An energy-momentum scheme for extended continuum models with rotational degrees of freedom

Michael Groß, Julian Dietzsch, Iniyan Kalaimani

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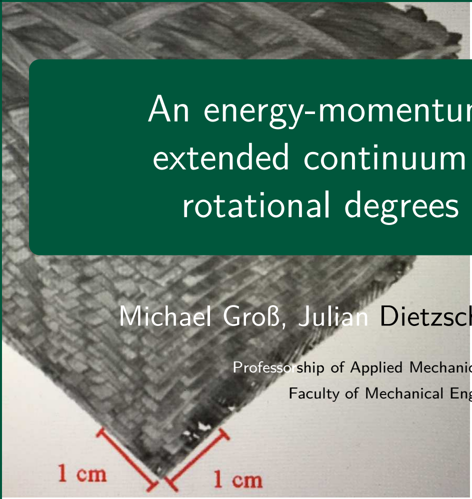

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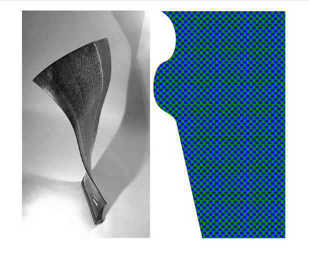
Motivation: Fiber roving composites in dynamics

Background: increasing application of fiber roving composites

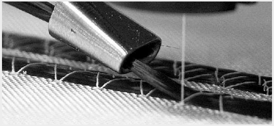
1. Turbine blades
2. Rotor shafts
3. Pump rotors

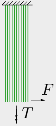
Macroscopic scale: turbine blade preform



De Luycker, Morestin, Boisse, Marsal [2009]

Micro scale: Roving with curvature stiffness





Kai Uhlig [2017] (IPF TU Dresden), recommended by IST TU Chemnitz

The need for extended continuum models

1. Higher gradient theory Asmanoglo, Menzel [2017]
2. Micropolar model Steinmann, Stein [1997]

Goals: Using mixed finite element methods for implementing extended continua

1. We introduce an independent axial rotation vector and no rotation tensor
2. We derive weak forms and balance laws using a mixed principle of virtual power
3. We design an energy-momentum time integration in the discrete setting

The new extended continuum formulation

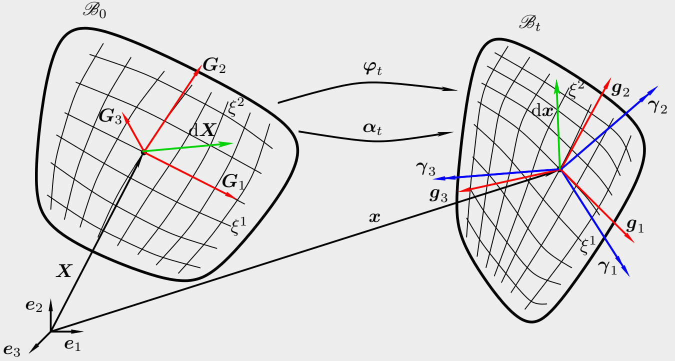
Local rotational degrees of freedom and rotation mapping

$$\frac{\mathrm{d}\boldsymbol{\gamma}(\boldsymbol{x})}{\mathrm{d}t} = -\frac{1}{2}\boldsymbol{\epsilon} : \frac{\partial \boldsymbol{v}(\boldsymbol{x})}{\partial \boldsymbol{x}} \qquad \frac{\partial \boldsymbol{\alpha}_t(\boldsymbol{X})}{\partial t} := \frac{\mathrm{d}\boldsymbol{\gamma}(\boldsymbol{x})}{\mathrm{d}t} \circ \boldsymbol{\varphi}_t(\boldsymbol{X})$$

Definition of rotational covariant basis vectors

$$\mathrm{d}\boldsymbol{\gamma} = \mathrm{d}\alpha^i(\xi^1,\xi^2,\xi^3)\,\boldsymbol{g}_i \equiv \alpha^i_{,j}\,\mathrm{d}\xi^j\,\boldsymbol{g}_i =: \mathrm{d}\xi^j\,\boldsymbol{\gamma}_j \qquad \boldsymbol{\gamma}_j := \alpha^k_{,j}\,\boldsymbol{g}_k$$

Covariant formulation of rotational degrees of freedom (cp. Wriggers [2001])



