A higher-order energy-momentum scheme for a non-isothermal two-phase dissipation model of fibrous composites

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MS07: Higher Order Numerical Methods in Space and Time

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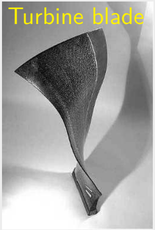
Motivation: Hybrid fiber roving composites

Combination of natural and inorganic fibers: Improved vibration damping

1. Natural fiber: flax, hemp, jute
2. Inorganic fiber: carbon, glass, aramid

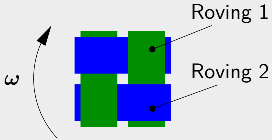
Michael Carus [2015] (nova-Institut GmbH)

Gareth Davies [2018] (Composites Evolution)

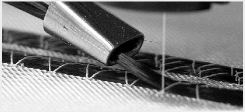


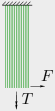
De Luycker, Morestin, Boisse, Marsal [2009]

Meso scale: Represent. volume element (RVE)



Micro scale: Fiber roving with curvature stiffn.





Kai Uhlig [2017] (IPF TU Dresden), recommended by IST TU Chemnitz

Goals: Variational formulations which take into account fiber damping

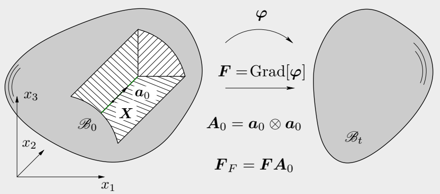
1. Design of energy-momentum schemes for stable dynamical FE simulations

Strategy (I): Matrix and fiber viscoelasticity

(cf. Nedjar [2007])

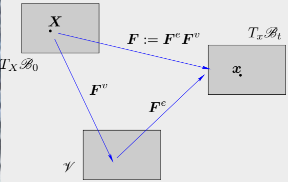
Modelling of anisotropic Cauchy continua with structural tensors

(see e.g. Reese et al. [2001], Klinkel et al. [2005], Schröder et al. [2005])

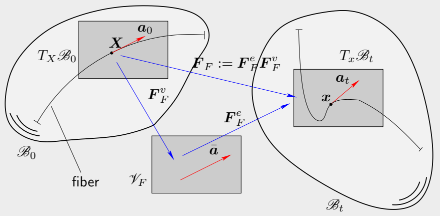


Matrix material with isotropic viscoelasticity

(see e.g. Govindjee & Reese [1997])



Fiber viscoelasticity with linear space $\mathscr{V}_F$ ('intermediate configuration')



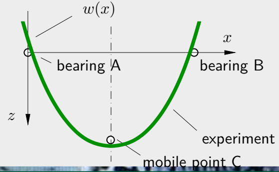
Numerical integration of the viscous evolution

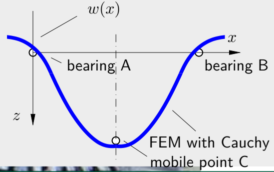
1. **Matrix material model:** solved pointwise (ODE) at spatial quadrature points
2. **Fiber material model:** solved elementwise with spatial element nodes

Strategy (II): Generalized continuum mechanics

(see e.g. Boisse et al. [2018], Asmanoglo & Menzel [2017], Madeo et al. [2015], Feretti et al. [2014], Spencer & Soldatos [2007])

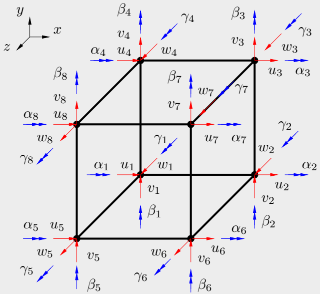
Three point bending test: experiment vs. Cauchy theory $\rightsquigarrow$ 'the need to a second-gradient theory' (see Charmetant et al. [2012], Madeo et al. [2015])



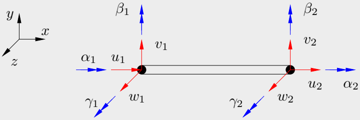


Constrained micropolar continuum with drilling degrees of freedom

(cf. Hughes & Brezzi [1989], Ibrahimbegovic et al. [1990,1991,1993], Choi et al. [2002], Boujelben & Ibrahimbegovic [2017])



Timoshenko beam (with secondary effects)



1. **Does not overestimate bending stiffness**
2. **Does not overestimate natural frequencies**

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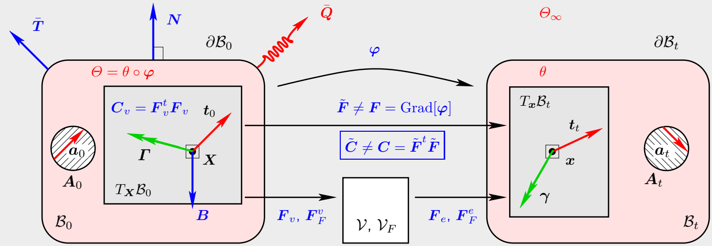
Cauchy continuum model (I)

Transversely isotropic (curvature-stiffness-free) material model



Deformation of fibers and matrix (cf. Klinkel et al. [2005], Nedjar [2007])

1. Deformation of the fibers

$$F_F := a_t \otimes a_0 = \tilde{F} A_0 \qquad A_0 := a_0 \otimes a_0 \qquad C_F := F_F^t F_F := C_F A_0$$

2. Fiber and volume dilatation

$$\tilde{C}_F \neq C_F := \tilde{C} : A_0 \qquad \tilde{C}_V \neq C_V := [\det(\tilde{C})]^{\frac{1}{n_{\dim}}}$$

3. Viscoelastic matrix and fibers

$$F = F_e F_v \qquad C_v = F_v^t F_v \qquad F_F = F_F^e F_F^v \qquad C_F^v = \tilde{C}_F^v A_0$$

Cauchy continuum model (II)

Viscous evolution equations (cf. Reese [2001], Nedjar [2007], Krüger et. al. [2011])

1. Viscous matrix evolution equation and non-equilibrium stress tensor

$$\boxed{\boldsymbol{Y} = \boldsymbol{\Sigma}_v} \qquad \boldsymbol{Y} := -\frac{\partial \Psi_M^{\text{vis}}}{\partial \boldsymbol{C}_v} \qquad D_M^{\text{int}} := \dot{\boldsymbol{C}}_v : \boldsymbol{\Sigma}_v = \dot{\boldsymbol{C}}_v : \mathbb{V}(\boldsymbol{C}_v) : \dot{\boldsymbol{C}}_v \geq 0$$

$$\mathbb{V}(\boldsymbol{C}_v) = \frac{1}{4}\left[V_{\text{vol}} - \frac{V_{\text{dev}}}{n_{\text{dim}}}\right] \boldsymbol{C}_v^{-1} \otimes \boldsymbol{C}_v^{-1} + \frac{V_{\text{dev}}}{4}\,\mathbb{I}^{\text{sym}} : \boldsymbol{C}_v^{-1} \otimes \boldsymbol{C}_v^{-1}$$

