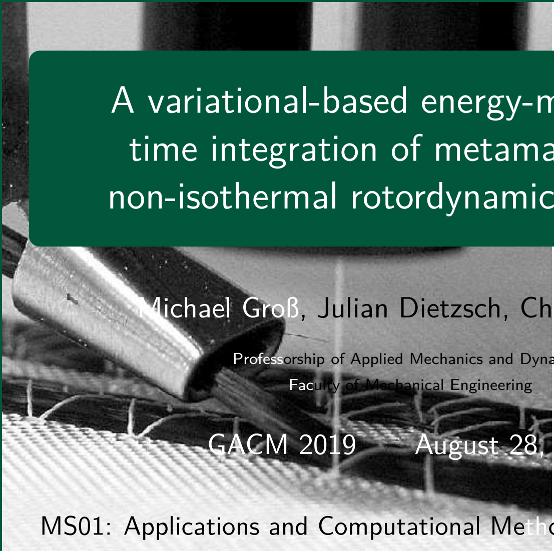

A variational-based energy-momentum time integration of metamaterials in non-isothermal rotordynamical systems

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GACM 2019 August 28, 2019

MS01: Applications and Computational Methods in Metamaterials

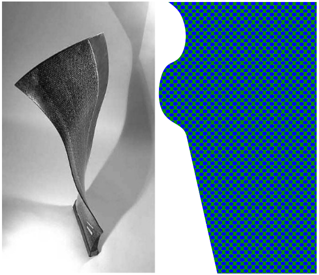
Acknowledgment: This research is provided by DFG under the grant GR 3297

Motivation: metamaterials in rotating systems

Motivation: increasing application of fiber roving composites

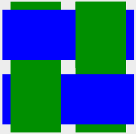
1. Hollow shafts
2. Turbine/pump rotors
3. Disc brake rotors

Macroscopic scale: turbine blade preform

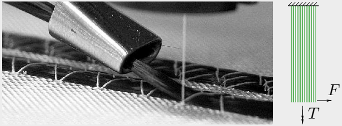


De Luycker, Morestin, Boisse, Marsal [2009]

Mesoscopic scale: Volume Element (RVE)



Micro scale: Roving with curvature stiffness



Kai Uhlig [2017] (IPF TU Dresden), recommended by IST TU Chemnitz

Goals: FE simulations taking into account the fiber bending/twisting (curvature) stiffness

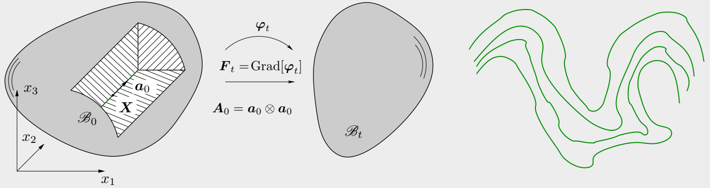
1. We design dynamic variational principles for continua with multi-scale effects
2. We derive an energy-momentum time integration for a more exact simulation

Strategy (I): Generalized continuum mechanics

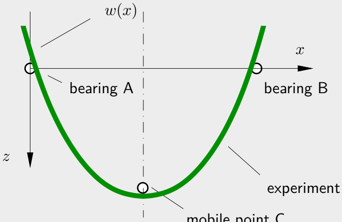
(see e.g. Boisse et al. [2018], Asmanoglo & Menzel [2017], Madeo et al. [2015], Feretti et al. [2014], Spencer & Soldatos [2007])

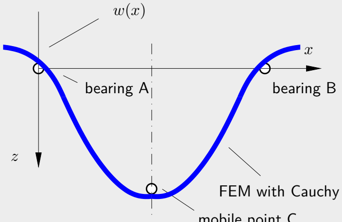
Formulation of anisotropic Cauchy continua with structural tensors

(see e.g. Reese, Raible & Wriggers [2001], Klinkel, Sansour & Wagner [2005], Schröder, Neff & Balzani [2005])



Three point bending test: experiment vs. Cauchy theory $\rightsquigarrow$ 'the need to a second-gradient theory' (see Charmetant et al. [2012], Madeo et al. [2015])





Strategy (II): Principle of virtual power

Principle of virtual work / Hamilton's principle (cf. Dimarogonas et al. [2013])