Kinematic analogy: local displacement/rotation

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Covariant basis vectors for continuum displacement/rotation

$$\boldsymbol{G}_i := \frac{\partial \boldsymbol{X}(\xi^1,\xi^2,\xi^3)}{\partial \xi^i} \qquad \boldsymbol{g}_i := \frac{\partial \boldsymbol{\varphi}_t(\boldsymbol{X}(\xi^1,\xi^2,\xi^3))}{\partial \xi^i} \qquad \boldsymbol{\gamma}_i := \frac{\partial \boldsymbol{\alpha}_t(\boldsymbol{X}(\xi^1,\xi^2,\xi^3))}{\partial \xi^i}$$

Translational/Rotational metric coefficients for stretch/curvature

$$g_{ij} := \boldsymbol{g}_i \cdot \boldsymbol{g}_j \qquad K_{ij} := \boldsymbol{g}_i \cdot \boldsymbol{\gamma}_j \equiv \boldsymbol{g}_i \cdot \alpha^k_{,j}\, \boldsymbol{g}_k \equiv g_{ik}\, \alpha^k_{,j}$$

Translational/Rotational metric tensors for stretch/curvature-twist

$$\boldsymbol{g} := \frac{\partial \boldsymbol{\varphi}_t}{\partial \boldsymbol{x}} = g_{ik}\, \boldsymbol{g}^k \otimes \boldsymbol{g}^i \qquad \boldsymbol{g}_\alpha := \frac{\partial \boldsymbol{\alpha}_t}{\partial \boldsymbol{x}} = K_{ki}\, \boldsymbol{g}^k \otimes \boldsymbol{g}^i$$

Deformation gradient and rotation gradient

$$\boldsymbol{F} := \frac{\partial \boldsymbol{\varphi}_t}{\partial \boldsymbol{X}} = \boldsymbol{g}_i \otimes \boldsymbol{G}^i \qquad \boldsymbol{G}_\alpha := \frac{\partial \boldsymbol{\alpha}_t}{\partial \boldsymbol{X}} = \boldsymbol{\gamma}_i \otimes \boldsymbol{G}^i$$

Material line/curvature-twist deformation

$$\mathrm{d}\boldsymbol{x} \cdot \boldsymbol{g} \cdot \mathrm{d}\boldsymbol{x} = \mathrm{d}\boldsymbol{X} \cdot \boldsymbol{C} \cdot \mathrm{d}\boldsymbol{X} \qquad \mathrm{d}\boldsymbol{x} \cdot \boldsymbol{g}_\alpha \cdot \mathrm{d}\boldsymbol{x} = \mathrm{d}\boldsymbol{X} \cdot \boldsymbol{K} \cdot \mathrm{d}\boldsymbol{X}$$

Right Cauchy-Green tensor and curvature-twist tensor

$$\boldsymbol{C} := \boldsymbol{F}^t \boldsymbol{g} \boldsymbol{F} = g_{ij}\, \boldsymbol{G}^i \otimes \boldsymbol{G}^j \qquad \boldsymbol{K} := \boldsymbol{F}^t \boldsymbol{g}_\alpha \boldsymbol{F} = K_{ij}\, \boldsymbol{G}^i \otimes \boldsymbol{G}^j$$

Kinematic analogy: fiber stretch/curvature-twist

Covariant fiber basis vectors

$$\boldsymbol{a}_0 = a^i\,\boldsymbol{G}_i \qquad \boldsymbol{a}_t := a^i\,\boldsymbol{g}_i \qquad \boldsymbol{a}_t \cdot \boldsymbol{a}_t = a^i\, g_{ij}\, a^j =: C_F$$

Fiber metric tensors (common assumption $\boldsymbol{a}_0 \cdot \boldsymbol{a}_0 = 1$)

$$\boldsymbol{a}_0 = \boldsymbol{A}_0\,\boldsymbol{a}_0 \qquad \boldsymbol{A}_0 = \boldsymbol{a}_0 \otimes \boldsymbol{a}_0 = a^i\,a^j\,\boldsymbol{G}_i \otimes \boldsymbol{G}_j$$

$$\boldsymbol{a}_t = \boldsymbol{g}_F\,\boldsymbol{a}_t \qquad \boldsymbol{g}_F = \boldsymbol{a}_t \otimes \boldsymbol{a}_t^\flat = \frac{a^i\,a^j}{C_F}\,\boldsymbol{g}_i \otimes \boldsymbol{g}_j$$

Fiber deformation gradient (cf. Klinkel, Sansour & Wagner [2005])

$$\boldsymbol{a}_t = \boldsymbol{F}_F\,\boldsymbol{a}_0 \qquad \boldsymbol{F}_F := \boldsymbol{a}_t \otimes \boldsymbol{a}_0 \qquad \boldsymbol{F}_F = \boldsymbol{F}\,\boldsymbol{A}_0$$