2. Viscous fiber evolution equation and non-equilibrium stress

$$\boxed{Y_F = \Sigma_F^v} \qquad Y_F := -\frac{\partial \Psi_F^{\text{vis}}}{\partial C_F^v} \qquad D_F^{\text{int}} := \dot{C}_F^v\,\Sigma_F^v = \dot{C}_F^v\,\frac{V_F}{4\,(C_F^v)^2}\,\dot{C}_F^v \geq 0$$

Thermo-viscoelastic free energy

1. Hyperelastic strain energy

$$\Psi^{\text{ela}}(\tilde{\boldsymbol{C}}, \tilde{C}_V, \tilde{C}_F; \boldsymbol{A}_0) = \hat{\Psi}_M^{\text{iso}}(I_1^{\tilde{C}}, I_2^{\tilde{C}}, I_3^{\tilde{C}}, I_4) + \hat{\Psi}_M^{\text{vol}}(\tilde{C}_V) + \hat{\Psi}_F^{\text{ela}}(\tilde{C}_F)$$

2. Thermoelastic free energy

$$\begin{aligned}\Psi^{\text{the}}(\tilde{C}_V, \tilde{C}_F, \Theta) = \hat{\Psi}^{\text{cap}}(\Theta) &- 2\sqrt{\tilde{C}_V^{\,2-n_{\text{dim}}}}\,\beta_M\,[\Theta - \Theta_\infty]\,\mathrm{D}\hat{\Psi}_M^{\text{vol}}(\tilde{C}_V)\\ &- 2\sqrt{\tilde{C}_F^{2-1}}\quad \beta_F\,[\Theta - \Theta_\infty]\,\mathrm{D}\hat{\Psi}_F^{\text{ela}}(\tilde{C}_F)\end{aligned}$$

3. Viscoelastic free energy

$$\Psi_M^{\text{vis}}(\tilde{\boldsymbol{C}}, \boldsymbol{C}_v) = \hat{\Psi}_M^{\text{ela}}(I_1^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}, I_2^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}, I_3^{\tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1}}) \qquad \Psi_F^{\text{vis}}(\tilde{C}_F, \tilde{C}_F^v) = \Psi_F^{\text{ela}}(\tilde{C}_F[\tilde{C}_F^v]^{-1})$$

Virtual power principle (I)

Total energy balance law

$$\dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}},\dot{\boldsymbol{v}},\dot{\boldsymbol{p}},\dot{\boldsymbol{\alpha}},\dot{\boldsymbol{\omega}},\dot{\boldsymbol{\pi}},\dot{\boldsymbol{C}}_v,\dot{\Theta},\dot{\eta},\dot{\tilde{\boldsymbol{F}}},\dot{\tilde{\boldsymbol{C}}},\dot{\tilde{C}}_V,\dot{\tilde{C}}_F,\dot{\tilde{C}}_F^v}_{\text{temporally continuous}},\underbrace{\tilde{\Theta},\boldsymbol{P},\boldsymbol{\tau}^t_{\mathbf{skw}},\boldsymbol{S},S_V,S_F,\boldsymbol{R},h,\lambda,\boldsymbol{Z},\hat{\omega}}_{\text{temporally discontinuous}})=0 \qquad \dot{\mathcal{H}}=\dot{\mathcal{T}}^{\mathrm{tra}}+\dot{\mathcal{T}}^{\mathrm{rot}}+\dot{\Pi}^{\mathrm{ext}}+\dot{\Pi}^{\mathrm{int}}$$

Kinetic power functionals (motivated by Altenbach et al. [2003], Askes & Aifantis [2011])

$$\dot{\mathcal{T}}^{\mathrm{tra}}(\dot{\boldsymbol{\varphi}},\dot{\boldsymbol{v}},\dot{\boldsymbol{p}}):=\int_{\mathcal{B}_0}[\rho_0\boldsymbol{I}\,\boldsymbol{v}-\boldsymbol{p}]\cdot\dot{\boldsymbol{v}}\,\mathrm{d}V-\int_{\mathcal{B}_0}\dot{\boldsymbol{p}}\cdot[\boldsymbol{v}-\dot{\boldsymbol{\varphi}}]\mathrm{d}V+\int_{\mathcal{B}_0}\boldsymbol{p}\cdot\ddot{\boldsymbol{\varphi}}\,\mathrm{d}V$$

$$\dot{\mathcal{T}}^{\mathrm{rot}}(\dot{\boldsymbol{\alpha}},\dot{\boldsymbol{\omega}},\dot{\boldsymbol{\pi}}):=\int_{\mathcal{B}_0}\left[\rho_0\left[(l_F^2-l_0^2)\,\boldsymbol{A}_0+l_0^2\boldsymbol{I}\right]\boldsymbol{\omega}-\boldsymbol{\pi}\right]\cdot\dot{\boldsymbol{\omega}}\,\mathrm{d}V-\int_{\mathcal{B}_0}\dot{\boldsymbol{\pi}}\cdot[\boldsymbol{\omega}-\dot{\boldsymbol{\alpha}}]\mathrm{d}V+\int_{\mathcal{B}_0}\boldsymbol{\pi}\cdot\ddot{\boldsymbol{\alpha}}\,\mathrm{d}V$$

External power functional

$D^{\mathrm{cdu}}=-\mathrm{Grad}[\ln\Theta]\cdot\boldsymbol{Q}$

$$\begin{aligned}\dot{\Pi}^{\mathrm{ext}}:=&-\int_{\mathcal{B}_0}\rho_0\boldsymbol{B}\cdot\dot{\boldsymbol{\varphi}}\,\mathrm{d}V-\int_{\partial_T\mathcal{B}_0}\bar{\boldsymbol{T}}\cdot\dot{\boldsymbol{\varphi}}\,\mathrm{d}A+\int_{\partial_Q\mathcal{B}_0}\frac{\tilde{\Theta}}{\Theta}\bar{Q}\,\mathrm{d}A\\
&+\int_{\mathcal{B}_0}\frac{1}{\Theta}\mathrm{Grad}[\tilde{\Theta}]\cdot\boldsymbol{Q}\,\mathrm{d}V+\int_{\mathcal{B}_0}\frac{\tilde{\Theta}}{\Theta}\left(D^{\mathrm{cdu}}+D^{\mathrm{int}}\right)\mathrm{d}V+\int_{\mathcal{B}_0}\boldsymbol{\Sigma}_v:\dot{\boldsymbol{C}}_v\,\mathrm{d}V\\
&+\int_{\partial_\Theta\mathcal{B}_0}\lambda\left[\tilde{\Theta}-\Theta_\infty\right]\mathrm{d}A-\int_{\partial_{\dot{\Theta}}\mathcal{B}_0}h\left[\dot{\Theta}-\dot{\bar{\Theta}}\right]\mathrm{d}A-\int_{\partial_\varphi\mathcal{B}_0}\boldsymbol{R}\cdot\left[\dot{\boldsymbol{\varphi}}-\dot{\bar{\boldsymbol{\varphi}}}\right]\mathrm{d}A\\
&-\int_{\partial_\alpha\mathcal{B}_0}\boldsymbol{Z}\cdot\left[\dot{\boldsymbol{\alpha}}-\dot{\bar{\boldsymbol{\alpha}}}\right]\mathrm{d}A-\int_{\partial_\alpha\mathcal{B}_0}\hat{\omega}\cdot\left[\boldsymbol{\epsilon}:\boldsymbol{\tau}^t_{\mathrm{skw}}\right]\mathrm{d}A-\int_{\partial_W\mathcal{B}_0}\bar{\boldsymbol{W}}\cdot\dot{\boldsymbol{\alpha}}\,\mathrm{d}A\\
&+\int_{\mathcal{B}_0}\frac{1}{2}\bar{\boldsymbol{S}}:\dot{\tilde{\boldsymbol{C}}}\,\mathrm{d}V+\int_{\mathcal{B}_0}\frac{1}{2}\bar{S}_V\,\dot{\tilde{C}}_V\,\mathrm{d}V+\int_{\mathcal{B}_0}\frac{1}{2}\bar{S}_F\,\dot{\tilde{C}}_F\,\mathrm{d}V\\
&+\int_{\mathcal{B}_0}\dot{\tilde{C}}_F^v\,\Sigma_F^v\,\mathrm{d}V+\int_{\mathcal{B}_0}\bar{M}_F^v\left[L_F(\dot{\tilde{C}}_F)-L_F(\dot{\tilde{C}}_F^v)\right]\mathrm{d}V\quad\text{with}\quad L_F(\dot{\epsilon})=\frac{\dot{\overline{\ln(\epsilon)}}}{2}\end{aligned}$$