1. Simo-Taylor-Pister functional (Simo, Taylor, Pister [1985])

$$\Pi_{\mathrm{STP}}(\boldsymbol{\varphi}, \tilde{J}, p) := \int_{\mathscr{B}_0} \Psi^{\mathrm{iso}}(\boldsymbol{\varphi})\,\mathrm{d}V + \int_{\mathscr{B}_0} \Psi^{\mathrm{vol}}(\tilde{J})\,\mathrm{d}V - \int_{\mathscr{B}_0} p\,[\tilde{J} - \det(\nabla\boldsymbol{\varphi})]\,\mathrm{d}V$$

2. Spatial weak form for $\tilde{J}$

$$\int_{\mathscr{B}_0} \delta p\, G(\boldsymbol{\varphi}, \tilde{J})\,\mathrm{d}t = 0 \quad \text{with} \quad G(\boldsymbol{\varphi}, \tilde{J}) := \tilde{J} - \det(\nabla\boldsymbol{\varphi})$$

Energy-momentum time integration (motivated by Betsch & Steinmann [2002], Armero [2008])

1. Preservation of balance law of total energy $H := T(\dot{\boldsymbol{\varphi}}) + \Psi^{\mathrm{iso}}(\boldsymbol{\varphi}) + \Psi^{\mathrm{vol}}(\tilde{J})$

$$\mathcal{H}_{t_{n+1}} - \mathcal{H}_{t_n} = \int_{t_n}^{t_{n+1}} \int_{\mathcal{B}_0} \left[\frac{\partial H}{\partial \dot{\boldsymbol{\varphi}}} \cdot \ddot{\boldsymbol{\varphi}} + \frac{\partial H}{\partial \boldsymbol{\varphi}} \cdot \dot{\boldsymbol{\varphi}} + \frac{\partial H}{\partial \tilde{J}}\,\dot{\tilde{J}} \right] \mathrm{d}V\,\mathrm{d}t$$

2. Time evolution of volume dilatation $\tilde{J}$

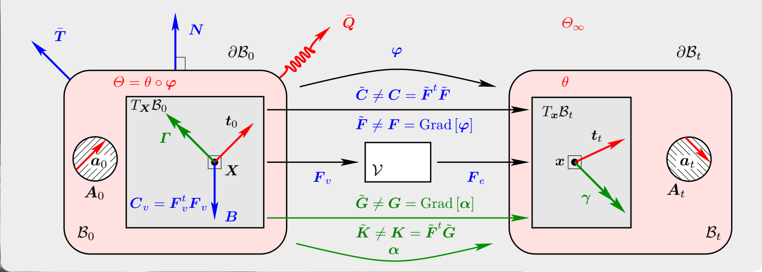
$$\int_{t_n}^{t_{n+1}} \int_{\mathscr{B}_0} \delta p\, \dot{G}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}})\,\mathrm{d}t = 0 \quad \text{with} \quad \dot{G}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}}) := \dot{\tilde{J}} - \overline{\det(\dot{\nabla\boldsymbol{\varphi}})}$$

Variational principle of choice (cf. Schröder & Kuhl [2015])

$$\int_{t_n}^{t_{n+1}} \delta_* \dot{\mathcal{H}}(\dot{\boldsymbol{\varphi}}, \dot{\tilde{J}}, p)\,\mathrm{d}t = 0 \quad \text{with} \quad \mathcal{H} := \frac{1}{2}\int_{\mathcal{B}_0} \rho_0\, \dot{\boldsymbol{\varphi}} \cdot \dot{\boldsymbol{\varphi}}\,\mathrm{d}V + \Pi_{\mathrm{STP}}(\boldsymbol{\varphi}, \tilde{J}, p)$$

Generalized continuum formulation (I)

Transversely isotropic constrained micropolar model (cf. Steinmann & Stein [1997])



Deformation of fibers and matrix (cf. Klinkel, Sansour & Wagner [2005])

1. Fiber stretch per unit length with right Cauchy-Green tensor

$$C_F := \boldsymbol{a}_t \cdot \boldsymbol{a}_t \equiv \boldsymbol{a}_0 \boldsymbol{C} \boldsymbol{a}_0 \equiv \mathrm{tr}\,[\boldsymbol{A}_0 \boldsymbol{C}] \qquad \boldsymbol{A}_0 := \boldsymbol{a}_0 \otimes \boldsymbol{a}_0 \qquad \boldsymbol{C} := \boldsymbol{F}^t \boldsymbol{F}$$