Fiber stretch/curvature-twist measurement

$$\boldsymbol{a}_t \cdot \boldsymbol{g} \cdot \mathrm{d}\boldsymbol{x} = \boldsymbol{a}_0 \cdot \boldsymbol{C}_A \cdot \mathrm{d}\boldsymbol{X} \qquad \boldsymbol{C}_A := \boldsymbol{F}_F^t\,\boldsymbol{g}\,\boldsymbol{F} \equiv \boldsymbol{A}_0\,\boldsymbol{C}$$

$$\boldsymbol{a}_t \cdot \boldsymbol{g}_\alpha \cdot \mathrm{d}\boldsymbol{x} = \boldsymbol{a}_0 \cdot \boldsymbol{K}_F \cdot \mathrm{d}\boldsymbol{X} \qquad \boldsymbol{K}_F := \boldsymbol{F}_F^t\,\boldsymbol{g}_\alpha\,\boldsymbol{F} \equiv \boldsymbol{A}_0\,\boldsymbol{K}$$

Fiber stretch/curvature-twist invariants in the strain energy

$$I_1(\boldsymbol{C}_A) := \boldsymbol{C}_A : \boldsymbol{G}^{-1} = C_F \qquad I_1(\boldsymbol{K}_F) := \boldsymbol{K}_F : \boldsymbol{G}^{-1} = T_F$$

$$J_2(\boldsymbol{C}_A) := \boldsymbol{C}_A : \boldsymbol{C}_A = J_5(\boldsymbol{C}, \boldsymbol{a}_0) \qquad J_2(\boldsymbol{K}_F) := \boldsymbol{K}_F : \boldsymbol{K}_F = \frac{B_F}{2}$$

Kinematic analogy: strain rate/stress tensors

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Material/Spatial translation/rotation velocity gradient

$$\dot{\boldsymbol{F}} = \boldsymbol{F}\,\boldsymbol{L} \qquad \dot{\boldsymbol{G}}_\alpha = \boldsymbol{G}_\alpha\,\boldsymbol{L}_\alpha \qquad \dot{\boldsymbol{F}} = \boldsymbol{l}\,\boldsymbol{F} \qquad \dot{\boldsymbol{G}}_\alpha = \boldsymbol{l}_\alpha\,\boldsymbol{G}_\alpha$$

Material deformation/curvature-twist rate tensor

$$\mathrm{d}\boldsymbol{X}\cdot\dot{\boldsymbol{C}}\cdot\mathrm{d}\boldsymbol{X} = \mathrm{d}\boldsymbol{X}\cdot 2\,\boldsymbol{D}\cdot\mathrm{d}\boldsymbol{X} \qquad 2\,\boldsymbol{D} := \boldsymbol{L}^t\,\boldsymbol{C} + \boldsymbol{C}\,\boldsymbol{L}$$

$$\mathrm{d}\boldsymbol{X}\cdot\dot{\boldsymbol{K}}\cdot\mathrm{d}\boldsymbol{X} = \mathrm{d}\boldsymbol{X}\cdot\boldsymbol{D}_\alpha\cdot\mathrm{d}\boldsymbol{X} \qquad \boldsymbol{D}_\alpha := \boldsymbol{L}^t\,\boldsymbol{K} + \boldsymbol{K}\,\boldsymbol{L}_\alpha$$

Spatial deformation/curvature-twist rate tensor

$$\dot{\overline{\mathrm{d}\boldsymbol{x}\cdot\boldsymbol{g}\cdot\mathrm{d}\boldsymbol{x}}} = \mathrm{d}\boldsymbol{x}\cdot 2\,\boldsymbol{d}\cdot\mathrm{d}\boldsymbol{x} \qquad 2\,\boldsymbol{d} := \boldsymbol{l}^t\boldsymbol{g} + \boldsymbol{g}\,\boldsymbol{l}$$

$$\dot{\overline{\mathrm{d}\boldsymbol{x}\cdot\mathrm{d}\boldsymbol{\gamma}}} = \mathrm{d}\boldsymbol{x}\cdot\boldsymbol{d}_\alpha\cdot\mathrm{d}\boldsymbol{x} \qquad \boldsymbol{d}_\alpha := \left(\boldsymbol{l}^t + \boldsymbol{l}_\alpha\right)\,\boldsymbol{g}_\alpha$$

Piola-Kirchhoff and couple/curvature-twist stress tensor

$$\boldsymbol{P} := \boldsymbol{F}\,\boldsymbol{S} \qquad \boldsymbol{S} := 2\,\frac{\partial\Psi_M}{\partial\boldsymbol{C}} \qquad \boldsymbol{P}_K := \boldsymbol{F}\,\boldsymbol{S}_K \qquad \boldsymbol{S}_K := \frac{\partial\Psi_K}{\partial\boldsymbol{K}}$$

