

A mixed
variational-based
dynamic simulation
method for
fiber-reinforced
continua in
non-isothermal
rotordynamical
systems

Michael Groß,
Julian Dietzsch,
Chris Rübiger

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Variational setting

Cauchy continuum model
Generalized power principle
Additional balance laws

Numerical studies

Driven dynamic torsion
Driven and heated pipe
Driven disc brake rotor
Driven turbine rotor

Summary

A mixed variational-based dynamic simulation method for fiber-reinforced continua in non-isothermal rotordynamical systems

Michael Groß, Julian Dietzsch, Chris Rübiger

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MS4: Coupled problems in rotating machinery

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Motivation: meta-materials in rotating machinery

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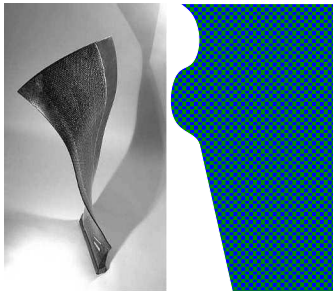
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Summary

Motivation: increasing application of 3D-woven composites

- 1 Turbine/pump rotors
- 2 Disc brake rotors
- 3 Hollow shafts

Macroscopic scale: turbine blade preform



De Luycker, Morestin, Boisse, Marsal [2009]

Mesoscopic scale: Volume Element (RVE)



Microscopic scale: Yarn with bending stiffness



Kai Uhlig [2017] (IPF TU Dresden), recommended by IST TU Chemnitz

Goals: FE simulations which take into account the fiber bending stiffness and micro inertia

- 1 We design **dynamic variational** principles for continua with **multi-scale effects**
- 2 We derive **energy-momentum** schemes which are preferable for rotor dynamics



Strategy (I): Generalized continuum mechanics

(see e.g. Boisse et al. [2018], Asmanoglo & Menzel [2017], Madeo et al. [2015], Feretti et al. [2014], Spencer & Soldatos [2007])

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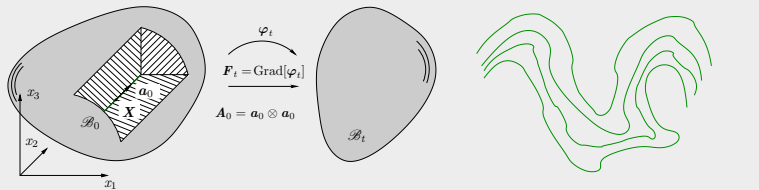
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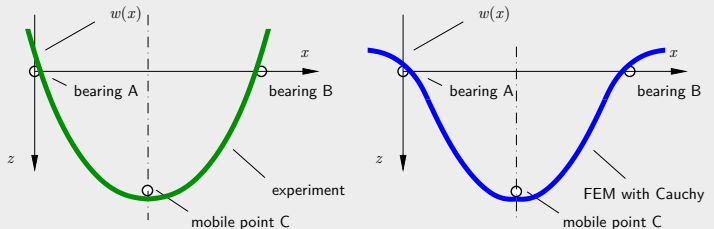
Formulation of anisotropic Cauchy continua with structural tensors

(see e.g. Reese, Raible & Wriggers [2001], Klinkel, Sansour & Wagner [2005], Schröder, Neff & Balzani [2005])



Three point bending test: experiment vs. Cauchy theory $\leadsto$ 'the need to a second-gradient theory'

(see Madeo et al. [2015], Charmetant et al. [2012])



Strategy (II): Principles of virtual power

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Principle of virtual work / Hamilton's principle (cf. Dimarogonas et al. [2013])

- 1 Simo-Taylor-Pister functional (Simo, Taylor, Pister [1985])

$$\Pi_{\text{STP}}(\varphi, \tilde{J}, p) := \int_{\mathcal{B}_0} \Psi^{\text{iso}}(\varphi) \, dV + \int_{\mathcal{B}_0} \Psi^{\text{vol}}(\tilde{J}) \, dV - \int_{\mathcal{B}_0} p [\tilde{J} - \det(\nabla \varphi)] \, dV$$

- 2 Spatial weak form for $\tilde{J}$

$$\int_{\mathcal{B}_0} \delta p \, G(\varphi, \tilde{J}) \, dt = 0 \quad \text{with} \quad G(\varphi, \tilde{J}) := \tilde{J} - \det(\nabla \varphi)$$

Energy-momentum time integration (motivated by Betsch & Steinmann [2002], Armero [2008])

- 1 Preservation of balance law of total energy $H := T(\dot{\varphi}) + \Psi^{\text{iso}}(\varphi) + \Psi^{\text{vol}}(\tilde{J})$

$$\mathcal{H}_{t_{n+1}} - \mathcal{H}_{t_n} = \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\frac{\partial H}{\partial \dot{\varphi}} \cdot \ddot{\varphi} + \frac{\partial H}{\partial \varphi} \cdot \dot{\varphi} + \frac{\partial H}{\partial \tilde{J}} \dot{\tilde{J}} \right] \, dV \, dt$$

- 2 Time evolution of volume dilatation $\tilde{J}$

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \delta p \, \dot{G}(\varphi, \dot{\tilde{J}}) \, dt = 0 \quad \text{with} \quad \dot{G}(\varphi, \dot{\tilde{J}}) := \dot{\tilde{J}} - \overline{\det(\dot{\nabla} \varphi)}$$

Variational principle

(cf. Schröder & Kuhl [2015])

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}(\dot{\varphi}, \dot{\tilde{J}}, p) \, dt = 0 \quad \text{with} \quad \mathcal{H} := \frac{1}{2} \int_{\mathcal{B}_0} \rho_0 \dot{\varphi} \cdot \dot{\varphi} \, dV + \Pi_{\text{STP}}(\varphi, \tilde{J}, p)$$