2. Fiber twist per unit length with curvature-twist tensor

$$T_F := \boldsymbol{a}_t \cdot \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{x}} \cdot \boldsymbol{a}_t \equiv \boldsymbol{a}_0 \cdot \boldsymbol{F}^t \boldsymbol{G} \cdot \boldsymbol{a}_0 \equiv \mathrm{tr}\,[\boldsymbol{A}_0 \boldsymbol{K}] \qquad \boldsymbol{K}_F := \boldsymbol{A}_0 \boldsymbol{K} \qquad \boldsymbol{K} := \boldsymbol{F}^t \boldsymbol{G}$$

3. Fiber bending per unit length

$$B_F := \boldsymbol{a}_0 \boldsymbol{K} \cdot \boldsymbol{a}_0 \boldsymbol{K} \equiv \boldsymbol{a}_0 \cdot \boldsymbol{K} \boldsymbol{K}^t \cdot \boldsymbol{a}_0 \equiv \boldsymbol{K}_F : \boldsymbol{K}_F$$

Generalized continuum formulation (II)

Thermo-viscoelastic free energy (cf. Beheshti [2017], Grbcic et al. [2018])

1. Hyperelastic strain energy $\Psi^{\text{ela}}(\tilde{\boldsymbol{C}}, \tilde{C}_V, \tilde{C}_F, \tilde{\boldsymbol{K}}; \boldsymbol{A}_0)$

$$\tilde{C}_V \neq C_V := [\det(\tilde{\boldsymbol{C}})]^{\frac{1}{n_{\text{dim}}}} \qquad \tilde{C}_F \neq C_F := \tilde{\boldsymbol{C}} : \boldsymbol{A}_0$$

$$\Psi^{\text{cur}}_{SVK}(\tilde{\boldsymbol{K}}; \boldsymbol{A}_0) = \frac{\mu_{\text{twi}}}{2}(T_F)^2 + \mu_{\text{ben}} B_F = \frac{\mu_{\text{twi}}}{2}(\text{tr}[\boldsymbol{K}_F])^2 + \mu_{\text{ben}} \boldsymbol{K}_F : \boldsymbol{K}_F$$

2. Thermoelastic free energy

$$\Psi^{\text{the}}(\Theta, \tilde{C}_V, \tilde{C}_F) = \hat{\Psi}^{\text{cap}}(\Theta) - 2\sqrt{\tilde{C}_V{}^{2-n_{\text{dim}}}}\, \beta_M [\Theta - \Theta_\infty]\, \mathrm{D}\hat{\Psi}^{\text{vol}}_M(\tilde{C}_V) - 2\sqrt{\tilde{C}_F^{2-1}}\, \beta_F [\Theta - \Theta_\infty]\, \mathrm{D}\hat{\Psi}^{\text{ela}}_F(\tilde{C}_F)$$

3. Viscoelastic free energy

$$\Psi^{\text{vis}}_M(\boldsymbol{C}_v) = \hat{\Psi}^{\text{ela}}_M(I_1^\Lambda, I_2^\Lambda, I_3^\Lambda) \qquad \boldsymbol{\Lambda} = \tilde{\boldsymbol{C}}\boldsymbol{C}_v^{-1} \qquad \boldsymbol{C}_e = \boldsymbol{F}_e^t \boldsymbol{F}_e$$

Viscous evolution equation of the matrix material (cf. Reese [2001])

1. Viscous evolution equation and non-equilibrium stress tensor

$$\boxed{\boldsymbol{Y} = \boldsymbol{\Sigma}_v} \qquad \boldsymbol{Y} := -\frac{\partial \Psi^{\text{vis}}_M}{\partial \boldsymbol{C}_v} \qquad D^{\text{int}} := \dot{\boldsymbol{C}}_v : \boldsymbol{\Sigma}_v = \dot{\boldsymbol{C}}_v : \mathbb{V}(\boldsymbol{C}_v) : \dot{\boldsymbol{C}}_v \geq 0$$