Virtual power principle (II)

Internal power functional

(motivated by Steinmann & Stein [1997])

$$\dot{\Pi}^{\text{int}} := \frac{1}{2}\int_{\mathscr{B}_0} \left\{ \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{\boldsymbol{C}}} + S_V\,\boldsymbol{A}^{\text{vol}} + S_F\,\boldsymbol{A}_0 - \boldsymbol{S} \right] : \dot{\tilde{\boldsymbol{C}}} + \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{C}_V} - S_V \right] \dot{\tilde{C}}_V \right\} \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\eta}\,[\tilde{\Theta} - \Theta]\,\mathrm{d}V$$

$$+ \frac{1}{2}\int_{\mathscr{B}_0} \left[2\,\frac{\partial \hat{\Psi}_F}{\partial \tilde{C}_F} - S_F \right] \dot{\tilde{C}}_F\,\mathrm{d}V + \int_{\mathscr{B}_0} \left\{ [\tilde{\boldsymbol{F}}\boldsymbol{S} - \boldsymbol{P}] : \dot{\tilde{\boldsymbol{F}}} + \frac{\partial \hat{\Psi}_M}{\partial \boldsymbol{C}_v} : \dot{\boldsymbol{C}}_v \right\} \mathrm{d}V + \int_{\mathscr{B}_0} \left[\eta + \frac{\partial \Psi}{\partial \Theta} \right] \dot{\Theta}\,\mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \boldsymbol{P} : \nabla \dot{\boldsymbol{\varphi}}\,\mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\tau}^t_{\text{skw}} : \left[\mathbb{I}^{\text{skw}} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}} \right] \mathrm{d}V + \int_{\mathscr{B}_0} \frac{\partial \Psi_F}{\partial \tilde{C}^v_F}\,\dot{\tilde{C}}^v_F\,\mathrm{d}V \quad \text{with } 2\,\mathbb{I}^{\text{skw}} = \boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}$$

Principle of virtual power

$$\delta_* \dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\boldsymbol{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F, \dot{\tilde{C}}^v_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \boldsymbol{P}, \boldsymbol{\tau}^t_{\text{skw}}, \boldsymbol{S}, S_V, S_F, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Non-standard weak forms

(compared to the Cauchy continuum)

$$\int_{\mathscr{B}_0} \delta_* \boldsymbol{\tau}^t_{\text{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\,\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] \mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \delta_* \boldsymbol{\tau}^t_{\text{skw}}\,\mathrm{d}A \qquad \int_{\partial_\alpha \mathscr{B}_0} \delta_* \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\,\mathrm{d}A = \boldsymbol{0}$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot [\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}]\,\mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \delta_* \dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A \qquad \int_{\partial_\alpha \mathscr{B}_0} \delta_* \boldsymbol{Z} : [\dot{\boldsymbol{\alpha}} - \dot{\bar{\boldsymbol{\alpha}}}]\,\mathrm{d}V = \boldsymbol{0}$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\omega}} \cdot [\rho_0\,[(l_F^2 - l_0^2)\,\boldsymbol{A}_0 + l_0^2 \boldsymbol{I}]\,\boldsymbol{\omega} - \boldsymbol{\pi}]\,\mathrm{d}V = \boldsymbol{0} \qquad \int_{\mathscr{B}_0} \delta_* \dot{\boldsymbol{\pi}} \cdot [\boldsymbol{\omega} - \dot{\boldsymbol{\alpha}}]\,\mathrm{d}V = \boldsymbol{0}$$

$$\int_{\mathscr{B}_0} \delta_* \dot{\tilde{\boldsymbol{F}}} : \left[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}} + \boldsymbol{\tau}^t_{\text{skw}} \tilde{\boldsymbol{F}}^{-t} - \boldsymbol{P} \right] \mathrm{d}V = \boldsymbol{0} \qquad \int_{\mathscr{B}_0} \delta_* \boldsymbol{P} : \left[\dot{\tilde{\boldsymbol{F}}} - \operatorname{Grad}[\dot{\boldsymbol{\varphi}}] \right] \mathrm{d}V = \boldsymbol{0}$$

Additional balance laws

Balance law of angular momentum $\mathcal{J}$ includes additively:

Balance law of spin angular momentum $\mathcal{J}^{\text{spn}}$

Set $\delta_* \dot{\boldsymbol{\alpha}} = \boldsymbol{c} = \text{const.}$:

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \boldsymbol{c} \cdot \left[\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\right] \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\partial_\alpha \mathscr{B}_0} \boldsymbol{c} \cdot \boldsymbol{Z}\,\mathrm{d}A\,\mathrm{d}t + \int_{t_n}^{t_{n+1}}\int_{\partial_W \mathscr{B}_0} \boldsymbol{c} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A\,\mathrm{d}t$$

$$\boxed{\int_{\mathscr{B}_0} \boldsymbol{\pi}_{t_{n+1}}\,\mathrm{d}V - \int_{\mathscr{B}_0} \boldsymbol{\pi}_{t_n}\,\mathrm{d}V = \int_{t_n}^{t_{n+1}}\left[-\int_{\mathscr{B}_0} \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\,\mathrm{d}V + \int_{\partial_\alpha \mathscr{B}_0} \boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \bar{\boldsymbol{W}}\,\mathrm{d}A\right]\mathrm{d}t}$$

Balance law of total energy $\mathcal{H}$ includes additively:

Balance law of rotational kinetic energy $\mathcal{T}^{\text{rot}}$

Set $\delta_* \dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}}$ and $\delta_* \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}$ and $\delta_* \dot{\boldsymbol{\pi}} = \dot{\boldsymbol{\pi}}$ and $\delta_* \boldsymbol{\tau}^t_{\text{skw}} = \boldsymbol{\tau}^t_{\text{skw}}$:

Eqn. (1): $$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \dot{\boldsymbol{\alpha}} \cdot \left[\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\right] \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\partial_\alpha \mathscr{B}_0} \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\,\mathrm{d}A\,\mathrm{d}t + \int_{t_n}^{t_{n+1}}\int_{\partial_W \mathscr{B}_0} \dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A\,\mathrm{d}t$$

Eqn. (2): $$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \boldsymbol{\tau}^t_{\text{skw}} : \left[\mathbb{I}^{\text{skw}} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \boldsymbol{\epsilon} \cdot \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \underbrace{\boldsymbol{\tau}^t_{\text{skw}}}_{\boldsymbol{0}}\,\mathrm{d}A\,\mathrm{d}t$$

Eqn. (3): $$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \dot{\boldsymbol{\omega}} \cdot \left[\boldsymbol{J}\boldsymbol{\omega} - \boldsymbol{\pi}\right] \mathrm{d}V\,\mathrm{d}t = \boldsymbol{0}$$

Eqn. (4): $$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \dot{\boldsymbol{\pi}} \cdot \left[\boldsymbol{\omega} - \dot{\boldsymbol{\alpha}}\right] \mathrm{d}V\,\mathrm{d}t = \boldsymbol{0}$$

$$\boxed{\begin{aligned}\mathcal{T}^{\text{rot}}_{n+1} - \mathcal{T}^{\text{rot}}_{n} \equiv{}& \frac{1}{2}\int_{\mathscr{B}_0} \boldsymbol{\omega}_{t_{n+1}} \cdot \boldsymbol{J}\boldsymbol{\omega}_{t_{n+1}}\mathrm{d}V - \frac{1}{2}\int_{\mathscr{B}_0} \boldsymbol{\omega}_{t_n} \cdot \boldsymbol{J}\boldsymbol{\omega}_{t_n}\mathrm{d}V \\ ={}& \int_{t_n}^{t_{n+1}}\left[\int_{\mathscr{B}_0} \boldsymbol{\tau}^t_{\text{skw}} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1}\,\mathrm{d}V + \int_{\partial_\alpha \mathscr{B}_0} \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A\right]\mathrm{d}t\end{aligned}}$$

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Discrete viscous fiber evolution equation

Incremental principle of virtual power

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}\, \mathrm{d}t \equiv \int_{t_n}^{t_{n+1}} \left[\delta_* \dot{\mathcal{T}} + \delta_* \dot{\mathit{\Pi}}^{\text{int}} + \delta_* \dot{\mathit{\Pi}}^{\text{ext}}\right] \mathrm{d}t = 0$$

Fully weak viscous fiber evolution equation

$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta_* \dot{\tilde{C}}^v_F \left[-Y_F - \frac{\bar{M}^v_F}{2\,\tilde{C}^v_F} + \frac{V_F}{2\,\tilde{C}^v_F}\, L_F(\dot{\tilde{C}}^v_F)\right] \mathrm{d}V\, \mathrm{d}t = 0$$

Discrete weak form on the reference element

$$\int_0^1 \int_{\mathscr{B}_\square} \delta_* \overset{\circ}{\overline{C^v_F}} \left[-Y_F - \frac{\bar{M}^v_F}{2\,\tilde{C}^v_F} + \frac{V_F}{2\,\tilde{C}^v_F}\, L_F(\dot{\tilde{C}}^v_F)\right] \mathrm{d}V_\square\, \mathrm{d}\alpha = 0$$

Galerkin approximations in space and time

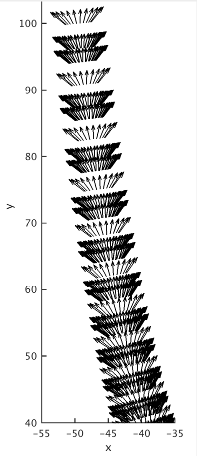
$$\delta_* \overset{\circ}{\overline{\tilde{C}^v_F}}(\boldsymbol{\chi}, \alpha) = \sum_{J=1}^{k} \sum_{B=1}^{\tilde{n}_{\text{node}}} \tilde{M}^J(\alpha) \tilde{N}_B(\boldsymbol{\chi})\, [\tilde{C}^v_F]^B_J \qquad \tilde{C}^v_F(\boldsymbol{\chi}, \alpha) = \sum_{I=1}^{k+1} \sum_{A=1}^{\tilde{n}_{\text{node}}} M^I(\alpha) \tilde{N}_A(\boldsymbol{\chi})\, [C^v_F]^A_I$$

Algorithmic fiber stress $\rightsquigarrow$ viscous evolution/equations of motion

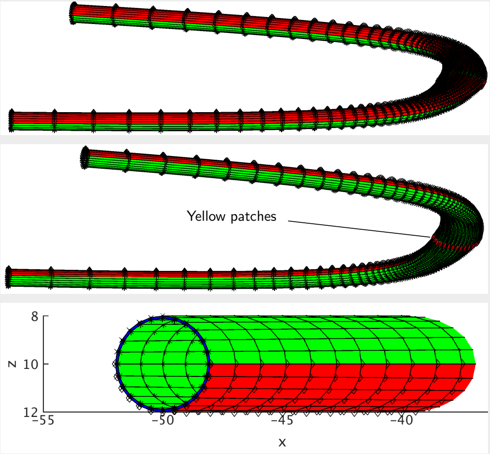
$$\bar{M}^v_F := \mu^v_F \left[L_F(\dot{\tilde{C}}_F) - L_F(\dot{\tilde{C}}^v_F)\right] \qquad \mu^v_F := \frac{\mathit{\Psi}^{\text{vis}}_{F_{t_{n+1}}} - \mathit{\Psi}^{\text{vis}}_{F_{t_n}} - \int_0^1 2\, Y_F\, \tilde{C}^v_F \left[L_F(\dot{\tilde{C}}_F) - L_F(\dot{\tilde{C}}^v_F)\right] \mathrm{d}\alpha}{\int_0^1 \left[L_F(\dot{\tilde{C}}_F) - L_F(\dot{\tilde{C}}^v_F)\right]^2 \mathrm{d}\alpha}$$

Dynamic bending/torsion of a long curved pipe
(Initial-boundary conditions; 121-em with H8-mixed-Bbar-drill)

Material Status Monitor



Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches: Temperature $\Theta^A = \Theta_\infty$, Position z-dofs fixed (bearing)
Blue patches : Temperature $\Theta^A = \Theta_\infty$, Torque load $W_x^A = \pm\hat{W}_x\, f(t)$
Green/red patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