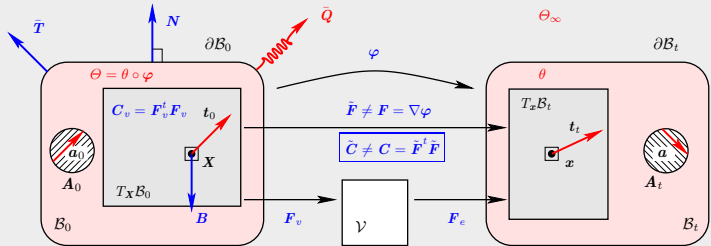
Cauchy continuum model (I)

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- Cauchy continuum model
- Generalized power principle
- Additional balance laws

Transversely isotropic first-gradient material law



Deformation of fibers and matrix

(cf. Klinkel, Sansour & Wagner [2005])

- ### 1 Deformation of the fibers

$$\mathbf{F}_F := \mathbf{a}_t \otimes \mathbf{a}_0 = \tilde{\mathbf{F}} \mathbf{A}_0 \quad \mathbf{A}_0 := \mathbf{a}_0 \otimes \mathbf{a}_0 \quad \mathbf{C}_F := \mathbf{F}_F^t \mathbf{F}_F := \mathbf{C}_F \mathbf{A}_0$$

- ## 2 Fiber and volume dilatation

$$\tilde{C}_F \neq C_F := \tilde{\mathbf{C}} : \mathbf{A}_0 \qquad \tilde{C}_V \neq C_V := [\det(\tilde{\mathbf{C}})]^{\frac{1}{n_{\text{dim}}}}$$

- ### 3 Isotropic viscoelastic matrix

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_v \quad \mathbf{C}_v = \mathbf{F}_v^t \mathbf{F}_v \quad \rightsquigarrow \text{symmetric internal variable}$$

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Thermo-viscoelastic free energy

1 Hyperelastic strain energy

$$\Psi^{\text{ela}}(\tilde{C}, \tilde{C}_V, \tilde{C}_F; \mathbf{A}_0) = \hat{\Psi}_M^{\text{iso}}(I_1^{\tilde{C}}, I_2^{\tilde{C}}, I_3^{\tilde{C}}, I_4) + \hat{\Psi}_M^{\text{vol}}(\tilde{C}_V) + \hat{\Psi}_F^{\text{ela}}(\tilde{C}_F)$$

2 Thermoelastic free energy

$$\begin{aligned} \Psi^{\text{the}}(\Theta, \tilde{C}_V, \tilde{C}_F) = & \hat{\Psi}^{\text{cap}}(\Theta) - 2\sqrt{\tilde{C}_V^{2-n_{\text{dim}}}} \beta_M [\Theta - \Theta_\infty] \text{D}\hat{\Psi}_M^{\text{vol}}(\tilde{C}_V) \\ & - 2\sqrt{\tilde{C}_F^{2-1}} \beta_F [\Theta - \Theta_\infty] \text{D}\hat{\Psi}_F^{\text{ela}}(\tilde{C}_F) \end{aligned}$$

3 Viscoelastic free energy

$$\Psi_M^{\text{vis}}(\mathbf{A}) = \hat{\Psi}_M^{\text{ela}}(I_1^{\mathbf{A}}, I_2^{\mathbf{A}}, I_3^{\mathbf{A}}) \quad \mathbf{A} = \tilde{\mathbf{C}} \mathbf{C}_v^{-1} \quad \mathbf{C}_e = \mathbf{F}_e^t \mathbf{F}_e$$

Non-equilibrium and total stress

(cf. Reese [2001])

1 Viscous evolution equation and non-equilibrium stress tensor

$$\boxed{\mathbf{Y} = \boldsymbol{\Sigma}_v} \quad \mathbf{Y} := -\frac{\partial \Psi_M^{\text{vis}}}{\partial \mathbf{C}_v} \quad D^{\text{int}} := \dot{\mathbf{C}}_v : \boldsymbol{\Sigma}_v = \dot{\mathbf{C}}_v : \mathbb{V}(\mathbf{C}_v) : \dot{\mathbf{C}}_v \geq 0$$

$$\mathbb{V}(\mathbf{C}_v) = \frac{1}{4} \left[V_{\text{vol}} - \frac{V_{\text{dev}}}{n_{\text{dim}}} \right] \mathbf{C}_v^{-1} \otimes \mathbf{C}_v^{-1} + \frac{V_{\text{dev}}}{4} \mathbb{I}^{\text{sym}} : \mathbf{C}_v^{-1} \otimes \mathbf{C}_v^{-1}$$

2 Total second Piola-Kirchhoff stress tensor

$$\mathbf{S} := \mathbf{S}_M^{\text{iso}} + \mathbf{S}_M^{\text{vis}} + \mathbf{S}_F \mathbf{A}_0 + \mathbf{S}_V \mathbf{A}^{\text{vol}} \quad \mathbf{A}^{\text{vol}} := \frac{1}{n_{\text{dim}}} \det(\tilde{\mathbf{C}})^{\frac{1}{n_{\text{dim}}}} \tilde{\mathbf{C}}^{-1}$$



Generalized Cauchy virtual power principle (I)

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Total energy **balance law** in functional form

$$\dot{\mathcal{H}}(\underbrace{\dot{\varphi}, \dot{v}, \dot{p}, \dot{\alpha}, \dot{\omega}, \dot{\pi}, \dot{C}_v, \dot{\Theta}, \dot{\eta}, \dot{\bar{F}}, \dot{\bar{C}}, \dot{C}_V, \dot{C}_F, \dot{\bar{\Theta}}}_{\text{temporally continuous}}, \underbrace{\mathbf{P}, \boldsymbol{\tau}_{\text{skw}}^t, \mathbf{S}, S_V, S_F, \mathbf{R}, h, \lambda, \mathbf{Z}}_{\text{temporally discontinuous}}) = 0 \quad \dot{\mathcal{H}} = \dot{\mathcal{T}}^{\text{tra}} + \dot{\mathcal{T}}^{\text{rot}} + \dot{H}^{\text{ext}} + \dot{H}^{\text{int}}$$