2. Viscosity tensor

$$\mathbb{V}(\boldsymbol{C}_v) = \frac{1}{4}\left[V_{\text{vol}} - \frac{V_{\text{dev}}}{n_{\text{dim}}}\right] \boldsymbol{C}_v^{-1} \otimes \boldsymbol{C}_v^{-1} + \frac{V_{\text{dev}}}{4} \mathbb{I}^{\text{sym}} : \boldsymbol{C}_v^{-1} \otimes \boldsymbol{C}_v^{-1}$$

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Principle of virtual power (I)

Total energy balance law $\dot{\mathcal{H}} = \dot{\mathcal{T}}^{\text{tra}} + \dot{\mathcal{T}}^{\text{rot}} + \dot{\Pi}^{\text{ext}} + \dot{\Pi}^{\text{int}}$

$$\dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\boldsymbol{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \boldsymbol{P}, \boldsymbol{P}_K, \boldsymbol{S}, S_V, S_F, \boldsymbol{S}_K, \boldsymbol{\tau}^t_{\mathbf{skw}}, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Kinetic power functionals (motivated by Altenbach et al. [2003], Askes & Aifantis [2011])

$$\dot{\mathcal{T}}^{\text{tra}}(\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}) := \int_{\mathscr{B}_0} [\rho_0 \boldsymbol{I}\, \boldsymbol{v} - \boldsymbol{p}] \cdot \dot{\boldsymbol{v}}\, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{p}} \cdot [\boldsymbol{v} - \dot{\boldsymbol{\varphi}}] \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{p} \cdot \ddot{\boldsymbol{\varphi}}\, \mathrm{d}V$$

$$\dot{\mathcal{T}}^{\text{rot}}(\dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}) := \int_{\mathscr{B}_0} \left[\rho_0 \left[(l_F^2 - l_0^2)\, \boldsymbol{A}_0 + l_0^2 \boldsymbol{I}\right] \boldsymbol{\omega} - \boldsymbol{\pi}\right] \cdot \dot{\boldsymbol{\omega}}\, \mathrm{d}V - \int_{\mathscr{B}_0} \dot{\boldsymbol{\pi}} \cdot [\boldsymbol{\omega} - \dot{\boldsymbol{\alpha}}] \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{\pi} \cdot \ddot{\boldsymbol{\alpha}}\, \mathrm{d}V$$

External power functional $D^{\text{cdu}} = -\text{Grad}[\ln \Theta] \cdot \boldsymbol{Q}$

$$\begin{aligned}
\dot{\Pi}^{\text{ext}} := {} & \int_{\mathscr{B}_0} \frac{1}{2}\, \bar{\boldsymbol{S}} : \dot{\tilde{\boldsymbol{C}}}\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2}\, \bar{S}_V\, \dot{\tilde{C}}_V\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{1}{2}\, \bar{S}_F\, \dot{\tilde{C}}_F\, \mathrm{d}V + \int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\, \mathrm{d}V \\
& - \int_{\mathscr{B}_0} \rho_0 \boldsymbol{B} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}V - \int_{\partial_T \mathscr{B}_0} \bar{\boldsymbol{T}} \cdot \dot{\boldsymbol{\varphi}}\, \mathrm{d}A + \int_{\partial_Q \mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta}\, \bar{Q}\, \mathrm{d}A \\
& + \int_{\mathscr{B}_0} \frac{1}{\Theta}\, \text{Grad}\left[\tilde{\Theta}\right] \cdot \boldsymbol{Q}\, \mathrm{d}V + \int_{\mathscr{B}_0} \frac{\tilde{\Theta}}{\Theta} \left(D^{\text{cdu}} + D^{\text{int}}\right) \mathrm{d}V + \int_{\mathscr{B}_0} \dot{\boldsymbol{C}}_v : \boldsymbol{\Sigma}_v\, \mathrm{d}V \\
& + \int_{\partial_\Theta \mathscr{B}_0} \lambda \left[\tilde{\Theta} - \Theta_\infty\right] \mathrm{d}A - \int_{\partial_{\dot{\Theta}} \mathscr{B}_0} h \left[\dot{\Theta} - \dot{\bar{\Theta}}\right] \mathrm{d}A - \int_{\partial_\varphi \mathscr{B}_0} \boldsymbol{R} \cdot \left[\dot{\boldsymbol{\varphi}} - \dot{\bar{\boldsymbol{\varphi}}}\right] \mathrm{d}A \\
& - \int_{\partial_\alpha \mathscr{B}_0} \boldsymbol{Z} \cdot \left[\dot{\boldsymbol{\alpha}} - \dot{\bar{\boldsymbol{\alpha}}}\right] \mathrm{d}A - \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \left[\boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\text{skw}}\right] \mathrm{d}A - \int_{\partial_W \mathscr{B}_0} \bar{\boldsymbol{W}} \cdot \dot{\boldsymbol{\alpha}}\, \mathrm{d}A
\end{aligned}$$