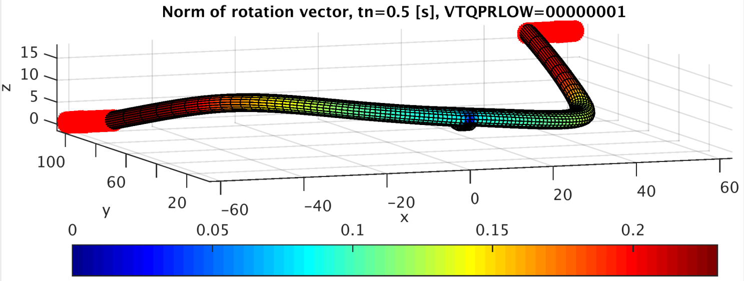
Initial conditions

$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \quad \boldsymbol{\alpha}_0^A = \mathbf{0} \quad \Theta_0^A = \Theta_\infty$

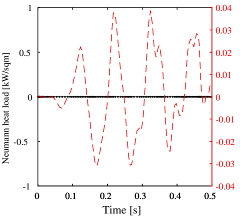
$\boldsymbol{v}_0^A = \mathbf{0} \quad \boldsymbol{\omega}_0^A = \mathbf{0} \quad \Theta_\infty = 298.15$

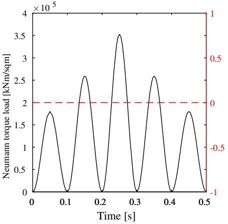
Dynamic bending/torsion of a long curved pipe
(Motion and loads/reactions; 121-em with H8-mixed-Bbar-drill)

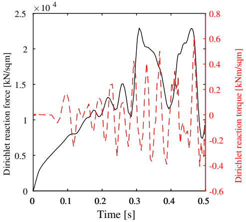
Current configuration with norm of rotation vector at $t_n = 0.5\,\mathrm{s}$



Time evolution of the loads





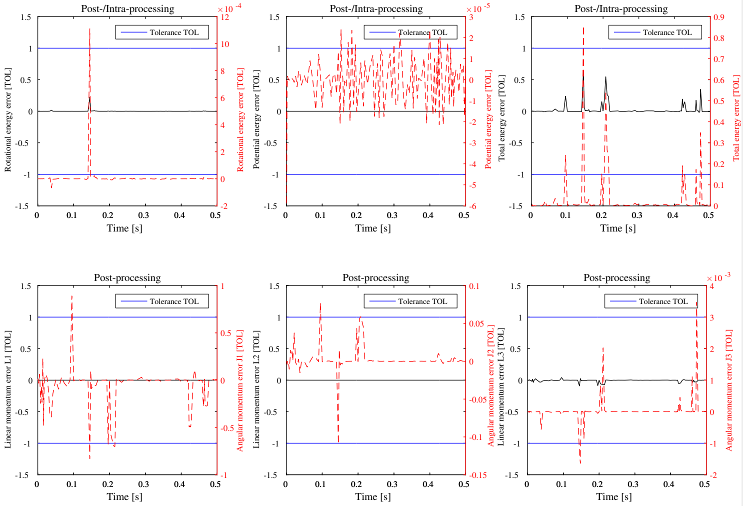


TECHNISCHE UNIVERSITÄT CHEMNITZ

Dynamic bending/torsion of a long curved pipe (Energy-momentum error; 121-em with H8-mixed-Bbar-drill)

Michael Groß, Julian Dietzsch, Chris Röbiger

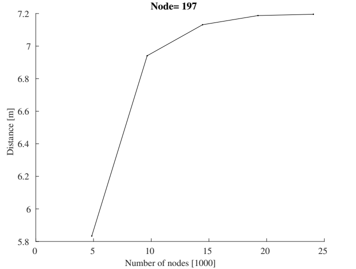
Errors of energy and momentum balance laws versus time (TOL = 1 J)

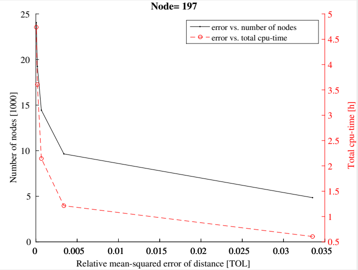


Dynamic bending/torsion of a long curved pipe

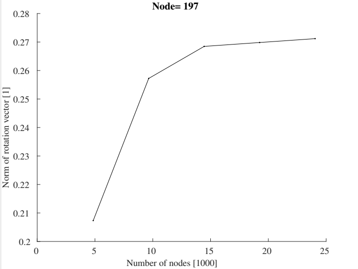
(Mesh convergence at $t_n = 0.5\,\mathrm{s}$; 121-em with H8-mixed-Bbar-drill)

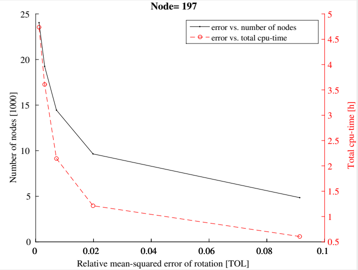
Displacement of right pipe end $(T = [0, 0.5]\,\mathrm{s})$





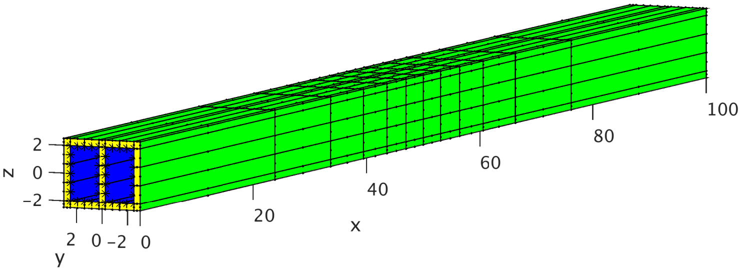
Rotation of right pipe end $(T = [0, 0.5]\,\mathrm{s})$





Driven rotation of a square pipe under pressure
(Initial-boundary conditions; 121-em with H20-mixed-Bbar-drill)

Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches $x = 0$: Temp. $\Theta^A = \hat{\Theta}\, f(t)$, Torque $\boldsymbol{W}^A = +\hat{\boldsymbol{W}}_{\text{dri}}\, f(t)$

Yellow patches $x = 100$: Temp. $\Theta^A = \hat{\Theta}\, f(t)$, Torque $\boldsymbol{W}^A = -\hat{\boldsymbol{W}}_{\text{fri}}\, f(t)$

Blue patches: 1) Outward heat flux (cooling) with $\bar{Q}^A(t) = \hat{Q}\, f(t)$

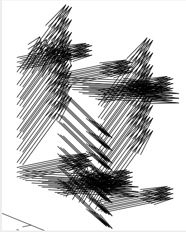
2) Pressure follower load $p^A(t) = \hat{p}\,|\sin(\omega_{\text{load,p}}\, t)|$

Green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

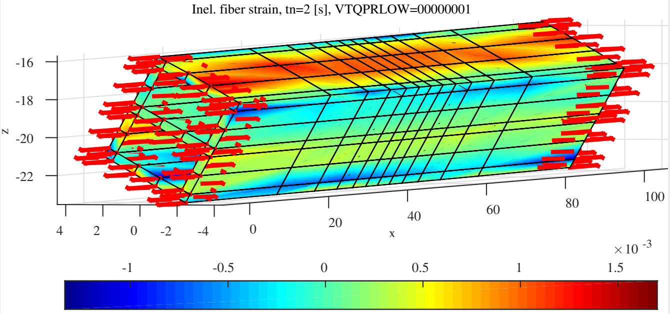
$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \quad \boldsymbol{\alpha}_0^A = \boldsymbol{0} \quad \Theta_0^A = \Theta_\infty \quad \boldsymbol{v}_0^A = \boldsymbol{0} \quad \boldsymbol{\omega}_0^A = \boldsymbol{0} \quad \Theta_\infty = 298.15$