Kinetic power functionals (motivated by Altenbach et al. [2003], Askes & Aifantis [2011])

$$\begin{aligned} \dot{\mathcal{T}}^{\text{tra}}(\dot{\varphi}, \dot{v}, \dot{p}) &:= \int_{\mathcal{B}_0} [\rho_0 \mathbf{I} \mathbf{v} - \mathbf{p}] \cdot \dot{\mathbf{v}} \, dV - \int_{\mathcal{B}_0} \dot{\mathbf{p}} \cdot [\mathbf{v} - \dot{\varphi}] \, dV + \int_{\mathcal{B}_0} \mathbf{p} \cdot \ddot{\varphi} \, dV \\ \dot{\mathcal{T}}^{\text{rot}}(\dot{\alpha}, \dot{\omega}, \dot{\pi}) &:= \int_{\mathcal{B}_0} [\rho_0 [(I_F^2 - I_0^2) \mathbf{A}_0 + I_0^2 \mathbf{I}] \boldsymbol{\omega} - \boldsymbol{\pi}] \cdot \dot{\boldsymbol{\omega}} \, dV - \int_{\mathcal{B}_0} \dot{\boldsymbol{\pi}} \cdot [\boldsymbol{\omega} - \dot{\alpha}] \, dV + \int_{\mathcal{B}_0} \boldsymbol{\pi} \cdot \ddot{\alpha} \, dV \end{aligned}$$

External **power functional**

$$D^{\text{cd}} = -\nabla(\ln \Theta) \cdot \mathbf{Q}$$

$$\begin{aligned} \dot{H}^{\text{ext}} := & \int_{\mathcal{B}_0} \frac{1}{2} \tilde{\mathbf{S}} : \dot{\bar{\mathbf{C}}} \, dV & + \int_{\mathcal{B}_0} \frac{1}{2} \tilde{S}_V \dot{C}_V \, dV & + \int_{\mathcal{B}_0} \frac{1}{2} \tilde{S}_F \dot{C}_F \, dV \\ & - \int_{\mathcal{B}_0} \rho_0 \mathbf{B} \cdot \dot{\varphi} \, dV & - \int_{\partial_T \mathcal{B}_0} \bar{\mathbf{T}} \cdot \dot{\varphi} \, dA & + \int_{\partial_Q \mathcal{B}_0} \frac{\tilde{\Theta}}{\bar{\Theta}} \bar{Q} \, dA \\ & + \int_{\mathcal{B}_0} \frac{1}{\bar{\Theta}} \nabla \tilde{\Theta} \cdot \mathbf{Q} \, dV & + \int_{\mathcal{B}_0} \frac{\tilde{\Theta}}{\bar{\Theta}} (D^{\text{cd}} + D^{\text{int}}) \, dV & + \int_{\mathcal{B}_0} \dot{\mathbf{C}}_v : \boldsymbol{\Sigma}_v \, dV \\ & + \int_{\partial_\Theta \mathcal{B}_0} \lambda [\tilde{\Theta} - \Theta_\infty] \, dA & - \int_{\partial_h \mathcal{B}_0} h [\dot{\Theta} - \dot{\bar{\Theta}}] \, dA & - \int_{\partial_\varphi \mathcal{B}_0} \mathbf{R} \cdot [\dot{\varphi} - \dot{\bar{\varphi}}] \, dA \\ & - \int_{\partial_\alpha \mathcal{B}_0} \mathbf{Z} \cdot [\dot{\alpha} - \dot{\bar{\alpha}}] \, dA & - \int_{\partial_\alpha \mathcal{B}_0} \dot{\mathbf{Z}} \cdot [\boldsymbol{\epsilon} : \boldsymbol{\tau}_{\text{skw}}^t] \, dA & - \int_{\partial_W \mathcal{B}_0} \bar{\mathbf{W}} \cdot \dot{\alpha} \, dA \end{aligned}$$



Generalized Cauchy virtual power principle (II)

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Internal power functional

(motivated by Steinmann & Stein [1997])

$$\begin{aligned} \dot{H}^{\text{int}} := & \frac{1}{2} \int_{\mathcal{B}_0} \left\{ \left[2 \frac{\partial \hat{\Psi}_M}{\partial \mathbf{C}} + S_V \mathbf{A}^{\text{vol}} + S_F \mathbf{A}_0 - \mathbf{S} \right] : \dot{\mathbf{C}} + \left[2 \frac{\partial \hat{\Psi}_M}{\partial \mathbf{C}_V} - S_V \right] : \dot{\mathbf{C}}_V \right\} dV - \int_{\mathcal{B}_0} \dot{\eta} [\bar{\Theta} - \Theta] dV \\ & + \frac{1}{2} \int_{\mathcal{B}_0} \left[2 \frac{\partial \hat{\Psi}_F}{\partial \mathbf{C}_F} - S_F \right] : \dot{\mathbf{C}}_F dV + \int_{\mathcal{B}_0} \left\{ [\tilde{\mathbf{F}} \mathbf{S} - \mathbf{P}] : \dot{\tilde{\mathbf{F}}} + \frac{\partial \hat{\Psi}_M}{\partial \mathbf{C}_v} : \dot{\mathbf{C}}_v \right\} dV + \int_{\mathcal{B}_0} \left[\eta + \frac{\partial \Psi}{\partial \Theta} \right] \dot{\Theta} dV \\ & + \int_{\mathcal{B}_0} \mathbf{P} : \nabla \dot{\boldsymbol{\varphi}} dV + \int_{\mathcal{B}_0} \boldsymbol{\tau}_{\text{skw}}^t : \left[\mathbb{I}^{\text{skw}} : \dot{\tilde{\mathbf{F}}} \tilde{\mathbf{F}}^{-1} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \right] dV \quad \text{with} \quad 2 \mathbb{I}^{\text{skw}} = \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon} \end{aligned}$$

Principle of virtual power

$$\delta_* \mathcal{H}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\mathbf{v}}, \dot{\mathbf{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\mathbf{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\mathbf{F}}}, \dot{\mathbf{C}}, \dot{\mathbf{C}}_V, \dot{\mathbf{C}}_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \mathbf{P}, \boldsymbol{\tau}_{\text{skw}}^t, \mathbf{S}, S_V, S_F, \mathbf{R}, h, \lambda, \mathbf{Z}}_{\text{temporally discontinuous}}) = 0$$