Kirchhoff and Kirchhoff couple/curvature-twist stress tensor

$$\boldsymbol{\tau} := \boldsymbol{F}\,\boldsymbol{S}\,\boldsymbol{F}^t \equiv \boldsymbol{F}\,\boldsymbol{P}^t \qquad \boldsymbol{\tau}_K^t := \boldsymbol{F}\,\boldsymbol{S}_K\,\boldsymbol{G}_\alpha^t =: \boldsymbol{F}\,\boldsymbol{N}_K^t$$

$$\boldsymbol{\mu}_K := \boldsymbol{F}\,\boldsymbol{S}_K\,\boldsymbol{F}^t \equiv \boldsymbol{F}\,\boldsymbol{P}_K^t \qquad \boldsymbol{\tau}_K^t = \frac{\partial\Psi_K}{\partial K_{ij}}\,\boldsymbol{g}_i\otimes\boldsymbol{\gamma}_j$$

Kinematic analogy: algorithmic stress tensor

Gradient theorem with respect line/curvature-twist deformations

$$\mathcal{G}_M := \Psi_M(\boldsymbol{C}(t_{n+1})) - \Psi_M(\boldsymbol{C}(t_n)) - \int_{t_n}^{t_{n+1}} \dot{\Psi}_M(\boldsymbol{C}(t))\,\mathrm{d}t = 0$$

$$\mathcal{G}_K := \Psi_K(\boldsymbol{K}(t_{n+1})) - \Psi_K(\boldsymbol{K}(t_n)) - \int_{t_n}^{t_{n+1}} \dot{\Psi}_K(\boldsymbol{K}(t))\,\mathrm{d}t = 0$$

Lagrange functional subject to a constraint

$$\mathcal{F}_M(\bar{\boldsymbol{S}}_M, \lambda_M) := \lambda_M\,\mathcal{G}_M(\bar{\boldsymbol{S}}_M) + \int_0^1 F_M(\bar{\boldsymbol{S}}_M(\alpha))\,\mathrm{d}\alpha$$

$$\mathcal{F}_K(\bar{\boldsymbol{S}}_K, \lambda_K) := \lambda_K\,\mathcal{G}_K(\bar{\boldsymbol{S}}_K) + \int_0^1 F_K(\bar{\boldsymbol{S}}_K(\alpha))\,\mathrm{d}\alpha$$

Minimization function

$$F_M(\bar{\boldsymbol{S}}) := \frac{1}{2}\,\boldsymbol{g}\,\bar{\boldsymbol{\tau}} : \bar{\boldsymbol{\tau}}\,\boldsymbol{g} = \frac{1}{2}\,\boldsymbol{C}\,\bar{\boldsymbol{S}} : \bar{\boldsymbol{S}}\,\boldsymbol{C} \qquad F_K(\bar{\boldsymbol{S}}_K) := \frac{1}{2}\,\boldsymbol{g}\,\bar{\boldsymbol{\mu}} : \bar{\boldsymbol{\mu}}\,\boldsymbol{g} = \frac{1}{2}\,\boldsymbol{C}\,\bar{\boldsymbol{S}}_K : \bar{\boldsymbol{S}}_K\,\boldsymbol{C}$$

Algorithmic stress tensors (cf. Armero & Zambrana-Rojas [2007])

$$\bar{\boldsymbol{S}} = \lambda_M\,\boldsymbol{C}^{-1}\,\mathring{\boldsymbol{C}}\;\boldsymbol{C}^{-1} \qquad \bar{\boldsymbol{S}}_K = \lambda_K\,\boldsymbol{C}^{-1}\,\mathring{\boldsymbol{K}}\;\boldsymbol{C}^{-1}$$

$$\lambda_K := \frac{\Psi_K(\boldsymbol{K}_{n+1}) - \Psi_K(\boldsymbol{K}_n) - \displaystyle\int_0^1 \frac{\partial \Psi_K(\boldsymbol{K})}{\partial \boldsymbol{K}} : \mathring{\boldsymbol{K}}(\alpha)\,\mathrm{d}\alpha}{\displaystyle\int_0^1 \boldsymbol{C}^{-1}\,\mathring{\boldsymbol{K}}(\alpha) : \mathring{\boldsymbol{K}}(\alpha)\,\boldsymbol{C}^{-1}\,\mathrm{d}\alpha}$$

Principle of virtual power (I)