Material Status Monitor

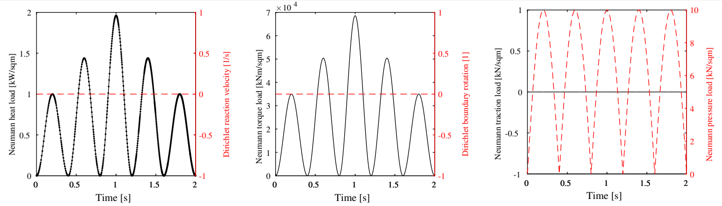


Driven rotation of a square pipe under pressure
(Motion and loads; 121-em with H20-mixed-Bbar-drill)

Current configuration with inelastic fiber strain at $t_n = 2.0\,\mathrm{s}$



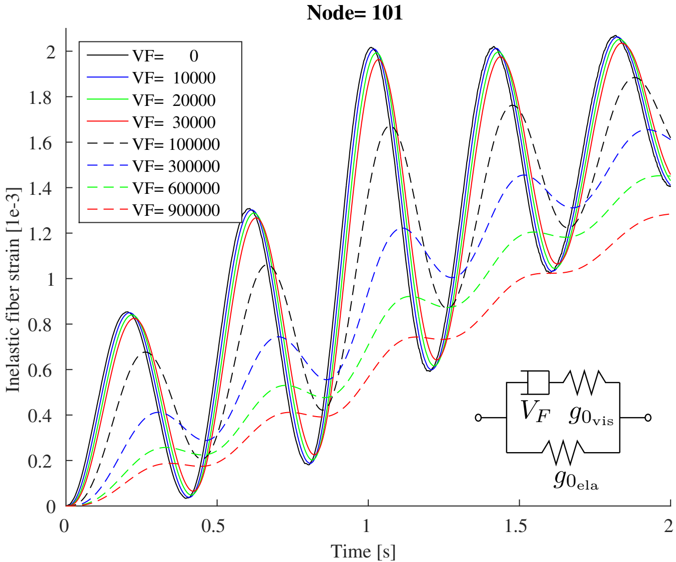
Time evolution of the loads



Driven rotation of a square pipe under pressure

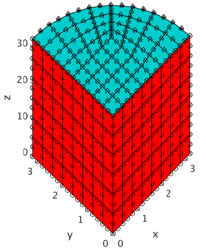
(Comparison of fiber viscosity; 121-em with H20-mixed-Bbar-drill)

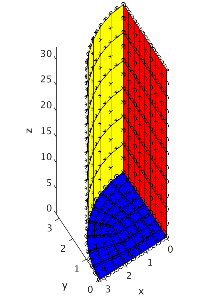
Inelastic fiber strain in $T = [0, 2.0]\,\mathrm{s}$ (Recall viscously damped oscillations)



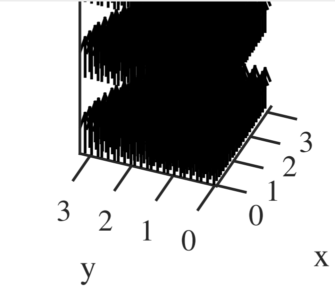
Dynamic impact test with Taylor's bar
(Initial-boundary conditions; 121-em with H20-mixed-Bbar-drill)

Boundary Status Monitor





Material Status Monitor



Dirichlet and Neumann boundary conditions

Yellow, blue patches: Temp. $\Theta^A = \Theta_\infty$

Red patches: 1) y-z face $\rightsquigarrow$ x-dofs fixed   2) x-z face $\rightsquigarrow$ y-dofs fixed

Blue patches: x-y face $\rightsquigarrow$ z-dofs fixed

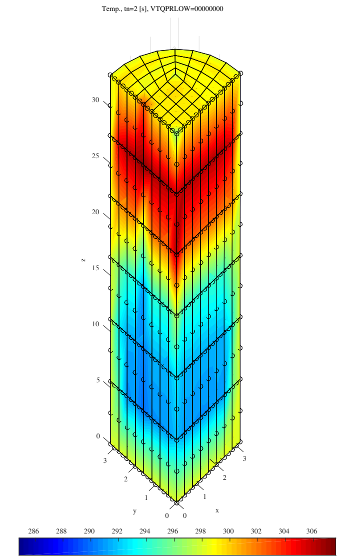
Red, cyan patches: thermal insulation $\bar{Q}^A := 0$ (no thermal b.c.)

Initial conditions ($\Theta_\infty = 298.15$)

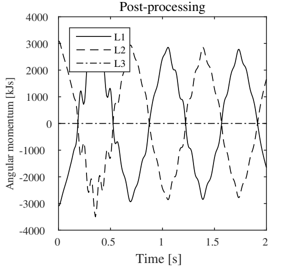
$$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \quad \boldsymbol{\alpha}_0^A = \boldsymbol{0} \quad \Theta_0^A = \Theta_\infty \quad \boldsymbol{v}_0^A = -8\,\boldsymbol{e}_z \quad \boldsymbol{\omega}_0^A = \boldsymbol{0}$$

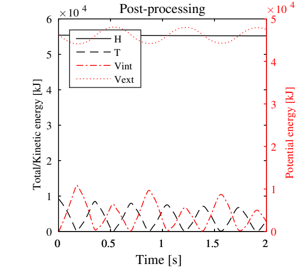
Dynamic impact test with Taylor's bar
(Motion and energies/momenta; 121-em with H20-mixed-Bbar-drill)

Current temperature at $t_n = 2.0\,\mathrm{s}$



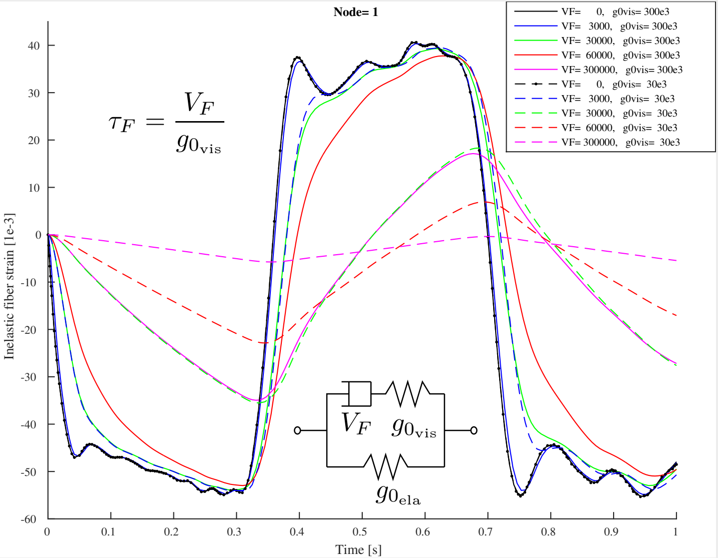
Time evolution of energies/momenta





Dynamic impact test with Taylor's bar
(Comparison of fiber stiffness; 121-em with H20-mixed-Bbar-drill)

