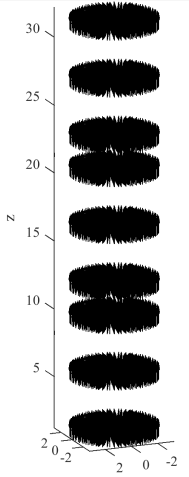
Dynamic torsion of Taylor's composite bar
(Boundary and initial conditions; 121-em with H20-mixed-Bbar)

Boundary conditions





Fiber directions



Dirichlet and Neumann boundary

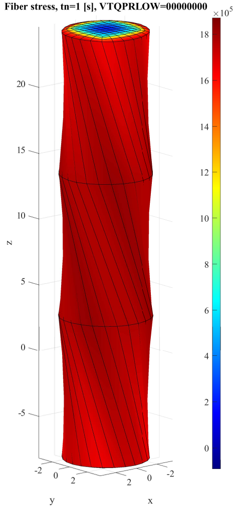
Yellow/blue patches: Torque load $W_z^A = \mp \hat{W}_z \, [\hat{f}_L \, \sin \omega_L t]^2$

Initial condition

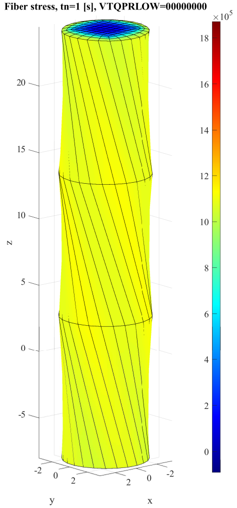
$\boldsymbol{v}_0^A = -8 \, \boldsymbol{e}_z$

Dynamic torsion of Taylor's composite bar
(Current configurations of different torsional length scales)

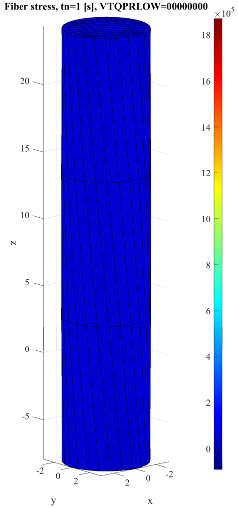
Fiber (tension) stress at $t_n = 1.0\,\mathrm{s}$ $(\mu_K = 25 \cdot 10^6,\ l_b^2 = 0.02)$



$l_t^2 = 0.4$



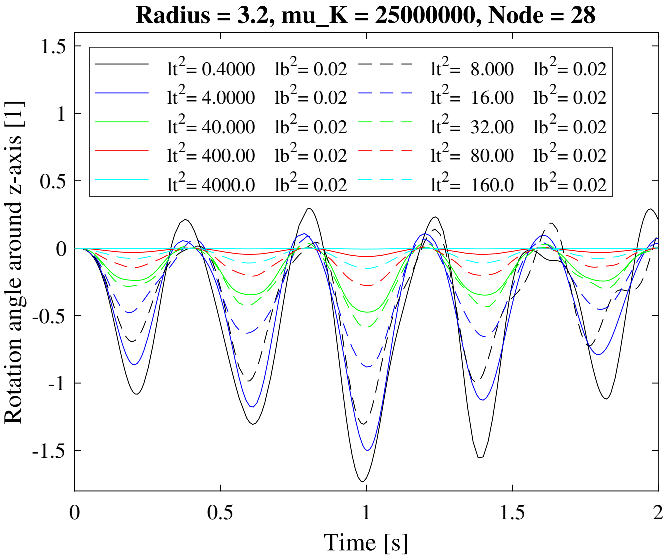
$l_t^2 = 4.0$



$l_t^2 = 40$

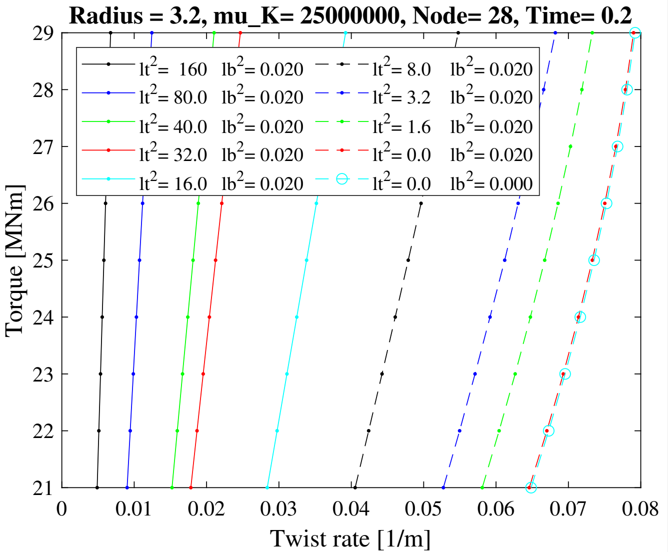
Dynamic torsion of Taylor's composite bar
(Time evolutions of different torsional length scales)