Total energy balance law $\dot{\mathcal{H}} = \dot{\mathcal{T}}^{\text{tra}} + \dot{\mathcal{T}}^{\text{rot}} + \dot{\Pi}^{\text{ext}} + \dot{\Pi}^{\text{int}}$

$$\dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\boldsymbol{C}}_v, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F, \dot{\tilde{C}}_F^v, \dot{\Theta}, \dot{\eta}}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \tilde{\boldsymbol{P}}, \tilde{\boldsymbol{S}}, \tilde{\boldsymbol{P}}_K, \tilde{\boldsymbol{S}}_K, \tilde{\boldsymbol{\tau}}^t_{\text{skw}}, \tilde{S}_V, \tilde{S}_F, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Kinetic power functionals (motivated by Altenbach et al. [2003], Askes & Aifantis [2011])

$$\dot{\mathcal{T}}^{\text{tra}}(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) := \int_{\mathscr{B}_0} [\rho_0 \boldsymbol{I}\, \boldsymbol{v} - \boldsymbol{p}] \cdot \dot{\boldsymbol{v}}\, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{p}} \cdot [\boldsymbol{v} - \dot{\boldsymbol{\varphi}}]\mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{p} \cdot \ddot{\boldsymbol{\varphi}}\, \mathrm{d}V$$

$$\dot{\mathcal{T}}^{\text{rot}}(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}) := \int_{\mathscr{B}_0} \left[\rho_0 \left[(l_F^2 - l_0^2)\, \boldsymbol{A}_0 + l_0^2 \boldsymbol{I}\right] \boldsymbol{\omega} - \boldsymbol{\pi}\right] \cdot \dot{\boldsymbol{\omega}}\, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{\pi}} \cdot [\boldsymbol{\omega} - \dot{\boldsymbol{\alpha}}]\mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\pi} \cdot \ddot{\boldsymbol{\alpha}}\, \mathrm{d}V$$

External power functional $D^{\text{cdu}} = -\mathrm{Grad}[\ln \Theta] \cdot \boldsymbol{Q}$

$$\begin{aligned}
\dot{\Pi}^{\text{ext}} := &-\int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A + \int_{\partial_Q \mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta} \bar{Q}\, \mathrm{d}A \\
&+ \int_{\mathscr{B}_0} \frac{1}{\Theta} \mathrm{Grad}[\tilde{\Theta}] \cdot \boldsymbol{Q}\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta} \left(D^{\text{cdu}} + D^{\text{int}}\right) \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\Sigma}_v : \dot{\boldsymbol{C}}_v\, \mathrm{d}V \\
&+ \int_{\partial_\Theta \mathscr{B}_0} \lambda \left[\tilde{\Theta} - \Theta_\infty\right] \mathrm{d}A - \int_{\partial_{\dot{\Theta}} \mathscr{B}_0} h \left[\dot{\Theta} - \dot{\bar{\Theta}}\right] \mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \cdot \left[\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}\right] \mathrm{d}A \\
&- \int_{\partial_\alpha \mathscr{B}_0} \boldsymbol{Z} \cdot \left[\dot{\boldsymbol{\alpha}} - \dot{\bar{\boldsymbol{\alpha}}}\right] \mathrm{d}A - \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \left[\boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\right] \mathrm{d}A - \int_{\partial_W \mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\alpha}}\, \mathrm{d}A \\
&+ \int_{\mathscr{B}_0} \frac{1}{2} \bar{\boldsymbol{S}} : \dot{\tilde{\boldsymbol{C}}}\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2} \bar{S}_V\, \dot{\tilde{C}}_V\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2} \bar{S}_F\, \dot{\tilde{C}}_F\, \mathrm{d}V + \int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\, \mathrm{d}V \\
&+ \int_{\mathscr{B}_0} \dot{\tilde{C}}_F^v\, \Sigma_F^v\, \mathrm{d}V + \int_{\mathscr{B}_0} \bar{M}_F^v \left[L_F(\dot{\tilde{C}}_F) - L_F(\dot{\tilde{C}}_F^v)\right] \mathrm{d}V \quad \text{with} \quad L_F(\dot{\epsilon}) = \frac{\dot{\overline{\ln(\epsilon)}}}{2}
\end{aligned}$$

Principle of virtual power (II)