Additional weak forms

$$\begin{aligned} \int_{\mathcal{B}_0} \delta_* \boldsymbol{\tau}_{\text{skw}}^t : \left[\mathbb{I}^{\text{skw}} : \dot{\tilde{\mathbf{F}}} \tilde{\mathbf{F}}^{-1} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \right] dV &= \int_{\partial_\alpha \mathcal{B}_0} \hat{\mathbf{Z}} \cdot \boldsymbol{\epsilon} : \delta_* \boldsymbol{\tau}_{\text{skw}}^t dA & \int_{\partial_\alpha \mathcal{B}_0} \delta_* \hat{\mathbf{Z}} \cdot \boldsymbol{\epsilon} : \boldsymbol{\tau}_{\text{skw}}^t dA &= 0 \\ \int_{\mathcal{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot [\hat{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}_{\text{skw}}^t] dV &= \int_{\partial_\alpha \mathcal{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \mathbf{Z} dA + \int_{\partial_W \mathcal{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \mathbf{W} dA & \int_{\partial_\alpha \mathcal{B}_0} \delta_* \mathbf{Z} : [\dot{\boldsymbol{\alpha}} - \hat{\boldsymbol{\alpha}}] dV &= 0 \\ \int_{\mathcal{B}_0} \delta_* \dot{\boldsymbol{\omega}} \cdot [\rho_0 [(I_F^2 - I_0^2) \mathbf{A}_0 + I_0^2 \mathbf{I}] \boldsymbol{\omega} - \boldsymbol{\pi}] dV &= 0 & \int_{\mathcal{B}_0} \delta_* \dot{\boldsymbol{\omega}} \cdot [\boldsymbol{\omega} - \hat{\boldsymbol{\alpha}}] dV &= 0 \\ \int_{\mathcal{B}_0} \delta_* \dot{\tilde{\mathbf{F}}} : [\tilde{\mathbf{F}} \tilde{\mathbf{S}} + \boldsymbol{\tau}_{\text{skw}}^t \tilde{\mathbf{F}}^{-t} - \mathbf{P}] dV &= 0 & \int_{\mathcal{B}_0} \delta_* \mathbf{P} : [\dot{\tilde{\mathbf{F}}} - \text{Grad}[\dot{\boldsymbol{\varphi}}]] dV &= 0 \end{aligned}$$



Additional balance laws

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Balance law of angular momentum $\mathcal{J}$ includes additively:

Balance law of rotational momentum $\mathcal{J}^{\text{rot}}$

Set $\delta_* \dot{\alpha} = \mathbf{c} = \text{const.}$:

$$\int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \mathbf{c} \cdot [\dot{\pi} + \epsilon : \tau_{\text{skw}}] dV dt = \int_{t_n}^{t_{n+1}} \int_{\partial_{\alpha} \mathcal{B}_0} \mathbf{c} \cdot \mathbf{Z} dA dt + \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathcal{B}_0} \mathbf{c} \cdot \mathbf{W} dA dt$$

$$\int_{\mathcal{B}_0} \pi_{t_{n+1}} dV - \int_{\mathcal{B}_0} \pi_{t_n} dV = (t_{n+1} - t_n) \int_{t_n}^{t_{n+1}} \left[- \int_{\mathcal{B}_0} \epsilon : \tau_{\text{skw}} dV + \int_{\partial_{\alpha} \mathcal{B}_0} \mathbf{Z} dA + \int_{\partial_W \mathcal{B}_0} \mathbf{W} dA \right] dt$$

Balance law of total energy $\mathcal{H}$ includes additively:

Balance law of rotational energy $\mathcal{T}^{\text{rot}}$

Set $\delta_* \dot{\alpha} = \dot{\alpha}$ and $\delta_* \dot{\omega} = \dot{\omega}$ and $\delta_* \tau_{\text{skw}}^t = \tau_{\text{skw}}^t$

$$\text{Eqn. (1): } \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \dot{\alpha} \cdot [\dot{\pi} + \epsilon : \tau_{\text{skw}}^t] dV dt = \int_{t_n}^{t_{n+1}} \int_{\partial_{\alpha} \mathcal{B}_0} \dot{\alpha} \cdot \mathbf{Z} dA dt + \int_{t_n}^{t_{n+1}} \int_{\partial_W \mathcal{B}_0} \dot{\alpha} \cdot \mathbf{W} dA dt$$

$$\text{Eqn. (2): } \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \tau_{\text{skw}}^t : [\mathbb{I}^{\text{skw}} : \dot{\mathbf{F}} \tilde{\mathbf{F}}^{-1} + \epsilon \cdot \dot{\alpha}] dV dt = \int_{t_n}^{t_{n+1}} \int_{\partial_{\alpha} \mathcal{B}_0} \hat{\mathbf{Z}} \cdot \underbrace{\epsilon : \tau_{\text{skw}}^t}_0 dA dt$$

$$\text{Eqn. (3): } \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \dot{\omega} \cdot [\mathbf{J} \dot{\omega} - \pi] dV dt = 0 \quad \text{with} \quad \mathbf{J} = \rho_0 [(I_F^2 - I_0^2) \mathbf{A}_0 + I_0^2 \mathbf{I}]$$

$$\begin{aligned} \mathcal{T}_{n+1}^{\text{rot}} - \mathcal{T}_n^{\text{rot}} &\equiv \frac{1}{2} \int_{\mathcal{B}_0} \omega_{t_{n+1}} \cdot \mathbf{J} \omega_{t_{n+1}} dV - \frac{1}{2} \int_{\mathcal{B}_0} \omega_{t_n} \cdot \mathbf{J} \omega_{t_n} dV \\ &= (t_{n+1} - t_n) \int_{t_n}^{t_{n+1}} \left[\int_{\mathcal{B}_0} \tau_{\text{skw}}^t : \dot{\mathbf{F}} \tilde{\mathbf{F}}^{-1} dV + \int_{\partial_{\alpha} \mathcal{B}_0} \dot{\alpha} \cdot \mathbf{Z} dA + \int_{\partial_W \mathcal{B}_0} \dot{\alpha} \cdot \mathbf{W} dA \right] dt \end{aligned}$$