Rotation angle around bar axis versus time



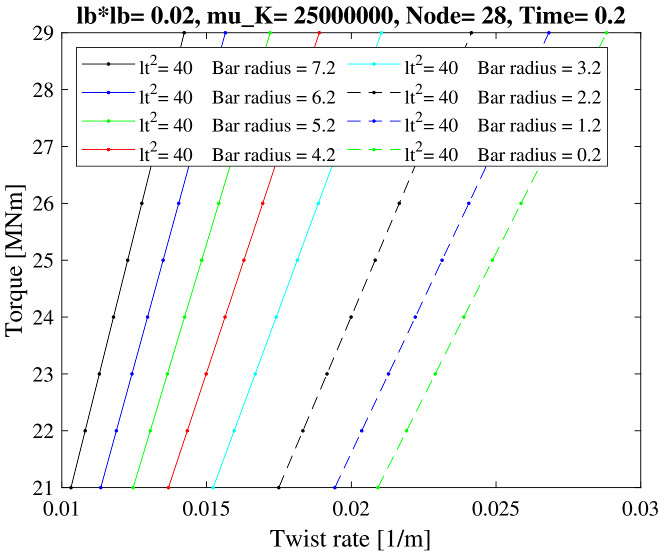
Dynamic torsion of Taylor's composite bar
(Load curves with different torsional length scales)

Torque $\hat{W}_z$ versus twist rate T_F



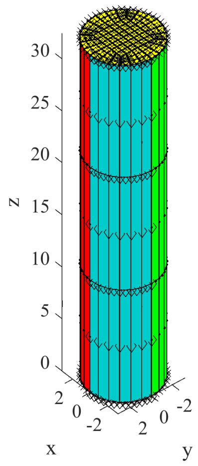
Dynamic torsion of Taylor's composite bar (Load curves with different bar diameters)

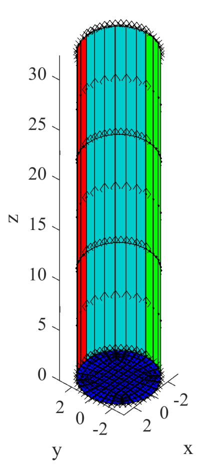
Torque $\hat{W}_z$ versus twist rate T_F



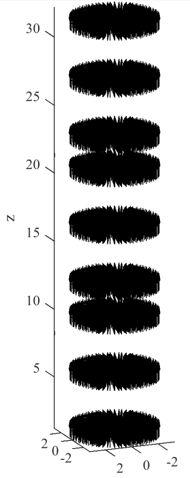
Dynamic bending of Taylor's composite bar
(Boundary and initial conditions; 121-em with H20-mixed-Bbar)

Boundary conditions





Fiber directions



Dirichlet and Neumann boundary

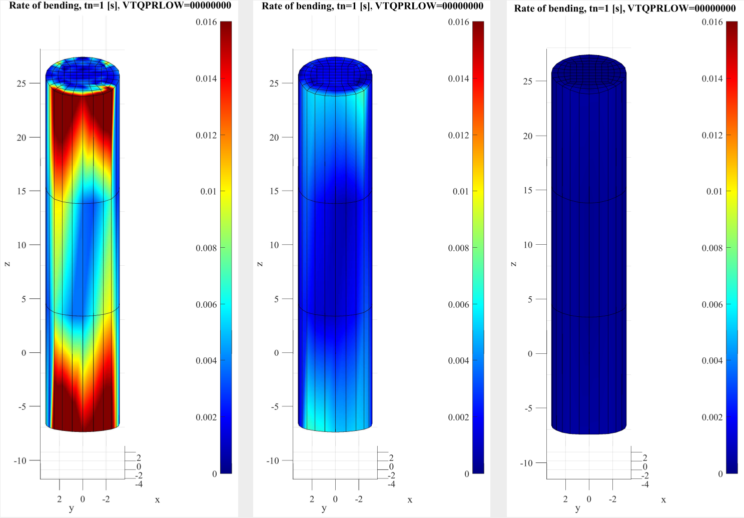
Yellow/blue patches: Torque load $W_y^A = \mp \hat{W}_y\, [\hat{f}_L\, \sin \omega_L t]^2$

Initial condition

$\boldsymbol{v}_0^A = -8\, \boldsymbol{e}_z$

Dynamic torsion of Taylor's composite bar
(Current configurations of different bending length scales)