Internal power functional (motivated by Steinmann & Stein [1997])

$$\dot{\Pi}^{\text{int}} := \int_{\mathscr{B}_0} \left\{ \left[2\frac{\partial \hat{\Psi}_M}{\partial \tilde{\boldsymbol{C}}} + S_V\, \boldsymbol{A}^{\text{vol}} + S_F\, \boldsymbol{A}_0 - \boldsymbol{S} \right] : \frac{1}{2}\dot{\tilde{\boldsymbol{C}}} + \left[2\frac{\partial \hat{\Psi}_M}{\partial \tilde{C}_V} - S_V \right] \frac{\dot{\tilde{C}}_V}{2} + \left[2\frac{\partial \hat{\Psi}_F}{\partial \tilde{C}_F} - S_F \right] \frac{\dot{\tilde{C}}_F}{2} \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ [\Theta - \tilde{\Theta}]\,\dot{\eta} + \left[\frac{\partial \Psi}{\partial \Theta} + \eta \right] \dot{\Theta} + \frac{\partial \hat{\Psi}_M}{\partial \boldsymbol{C}_v} : \dot{\boldsymbol{C}}_v + \frac{\partial \Psi_F}{\partial \tilde{C}^v_F}\, \dot{\tilde{C}}^v_F + [\tilde{\boldsymbol{F}}\boldsymbol{S} + \boldsymbol{N}_K - \boldsymbol{P}] : \dot{\tilde{\boldsymbol{F}}} + \boldsymbol{P} : \mathrm{Grad}\,[\dot{\boldsymbol{\varphi}}] \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ \boldsymbol{P}_K : \mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] + \boldsymbol{\tau}^t_{\text{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] + [\tilde{\boldsymbol{F}}\boldsymbol{S}_K - \boldsymbol{P}_K] : \dot{\tilde{\boldsymbol{G}}} + \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} - \boldsymbol{S}_K \right] : \dot{\tilde{\boldsymbol{K}}} \right\} \mathrm{d}V$$

Principle of virtual power

$$\delta_* \dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\boldsymbol{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \boldsymbol{P}, \boldsymbol{P}_K, \boldsymbol{S}, S_V, S_F, \boldsymbol{S}_K, \boldsymbol{\tau}^t_{\text{skw}}, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Rotational weak forms (here only the volume weak forms)

$$\int_{\mathscr{B}_0} \delta_*\boldsymbol{\tau}^t_{\text{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] \mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \delta_*\boldsymbol{\tau}^t_{\text{skw}}\, \mathrm{d}A \qquad \int_{\mathscr{B}_0} \delta_*\dot{\tilde{\boldsymbol{K}}} : \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} + \bar{\boldsymbol{S}}_K - \boldsymbol{S}_K \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\tilde{\boldsymbol{G}}} : [\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}}_K - \boldsymbol{P}_K]\, \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\boldsymbol{P}_K : \left[\mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] - \dot{\tilde{\boldsymbol{G}}} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\pi}} \cdot [\dot{\boldsymbol{\alpha}} - \boldsymbol{\omega}]\, \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\omega}} \cdot \left[\rho_0 \left[(l_F^2 - l_0^2)\, \boldsymbol{A}_0 + l_0^2 \boldsymbol{I} \right] \boldsymbol{\omega} - \boldsymbol{\pi} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} + \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}} - \dot{\tilde{\boldsymbol{K}}} \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot [\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}]\, \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{P}_K : \mathrm{Grad}\,[\delta_*\dot{\boldsymbol{\alpha}}]\, \mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\, \mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\, \mathrm{d}A$$

Vibration analysis of thin laminated beams (Initial-boundary conditions; 121-em with H20-mixed-Bbar)

Quadratic St. Venant-Kirchhoff curvature-twist strain energy

$$\varPsi^{\text{ctw}}_{\text{SVK}} := \mu_K \left[\frac{(l_t)^2}{2} \, [I_1(\boldsymbol{K}_F)]^2 + (l_b)^2 \, J_2(\boldsymbol{K}_F) \right] \equiv \mu_K \left[\frac{(l_K)^2}{2} \, [I_1(\boldsymbol{K}_F)]^2 - 2 \, (l_b)^2 \, J_2^{\text{dev}}(\boldsymbol{K}_F) \right]$$

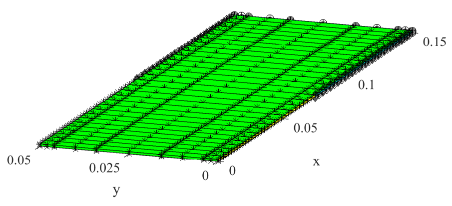
Nonlinear Kauderer-type strain energy function (cf. Menzel & Steinmann [2001])

$$\varPsi^{\text{ctw}}_{\text{Kau}} := \varPsi^{\text{sph}}_{\text{Kau}}(T_F) + \varPsi^{\text{dev}}_{\text{Kau}}(B_F)$$

$$\varPsi^{\text{sph}}_{\text{Kau}}(T_F) := \mu_K (l_K)^2 \left[\frac{(T_F)^2}{2} + \kappa_1^{\text{sph}} \frac{(T_F)^3}{3} + \kappa_2^{\text{sph}} \frac{(T_F)^4}{4} + \dots \right]$$

$$\varPsi^{\text{dev}}_{\text{Kau}}(B_F) := \mu_K (l_b)^2 \left[\frac{(B_F)^1}{2} + \kappa_2^{\text{dev}} \frac{(B_F)^2}{4} + \kappa_4^{\text{dev}} \frac{(B_F)^3}{6} + \dots \right]$$