Principle of virtual power (II)

Internal power functional

(motivated by Steinmann & Stein [1997])

$$\dot{\Pi}^{\mathrm{int}} := \int_{\mathscr{B}_0} \left\{ \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{\boldsymbol{C}}} + S_V\,\boldsymbol{A}^{\mathrm{vol}} + S_F\,\boldsymbol{A}_0 - \boldsymbol{S} \right] : \frac{1}{2}\dot{\tilde{\boldsymbol{C}}} + \left[2\,\frac{\partial \hat{\Psi}_M}{\partial \tilde{C}_V} - S_V \right] \frac{\dot{\tilde{C}}_V}{2} + \left[2\,\frac{\partial \hat{\Psi}_F}{\partial \tilde{C}_F} - S_F \right] \frac{\dot{\tilde{C}}_F}{2} \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ [\Theta - \tilde{\Theta}]\,\dot{\eta} + \left[\frac{\partial \Psi}{\partial \Theta} + \eta \right] \dot{\Theta} + \frac{\partial \hat{\Psi}_M}{\partial \boldsymbol{C}_v} : \dot{\boldsymbol{C}}_v + \left[\tilde{\boldsymbol{F}}\boldsymbol{S} + \tilde{\boldsymbol{G}}(\boldsymbol{S}_K)^t - \boldsymbol{P} \right] : \dot{\tilde{\boldsymbol{F}}} + \boldsymbol{P} : \mathrm{Grad}\,[\dot{\boldsymbol{\varphi}}] \right\} \mathrm{d}V$$

$$+ \int_{\mathscr{B}_0} \left\{ \boldsymbol{P}_K : \mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] + \boldsymbol{\tau}^t_{\mathrm{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\,\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] + \left[\tilde{\boldsymbol{F}}\boldsymbol{S}_K - \boldsymbol{P}_K \right] : \dot{\tilde{\boldsymbol{G}}} + \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} - \boldsymbol{S}_K \right] : \dot{\tilde{\boldsymbol{K}}} \right\} \mathrm{d}V$$

Principle of virtual power

$$\delta_* \dot{\mathcal{H}}(\underbrace{\dot{\boldsymbol{\varphi}}, \dot{\boldsymbol{v}}, \dot{\boldsymbol{p}}, \dot{\boldsymbol{\alpha}}, \dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\pi}}, \dot{\boldsymbol{C}}_v, \dot{\Theta}, \dot{\eta}, \dot{\tilde{\boldsymbol{F}}}, \dot{\tilde{\boldsymbol{G}}}, \dot{\tilde{\boldsymbol{C}}}, \dot{\tilde{\boldsymbol{K}}}, \dot{\tilde{C}}_V, \dot{\tilde{C}}_F}_{\text{temporally continuous}}, \underbrace{\tilde{\Theta}, \boldsymbol{P}, \boldsymbol{P}_K, \boldsymbol{S}, S_V, S_F, \boldsymbol{S}_K, \boldsymbol{\tau}^t_{\mathrm{skw}}, \boldsymbol{R}, h, \lambda, \boldsymbol{Z}, \hat{\boldsymbol{\omega}}}_{\text{temporally discontinuous}}) = 0$$

Rotational weak forms

(here only the volume weak forms)