Inelastic fiber strain at bottom
(Recall meaning of the damping factor)



TECHNISCHE UNIVERSITÄT CHEMNITZ

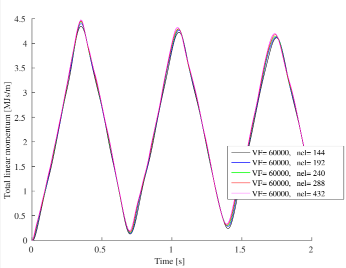
A higher-order energy-momentum scheme for a non-isothermal two-phase dissipation model of fibrous composites

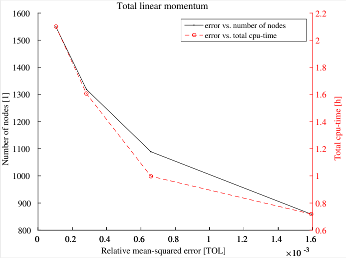
Michael Groß, Julian Dietzsch, Chris Röbiger

Dynamic impact test with Taylor's bar

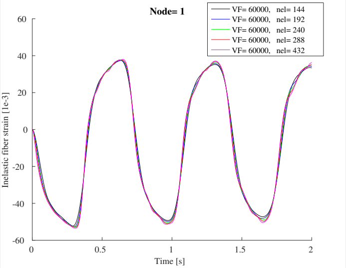
(Mesh convergence at $t_n = 2\,\mathrm{s}$; 121-em with H20-mixed-Bbar-drill)

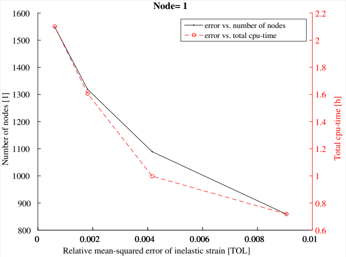
Total linear momentum: Time evolutions and relative error





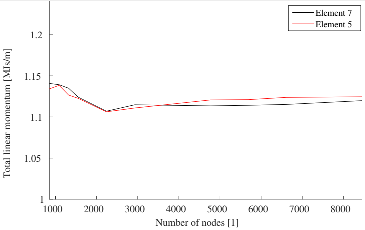
Inelastic fiber strain at bottom: Time evolutions and rela. error

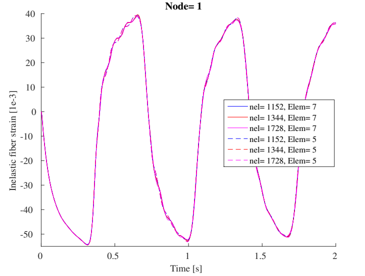




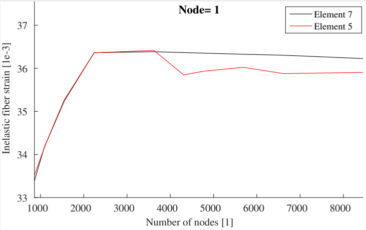
Dynamic impact test with Taylor's bar
(Mesh convergence at $t_n = 2\,\mathrm{s}$; 121-em with H20-mixed-Bbar-drill)

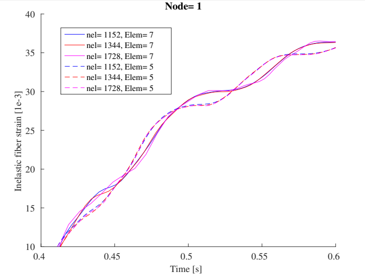
Total linear momentum: Elem. 7 = drill, Element 5 = no drill





Inelastic fiber strain: Elem. 7 = drill, Element 5 = no drill



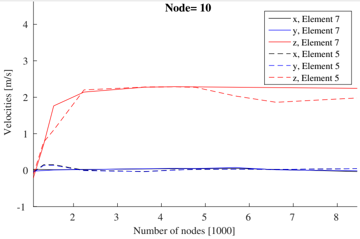


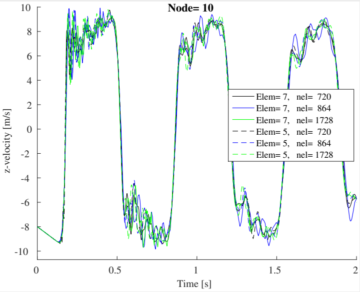
TECHNISCHE UNIVERSITÄT CHEMNITZ

Dynamic impact test with Taylor's bar

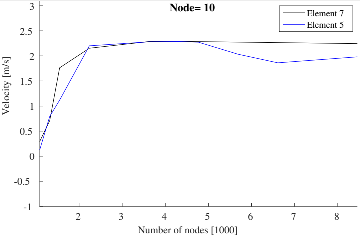
(Mesh convergence at $t_n = 2\,\mathrm{s}$; 121-em with H20-mixed-Bbar-drill)

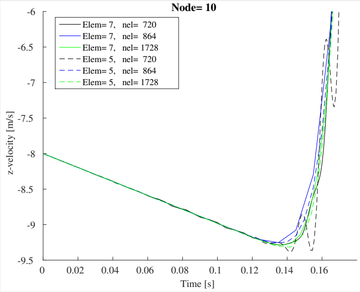
Material velocity: Element 7 = drill, Element 5 = no drill





Material velocity: Element 7 = drill, Element 5 = no drill

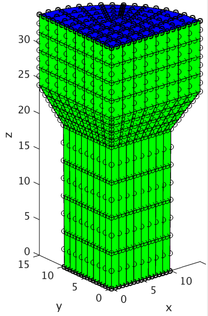


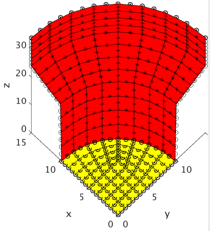


Tension bar: Displacement-controlled tensile test
(Initial-boundary conditions; 121-em with H20-mixed-Bbar-drill)

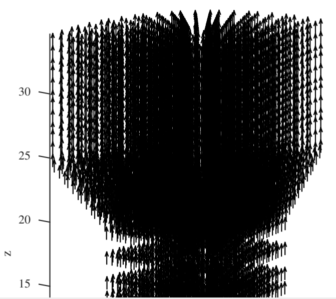
Michael Groß, Julian Dietzsch, Chris Röbiger

Boundary Status Monitor





Material Status Monitor



Dirichlet and Neumann boundary conditions

Blue patches: Temp. $\Theta^A = \Theta_\infty$, x^A,y^A-dofs fixed, $u_z^A = 0.1\, f(t)$

Yellow patches: Temp. $\Theta^A = \Theta_\infty$, x^A,y^A-dofs fixed, $u_z^A = -0.1\, f(t)$

Green patches: 1) y-z face $\rightsquigarrow$ x-dofs fixed, 2) x-z face $\rightsquigarrow$ y-dofs fixed