Boundary conditions



Initial conditions

$$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \qquad \boldsymbol{\alpha}_0^A = \boldsymbol{0}$$
$$\boldsymbol{v}_0^A = \boldsymbol{0} \qquad \boldsymbol{\omega}_0^A = \boldsymbol{0}$$
$$\varTheta_0^A = \varTheta_\infty \qquad \eta_0^A = 0$$
$$\boldsymbol{\tau}^t_{\text{skw}} = \boldsymbol{O} \qquad \varTheta_\infty = 298.15$$

Loads

$g = 9.81$ with $\boldsymbol{e}_g = -\boldsymbol{e}_z$

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Introduction

Continuum model
Local rotation
Fiber curvature-twist
Fiber strain rate/stress
Algorithmic stress tensors
Virtual power principle

Numerical studies
Initial-boundary conditions
Compared fiber directions
Vertical displacement
Norm of rotation vector
Current configurations

Summary

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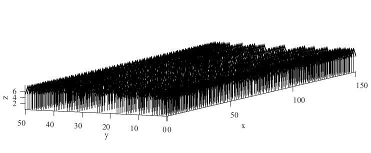
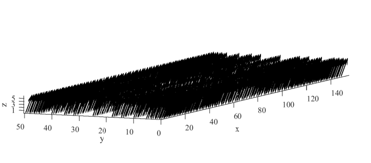
Vibration analysis of thin laminated beams (Compared fiber directions; 121-em with H20-mixed-Bbar)

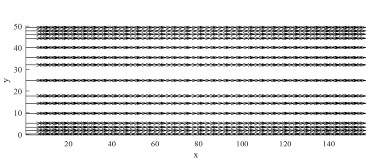
Fiber direction fields $\quad {}_{ij}\boldsymbol{a}_0 := \sin\phi_l^s\, \boldsymbol{e}_i + \cos\phi_l^s\, \boldsymbol{e}_j$



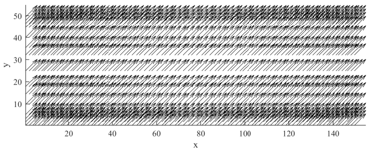
a ${}_{xz}\boldsymbol{a}_0$ with $\phi_1^a = 0 = \phi_1^b$



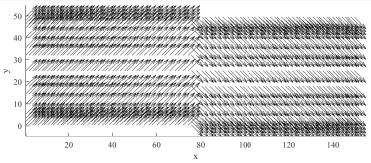
b ${}_{xz}\boldsymbol{a}_0$ with $\phi_1^a = \pi/4 = \phi_1^b$



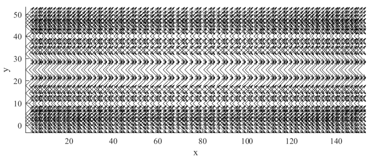
c ${}_{xz}\boldsymbol{a}_0$ with $\phi_1^a = \pi/2 = \phi_1^b$



d ${}_{xy}\boldsymbol{a}_0$ with $\phi_1^a = \pi/4 = \phi_1^b$



e ${}_{xy}\boldsymbol{a}_0$ with $\phi_1^a/3 = \pi/4 = \phi_1^b$

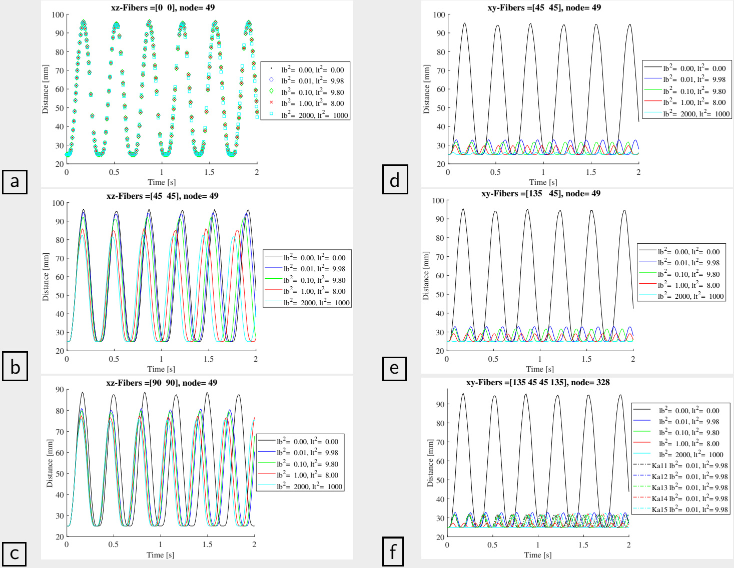


f ${}_{xy}\boldsymbol{a}_0$ with $\phi_1^a/3 = \pi/4 = \phi_1^b,\ \ \phi_2^a = \pi/4 = \phi_2^b/3$