Dynamic torsion of a long composite square pipe (Boundary and initial conditions; 121-em with H2O-mixed-Bbar)

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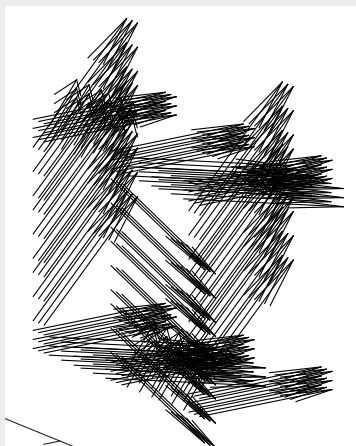
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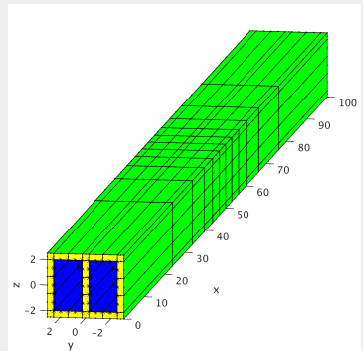
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Material Status Monitor



Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches: Temperature $\theta^A = \hat{\theta} f(t)$, Torque load $W_x^A = \pm \hat{W}_x f(t)$

Blue and green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

$$\begin{aligned} \varphi_0^A &= \mathbf{X}^A & \alpha_0^A &= 0 & \theta_0^A &= \theta_\infty \\ \mathbf{v}_0^A &= 0 & \omega_0^A &= 0 & \theta_\infty &= 298.15 \end{aligned}$$



Dynamic torsion of a long composite square pipe (Motion and loads; 121-em with H2O-mixed-Bbar)

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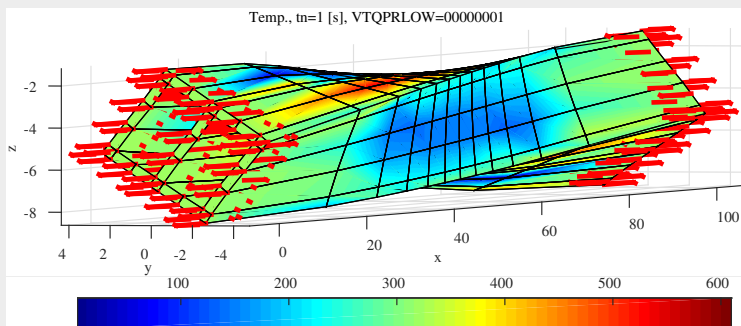
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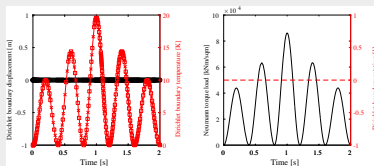
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Current temperature at $t_n = 1.0$ s



Time evolution of the loads



Movies

$$\tau_{skw}^t(X)$$

$$\Theta(X)$$

$$S_V(X)$$

$$\bar{C}_V(X)$$

$$\det \bar{F}(X)$$

$$C_i(X)$$

Dynamic torsion of a long composite square pipe (Solution versus time; 121-em with H2O-mixed-Bbar)

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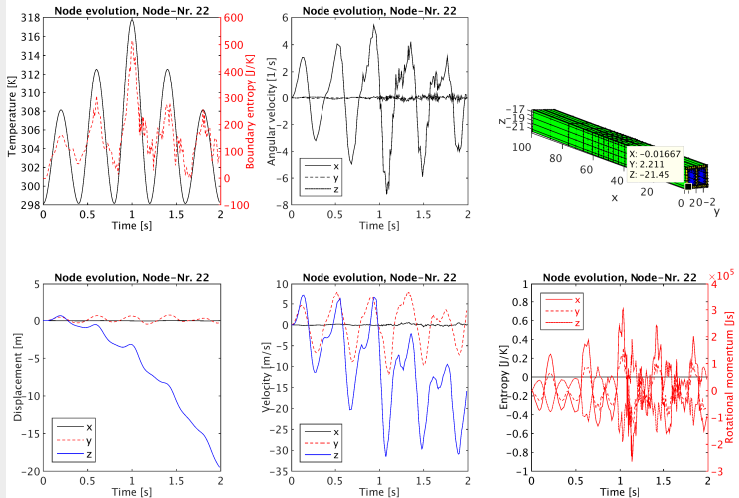
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Driven disc brake rotor
Driven turbine rotor

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Time evolutions of a node on the boundaries $\partial_{\dot{\mathcal{B}}_0}$ and $\partial_W \mathcal{B}_0$





Driven rotation of the heated pipe under pressure (Boundary and initial conditions; 121-em with H2O-mixed-Bbar)

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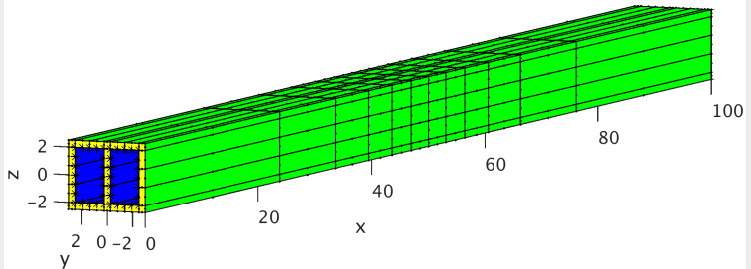
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Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches $x = 0$: Temperature $\theta^A = \hat{\theta} f(t)$