$$\int_{\mathscr{B}_0} \delta_*\boldsymbol{\tau}^t_{\mathrm{skw}} : \boldsymbol{\epsilon} \cdot \left[\frac{1}{2}\boldsymbol{\epsilon} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\alpha}} \right] \mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \hat{\boldsymbol{\omega}} \cdot \boldsymbol{\epsilon} : \delta_*\boldsymbol{\tau}^t_{\mathrm{skw}}\,\mathrm{d}A \qquad \int_{\mathscr{B}_0} \delta_*\dot{\tilde{\boldsymbol{K}}} : \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} + \bar{\boldsymbol{S}}_K - \boldsymbol{S}_K \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\tilde{\boldsymbol{G}}} : \left[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}}_K - \boldsymbol{P}_K \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\boldsymbol{P}_K : \left[\mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] - \dot{\tilde{\boldsymbol{G}}} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\pi}} \cdot [\dot{\boldsymbol{\alpha}} - \boldsymbol{\omega}]\,\mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\omega}} \cdot \left[\rho_0 \left[(l_F^2 - l_0^2)\,\boldsymbol{A}_0 + l_0^2 \boldsymbol{I} \right] \boldsymbol{\omega} - \boldsymbol{\pi} \right] \mathrm{d}V = 0 \qquad \int_{\mathscr{B}_0} \delta_*\boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{F}}}^t\tilde{\boldsymbol{G}} + \tilde{\boldsymbol{F}}^t\dot{\tilde{\boldsymbol{G}}} - \dot{\tilde{\boldsymbol{K}}} \right] \mathrm{d}V = 0$$

$$\int_{\mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot \left[\dot{\boldsymbol{\pi}} + \boldsymbol{\epsilon} : \boldsymbol{\tau}^t_{\mathrm{skw}} \right] \mathrm{d}V + \int_{\mathscr{B}_0} \boldsymbol{P}_K : \mathrm{Grad}\,[\delta_*\dot{\boldsymbol{\alpha}}]\,\mathrm{d}V = \int_{\partial_\alpha \mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot \boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W \mathscr{B}_0} \delta_*\dot{\boldsymbol{\alpha}} \cdot \bar{\boldsymbol{W}}\,\mathrm{d}A$$

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Rotational balance laws

Balance law of angular momentum $\mathcal{J}$ includes additively:

Balance law of spin angular momentum $\mathcal{J}^{\mathrm{spn}}$

Set $\delta_*\dot{\boldsymbol{\alpha}} = \boldsymbol{c} = $ const.:

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\{\boldsymbol{c}\cdot[\dot{\boldsymbol{\pi}}+\boldsymbol{\epsilon}:\boldsymbol{\tau}^t_{\mathrm{skw}}] + \boldsymbol{P}_K : \overbrace{\mathrm{Grad}\,[\boldsymbol{c}]}^{0}\}\,\mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\Big\{\int_{\partial_\alpha\mathscr{B}_0}\boldsymbol{c}\cdot\boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W\mathscr{B}_0}\boldsymbol{c}\cdot\boldsymbol{W}\,\mathrm{d}A\Big\}\,\mathrm{d}t$$

$$\boxed{\int_{\mathscr{B}_0}\boldsymbol{\pi}_{t_{n+1}}\,\mathrm{d}V - \int_{\mathscr{B}_0}\boldsymbol{\pi}_{t_n}\,\mathrm{d}V = \int_{t_n}^{t_{n+1}}\Big[-\int_{\mathscr{B}_0}\boldsymbol{\epsilon}:\boldsymbol{\tau}^t_{\mathrm{skw}}\,\mathrm{d}V + \int_{\partial_\alpha\mathscr{B}_0}\boldsymbol{Z}\,\mathrm{d}A+\int_{\partial_W\mathscr{B}_0}\boldsymbol{W}\,\mathrm{d}A\Big]\,\mathrm{d}t}$$

Balance law of total energy $\mathcal{H}$ includes additively:

Balance law of rotational kinetic energy $\mathcal{T}^{\mathrm{rot}}$

Set $\delta_*\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}}$ and $\delta_*\dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}$ and $\delta_*\dot{\boldsymbol{\pi}} = \dot{\boldsymbol{\pi}}$ and $\delta_*\boldsymbol{\tau}^t_{\mathrm{skw}} = \boldsymbol{\tau}^t_{\mathrm{skw}}$ and $\delta_*\dot{\tilde{\boldsymbol{G}}} = \dot{\tilde{\boldsymbol{G}}}$ and $\delta_*\boldsymbol{P}_K = \boldsymbol{P}_K$:

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\{\dot{\boldsymbol{\alpha}}\cdot[\dot{\boldsymbol{\pi}}+\boldsymbol{\epsilon}:\boldsymbol{\tau}^t_{\mathrm{skw}}] + \boldsymbol{P}_K : \mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}]\}\,\mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\Big\{\int_{\partial_\alpha\mathscr{B}_0}\dot{\boldsymbol{\alpha}}\cdot\boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W\mathscr{B}_0}\dot{\boldsymbol{\alpha}}\cdot\boldsymbol{W}\,\mathrm{d}A\Big\}\,\mathrm{d}t$$