Vibration analysis of thin laminated beams

(Effects of curvature-twist stiffness I; 121-em with H20-mixed-Bbar)

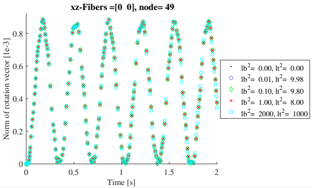
Time evolutions of the vertical displacement ($\mu_K = 0.250 \cdot 10^9$)



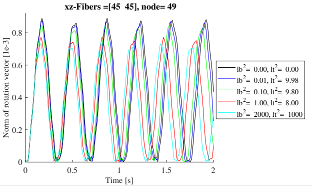
Vibration analysis of thin laminated beams

(Effects of curvature-twist stiffness II; 121-em with H20-mixed-Bbar)

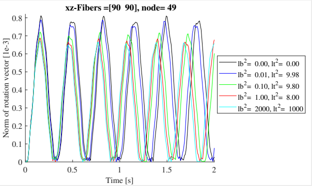
Time evolutions of the norm of rotation vector ($\mu_K = 0.250 \cdot 10^9$)



a



b



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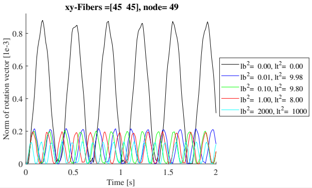
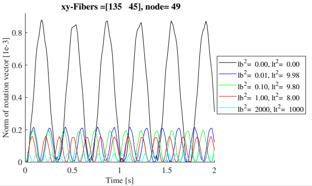
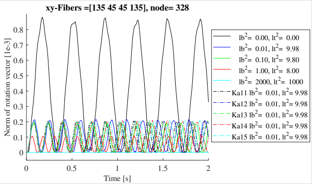


d



e

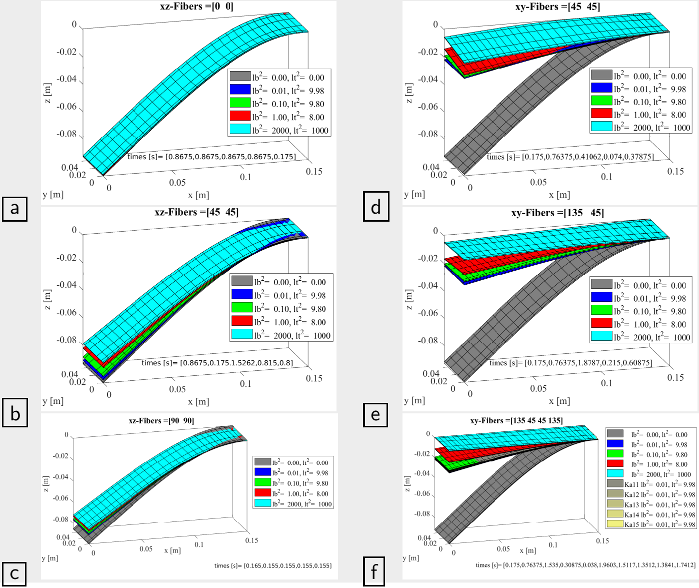


f

Vibration analysis of thin laminated beams

(Anisotropic deformation behaviour; 121-em with H20-mixed-Bbar)

Snapshots at times with maximum deflection ($\mu_K = 0.250 \cdot 10^9$)



Summary

1. Motivation:
   - Precise simulations of fiber roving composites with
   - fiber twisting and bending stiffness on the micro scale
2. Goals:
   - Energy-momentum schemes for constrained micropolar continua
   - derived by a mixed principle of virtual power for any continua
3. Strategy:
   - Introduction of independent fields for the continuum rotation
   - Discretization by using a new mixed finite element formulation
4. Important results:
   - curvature-twist stiffness of rovings can be prescribed
   - even separately, and increases the material stiffness
5. Outlook: This formulation allows to model
   - non-isothermal rigid bodies by a constitutive law
   - liquid crystal elastomers and micropolar fluids