Torque load $\mathbf{W}^A = + \hat{\mathbf{W}}_{\text{drive}} f(t)$

Yellow patches $x = 100$: Temperature $\theta^A = \hat{\theta} f(t)$

Torque load $\mathbf{W}^A = - \hat{\mathbf{W}}_{\text{friction}} f(t)$

Blue patches: 1) Transient pressure $p^A(t) = \hat{p} |\sin(\omega_{\text{load},p} t)|$

2) Outward heat load (cooling) with $\bar{Q}^A(t) = \hat{Q} f(t)$

Green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

$$\varphi_0^A = \mathbf{X}^A \quad \alpha_0^A = 0 \quad \theta_0^A = \theta_\infty \\ \mathbf{v}_0^A = 0 \quad \omega_0^A = 0 \quad \theta_\infty = 298.15$$

Movies

$S_F(\mathbf{X})$

$\Theta(\mathbf{X})$

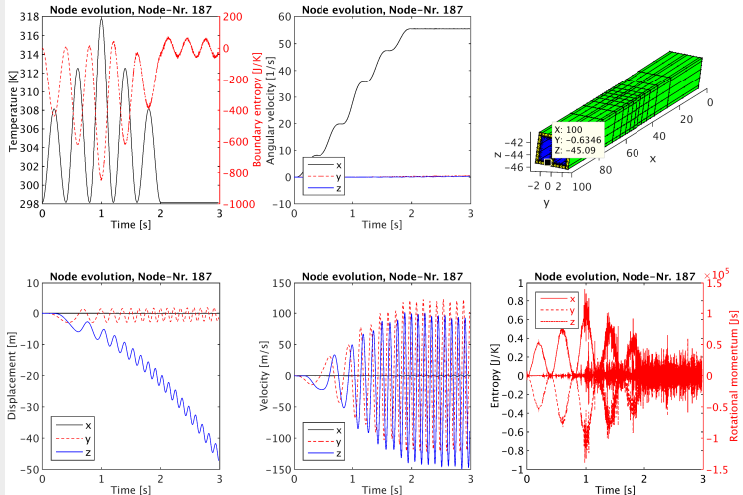
$S_V(\mathbf{X})$

$\bar{C}_V(\mathbf{X})$



Driven rotation of the heated pipe under pressure (Solution versus time; 121-em with H2O-mixed-Bbar)

Time evolutions of a node on the boundaries $\partial_{\Theta}\mathcal{B}_0$ and $\partial_W\mathcal{B}_0$



Weidenhammer [1958]; Gasch, Markert & Pfützner [1979]; Wauer [1981]
Markert & Pfützner [1981]; Dresig & Holzweilig [2011]; Dresig & Fidlin [2014]



Driven rotation of the heated pipe under pressure (Energy-momentum consistency; 121-em with H2O-mixed-Bbar)

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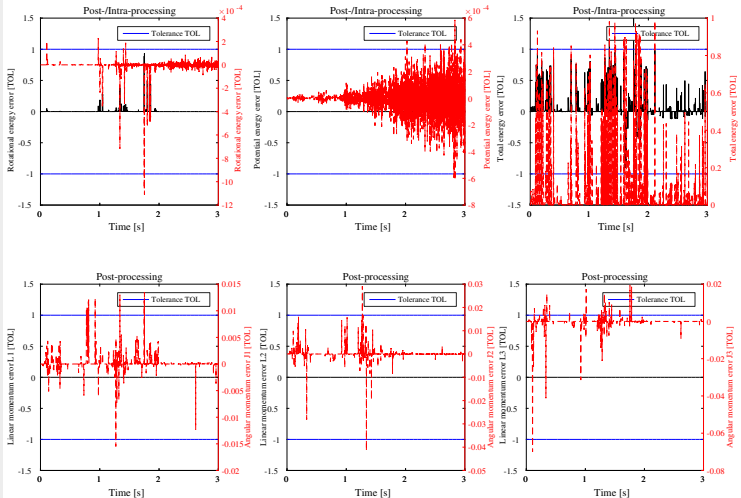
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Error of energy and momentum balance laws versus time



Driven rotation of a composite disc brake rotor

(Boundary and initial conditions; 121-em with H2O-mixed-Bbar)

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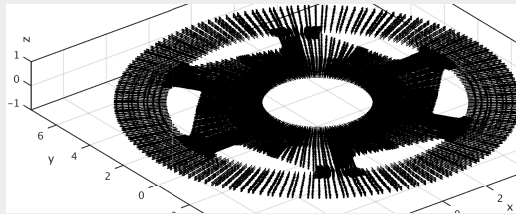
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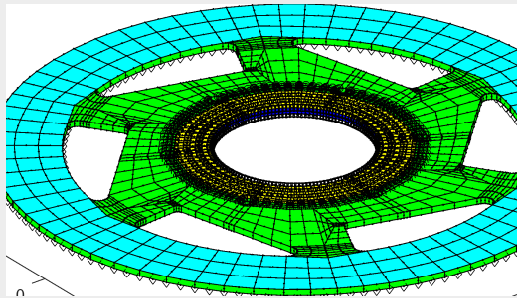
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Material Status Monitor



Boundary Status Monitor



Mechanical boundary

Diamond marker:

$$\partial_T \mathcal{B}_0: \text{traction } \bar{T}_z^A = \hat{T} f(t)$$

Circle and square marker:

$$\partial_\varphi \mathcal{B}_0 \text{ with fixed } z\text{-dof}$$

Cross marker:

$$\partial_W \mathcal{B}_0: \text{torque } \bar{W}_z^A = \hat{W} f(t)$$

Mechanical initial conditions

$$\varphi_0^A = X^A \quad \alpha_0^A = v_0^A = \omega_0^A = 0$$

Thermal boundary

Cyan patches:

$$\partial_Q \mathcal{B}_0: \text{head flux } \bar{Q}^A = \hat{Q} f(t)$$

Green patches:

$$\text{no b.c. (insulation } \bar{Q}^A = 0)$$

Blue patches:

$$\partial_\Theta \mathcal{B}_0: \Theta^A = \Theta_\infty \text{ (heat sink)}$$

Yellow patches:

$$\partial_{\hat{\Theta}} \mathcal{B}_0: \Theta^A = \Theta_\infty \text{ (heat sink)}$$