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{\tau}^t_{\mathrm{skw}}:\boldsymbol{\epsilon}\cdot\left[\frac{1}{2}\boldsymbol{\epsilon}:\dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1}+\dot{\boldsymbol{\alpha}}\right]\mathrm{d}V\,\mathrm{d}t = 0 \qquad 0 = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\dot{\tilde{\boldsymbol{G}}}:[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}}_K - \boldsymbol{P}_K]\,\mathrm{d}V\,\mathrm{d}t$$

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\dot{\boldsymbol{\omega}}\cdot[\boldsymbol{J}\boldsymbol{\omega}-\boldsymbol{\pi}]\,\mathrm{d}V\,\mathrm{d}t = 0 \qquad 0 = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{P}_K : \left[\mathrm{Grad}\,[\dot{\boldsymbol{\alpha}}] - \dot{\tilde{\boldsymbol{G}}}\right]\mathrm{d}V\,\mathrm{d}t$$

$$\boxed{\begin{aligned}\mathcal{T}^{\mathrm{rot}}_{n+1}-\mathcal{T}^{\mathrm{rot}}_{n} &\equiv \frac{1}{2}\int_{\mathscr{B}_0}\boldsymbol{\omega}_{t_{n+1}}\cdot\boldsymbol{J}\boldsymbol{\omega}_{t_{n+1}}\mathrm{d}V - \frac{1}{2}\int_{\mathscr{B}_0}\boldsymbol{\omega}_{t_n}\cdot\boldsymbol{J}\boldsymbol{\omega}_{t_n}\mathrm{d}V + \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{S}_K:\tilde{\boldsymbol{F}}^t\dot{\tilde{\boldsymbol{G}}}\,\mathrm{d}V\,\mathrm{d}t\\ &= \int_{t_n}^{t_{n+1}}\left[\int_{\mathscr{B}_0}\boldsymbol{\tau}^t_{\mathrm{skw}}:\mathbb{I}^{\mathrm{skw}}:\dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1}\mathrm{d}V + \int_{\partial_\alpha\mathscr{B}_0}\dot{\boldsymbol{\alpha}}\cdot\boldsymbol{Z}\,\mathrm{d}A + \int_{\partial_W\mathscr{B}_0}\dot{\boldsymbol{\alpha}}\cdot\boldsymbol{W}\,\mathrm{d}A\right]\mathrm{d}t\end{aligned}}$$

Translational balance laws

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Balance law of orbital angular momentum $\mathcal{J}^{\mathrm{orb}}$

Set $\delta_*\dot{\boldsymbol{\varphi}} = \boldsymbol{c}\times\boldsymbol{\varphi}$ and $\delta_*\boldsymbol{P} = \boldsymbol{c}\times\boldsymbol{P}$ and $\delta_*\dot{\tilde{\boldsymbol{F}}} = \boldsymbol{c}\times\tilde{\boldsymbol{F}}$ and $\delta_*\dot{\boldsymbol{v}} = \boldsymbol{c}\times\dot{\boldsymbol{\varphi}}$ and $\delta_*\dot{\boldsymbol{p}} = \boldsymbol{c}\times\rho_0\dot{\boldsymbol{\varphi}}$:

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\cdot\boldsymbol{\varphi}\times\dot{\boldsymbol{p}}\,\mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\mathrm{Grad}\,[\boldsymbol{\varphi}] : [\boldsymbol{c}\times\boldsymbol{P}]\,\mathrm{d}V\,\mathrm{d}t + \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\cdot[\boldsymbol{\varphi}\times\rho_0\,\boldsymbol{B}]\,\mathrm{d}V\,\mathrm{d}t + \int_{t_n}^{t_{n+1}}\int_{\partial_T\mathscr{B}_0}\boldsymbol{c}\cdot[\boldsymbol{\varphi}\times\bar{\boldsymbol{T}}]\,\mathrm{d}A\,\mathrm{d}t + \int_{t_n}^{t_{n+1}}\int_{\partial_\varphi\mathscr{B}_0}\boldsymbol{c}\cdot[\boldsymbol{\varphi}\times\boldsymbol{R}]\,\mathrm{d}A\,\mathrm{d}t$$