Green/Red patches: thermal insulation $\bar{Q}^A := 0$

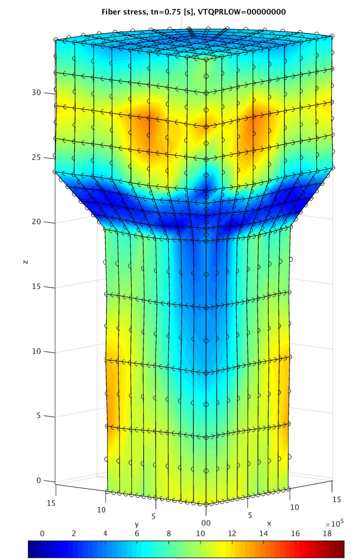
Initial conditions

$$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \quad \boldsymbol{\alpha}_0^A = \boldsymbol{0} \quad \Theta_0^A = \Theta_\infty \quad \boldsymbol{v}_0^A = \boldsymbol{0} \quad \boldsymbol{\omega}_0^A = \boldsymbol{0} \quad \Theta_\infty = 298.15$$

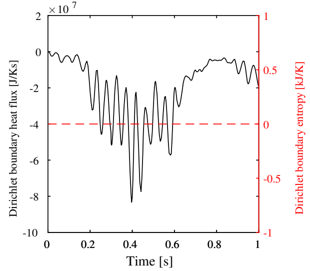
Tension bar: Displacement-controlled tensile test

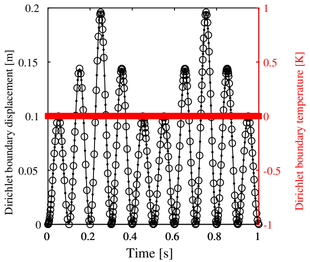
(Motion and loads; 121-em with H20-mixed-Bbar-drill)

Total fiber stress at $t_n = 0.75\,\mathrm{s}$



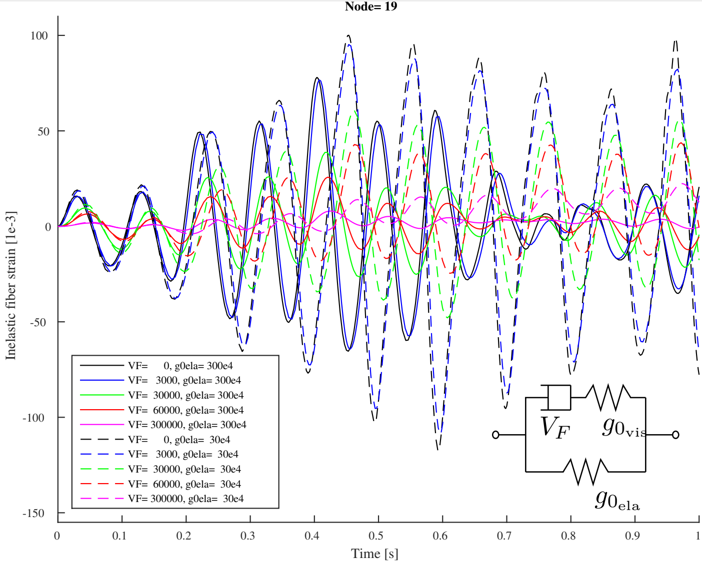
Time evolution of loads/reactions





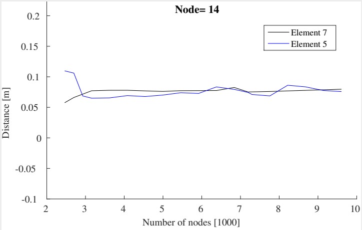
Tension bar: Displacement-controlled tensile test
(Comparison of fiber stiffness; 121-em with H20-mixed-Bbar-drill)

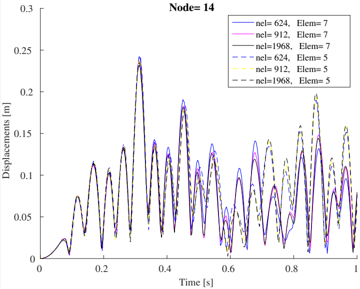
Inelastic fiber strain at bottom (Recall driven damped oscillations)



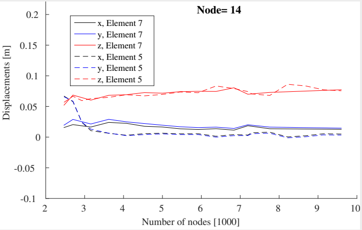
Tension bar: Displacement-controlled tensile test
(Mesh convergence at $t_n = 1\,\mathrm{s}$; 121-em with H20-mixed-Bbar-drill)

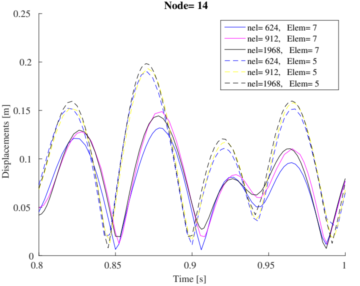
Current position: Element 7 = drill, Element 5 = no drill





Current position: Element 7 = drill, Element 5 = no drill

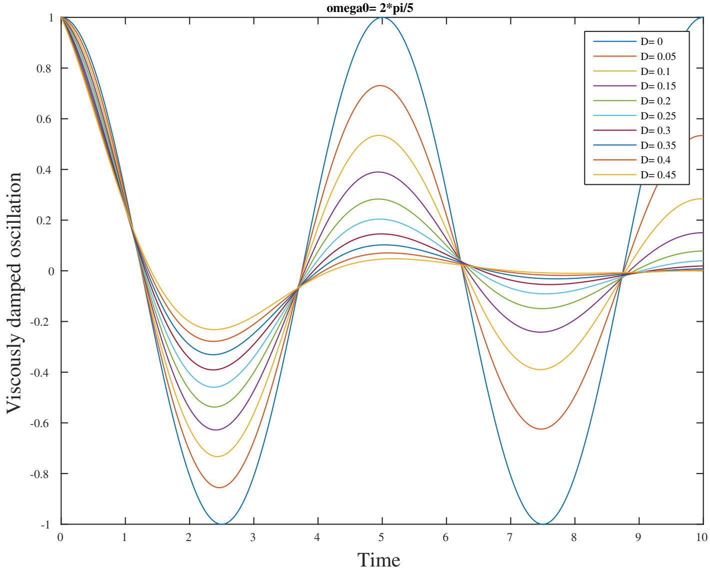




Summary

1. Motivation:
   - Dynamical FE simulations for hybrid fiber roving composites
   - with fiber damping and secondary effects (curvature/inertia)
2. Goals:
   - Energy-momentum schemes for generalized continua
   - derived by a variational principle with drilling DoF's
3. Strategy:
   - Variational-based two-phase thermo-viscoelasticity model with
   - mixed fields for rotation vectors and a skew-symmetric stress
4. Current results:
   - Finite damping behaviour in fiber direction can be simulated by
   - finite elements with drilling DoF's and well mesh convergence
   - This new fiber damping model also generalizes linear effects
5. Next step:
   - Introduction of strain energy function for fiber curvature effects

Viscously damped oscillation



Driven viscously damped oscillation