Thermal initial conditions

$$\Theta_0^A = \Theta_\infty \text{ with } \Theta_\infty = 298.15$$

Driven rotation of a composite disc brake rotor (Solution versus time; 121-em with H2O-mixed-Bbar)

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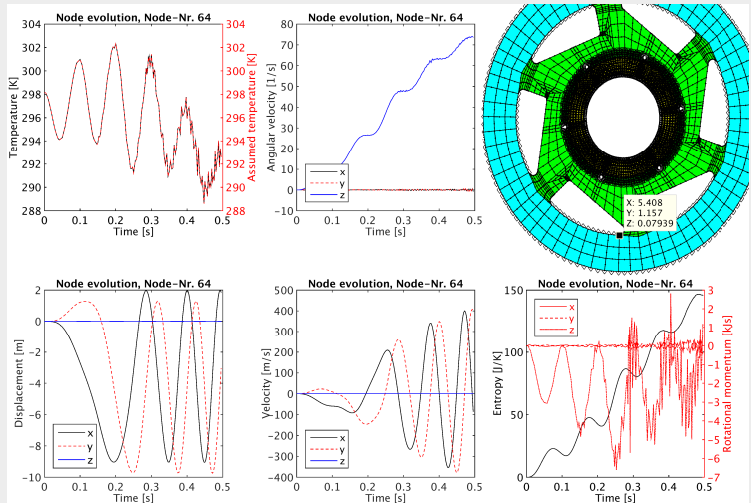
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Time evolutions of a node on the boundary $\partial_Q \mathcal{B}_0$ on the top



Circumferential elongation w.r.t. z -axis

Longitudinal elongation w.r.t. z -axis

Driven rotation of a composite disc brake rotor (Solution versus time; 121-em with H20-mixed-Bbar)

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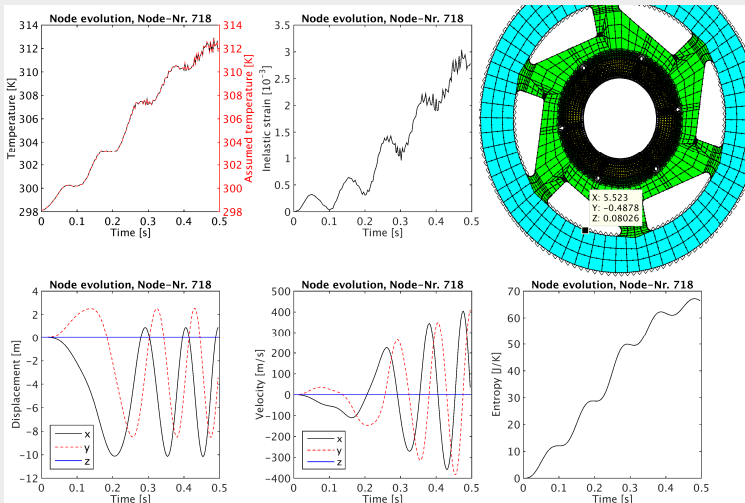
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Driven rotation of a composite turbine rotor (Boundary Status Monitor: back; 121-em with H8-mixed-Bbar)

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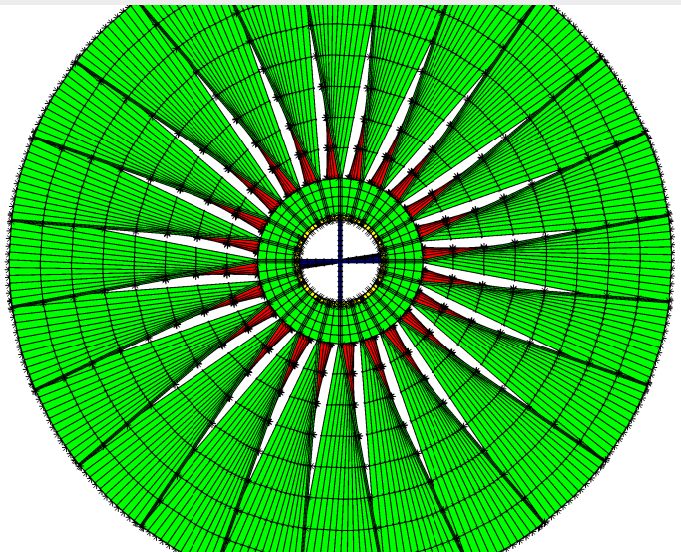
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Driven turbine rotor

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Boundaries: Yellow patches = $\partial_{\partial} \mathcal{B}_0$ (heating); Cross marker = $\partial_W \mathcal{B}_0$ (drive); Green patches = no b.c. (insulation)



derived from the proceeding Noels, Stainier, Ponthot [2004]: A Consistent dissipative time integration scheme for structural dynamics.



Driven rotation of a composite turbine rotor

(Boundary Status Monitor: front; 121-em with H8-mixed-Bbar)

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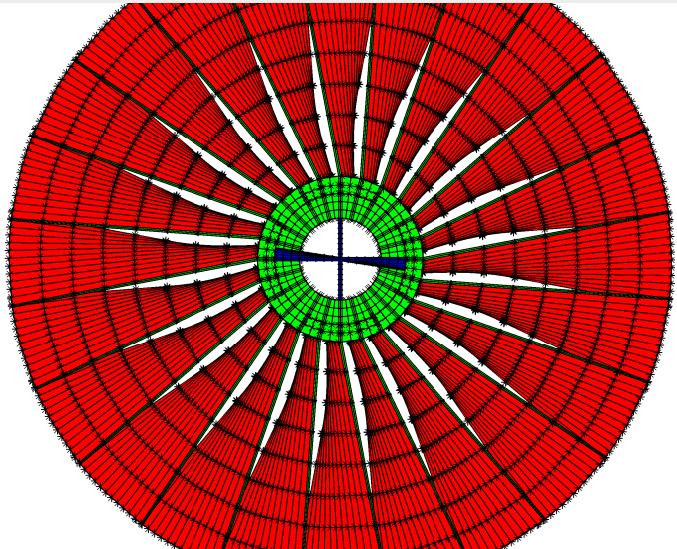
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Boundaries: Red patches = $\partial_Q \mathcal{B}_0$ (cooling); Star marker = $\partial_T \mathcal{B}_0$ (pressure $\rightarrow$ brake); Blue patches = $\partial_\Theta \mathcal{B}_0$ (heat sink)