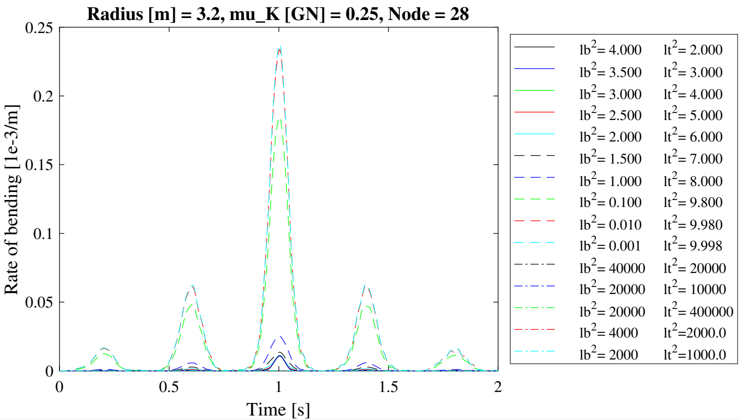
Rate of bending B_F at $t_n = 1.0\,\mathrm{s}$ $\quad(\mu_K = 250 \cdot 10^6)$



$l_b^2 = 0.01, l_k^2 = 10$ $\quad$ $l_b^2 = 1, l_k^2 = 10$ $\quad$ $l_b^2 = 2\cdot 10^3,\ l_k^2 = 5\cdot 10^3$

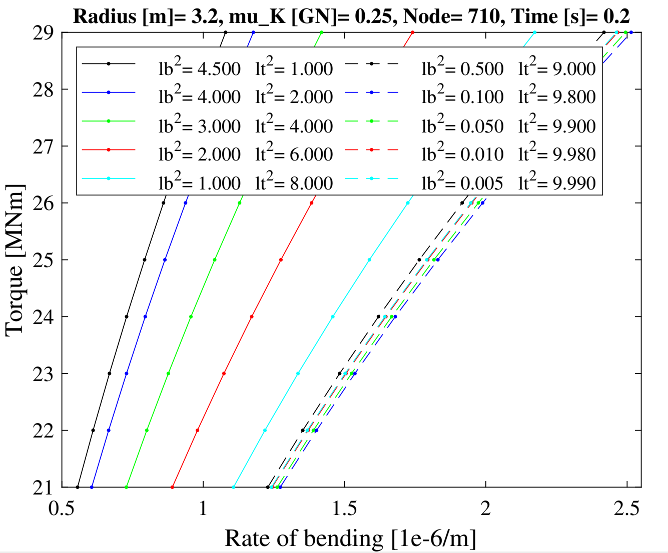
Dynamic bending of Taylor's composite bar
(Time evolutions of different bending length scales)

Rate of bending versus time



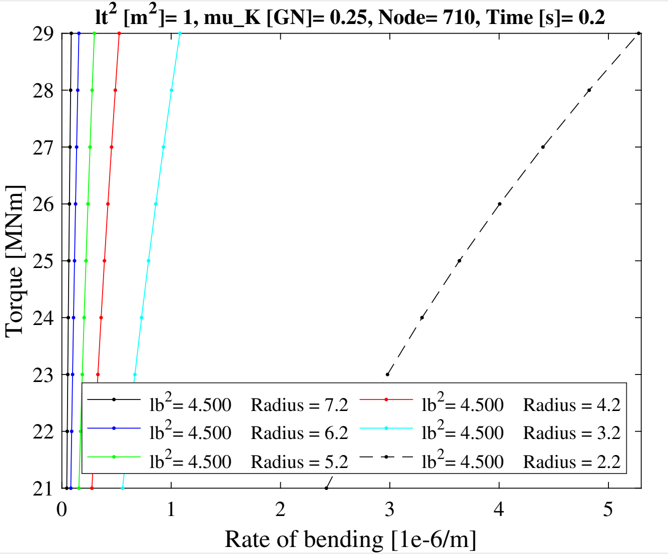
Dynamic bending of Taylor's composite bar (Load curves of different bending length scales)

Torque $\hat{W}_y$ versus bending rate B_F



Dynamic bending of Taylor's composite bar
(Load curves with different bar diameters)

Torque $\hat{W}_y$ versus bending rate B_F



Michael Groß, Julian Dietzsch, Iniyan Kalaimani

Summary

1. Motivation:
   - Precise simulations of fiber roving composites with
   - fiber twisting and bending stiffness on the micro scale
2. Goals:
   - Energy-momentum schemes for constrained micropolar continua
   - derived by a principle of virtual power for continuum mechanics
3. Strategy:
   - Introduction of independent fields for the continuum rotation
   - Discretization by using a new mixed finite element formulation
4. Important results:
   - Torsional and flexural rigidity of fibers can be prescribed
   - even separately, and increases the total material stiffness
5. Next design steps:
   - a nonlinear curvature-twist strain energy function
   - an algorithmic curvature-twist stress tensor