Driven rotation of a composite turbine rotor (Material Status Monitor; 121-em with H8-mixed-Bbar)

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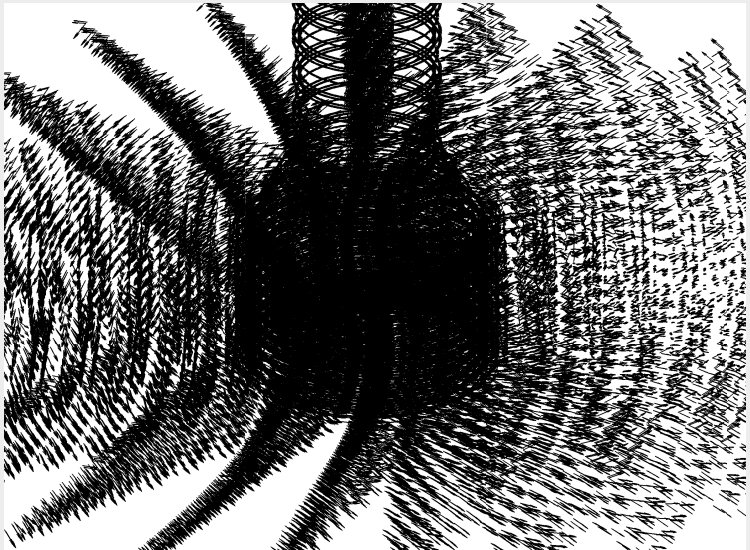
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Fiber directions: Normal in blades, tangential in shaft, crossed in struts



Driven rotation of a composite turbine rotor (Solution versus time; 121-em with H8-mixed-Bbar)

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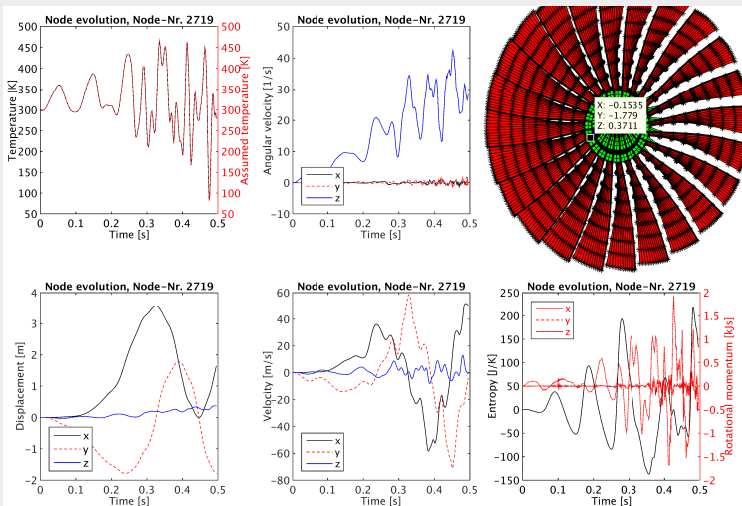
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Time evolutions of a node on the boundary $\partial_W \mathcal{B}_0$ on the front



$$S_F(\mathbf{X})$$

$$S_V(\mathbf{X})$$

Longitudinal elongation w.r.t. z -axis

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Summary

1 Motivation:

- ▶ Rotor dynamic simulations with **meta-materials**
- ▶ Woven composites with a **fiber bending stiffness**

2 Goals:

- ▶ Energy-momentum schemes for **second-gradient** continua
- ▶ derived by a **variational principle** for generalized continua

3 Strategy:

- ▶ **Independent fields** for **rotations** and deformation gradient
- ▶ Discretization of a **mixed** principle of virtual **power**

4 Important result for rotor dynamic applications: **non-standard** boundary conditions

- ▶ **Torque load** from a drive or a bearing with friction
- ▶ Outlook: **Pressure from the Bernoulli effect** $\left[p = \frac{\rho_F}{2} |\omega^A|^2 r^2\right]$

5 Next steps:

- ▶ Investigation of the influences of the **micro-inertia**
- ▶ Introduction of **second-gradient** continuum models



Driven rotation of a composite turbine rotor (Energy-momentum versus time; 121-em with H8-mixed-Bbar)

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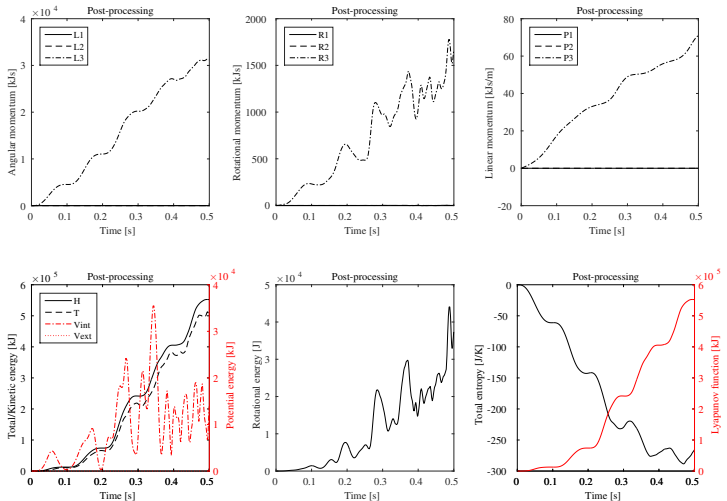
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Time evolutions of energies and momenta



$$\tilde{C}_F(X)$$

$$\tilde{C}_V(X)$$

Circumferential elongation w.r.t. z -axis