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\times\boldsymbol{P} : \left[\mathrm{Grad}\,[\boldsymbol{\varphi}] - \tilde{\boldsymbol{F}}\right]\mathrm{d}V\,\mathrm{d}t = 0 \qquad 0 = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\times\dot{\boldsymbol{\varphi}}\cdot[\rho_0\,\boldsymbol{v} - \boldsymbol{p}]\,\mathrm{d}V\,\mathrm{d}t$$

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\times\tilde{\boldsymbol{F}} : \left[\tilde{\boldsymbol{F}}\tilde{\boldsymbol{S}} + \tilde{\boldsymbol{G}}(\boldsymbol{S}_K)^t + \boldsymbol{\tau}_{\mathrm{skw}}^t\tilde{\boldsymbol{F}}^{-t} - \boldsymbol{P}\right]\mathrm{d}V\,\mathrm{d}t = 0 \qquad 0 = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\boldsymbol{c}\times\rho_0\dot{\boldsymbol{\varphi}}\cdot[\boldsymbol{v} - \dot{\boldsymbol{\varphi}}]\,\mathrm{d}V\,\mathrm{d}t$$

$$\boxed{\begin{aligned}\mathcal{J}^{\mathrm{orb}}_{n+1} - \mathcal{J}^{\mathrm{orb}}_{n} \equiv \int_{\mathscr{B}_0}\boldsymbol{\varphi}_{n+1}\times\boldsymbol{p}_{n+1}\,\mathrm{d}V - \int_{\mathscr{B}_0}\boldsymbol{\varphi}_{n}\times\boldsymbol{p}_{n}\,\mathrm{d}V &= \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\left[\boldsymbol{\epsilon} : \boldsymbol{\tau}_{\mathrm{skw}}^t + \tilde{\boldsymbol{G}}(\boldsymbol{S}_K)^t\times\tilde{\boldsymbol{F}}\right]\mathrm{d}V\,\mathrm{d}t \\ &+ \int_{t_n}^{t_{n+1}}\left[\int_{\mathscr{B}_0}\boldsymbol{\varphi}\times\rho_0\,\boldsymbol{B}\,\mathrm{d}V + \int_{t_n}^{t_{n+1}}\int_{\partial_T\mathscr{B}_0}\boldsymbol{\varphi}\times\bar{\boldsymbol{T}}\,\mathrm{d}A + \int_{t_n}^{t_{n+1}}\int_{\partial_\varphi\mathscr{B}_0}\boldsymbol{\varphi}\times\boldsymbol{R}\,\mathrm{d}A\right]\mathrm{d}t\end{aligned}}$$

Balance law of transational kinetic energy $\mathcal{T}^{\mathrm{tra}}$

Set $\delta_*\dot{\boldsymbol{\varphi}} = \dot{\boldsymbol{\varphi}}$ and $\delta_*\dot{\boldsymbol{v}} = \dot{\boldsymbol{v}}$ and $\delta_*\dot{\boldsymbol{p}} = \dot{\boldsymbol{p}}$ and $\delta_*\boldsymbol{P} = \boldsymbol{P}$ and $\delta_*\dot{\tilde{\boldsymbol{F}}} = \dot{\tilde{\boldsymbol{F}}}$:

$$\boxed{\begin{aligned}\mathcal{T}^{\mathrm{tra}}_{n+1} - \mathcal{T}^{\mathrm{tra}}_{n} &\equiv \frac{1}{2}\int_{\mathscr{B}_0}\left[\boldsymbol{v}_{t_{n+1}}\cdot\rho_0\boldsymbol{v}_{t_{n+1}} - \boldsymbol{v}_{t_n}\cdot\rho_0\boldsymbol{v}_{t_n}\right]\mathrm{d}V + \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0}\left[\tilde{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{F}}}^t\tilde{\boldsymbol{G}} + \tilde{\boldsymbol{S}} : \tilde{\boldsymbol{F}}^t\dot{\tilde{\boldsymbol{F}}}\right]\mathrm{d}V\,\mathrm{d}t \\ &= \int_{t_n}^{t_{n+1}}\left\{\int_{\mathscr{B}_0}\left[-\boldsymbol{\tau}_{\mathrm{skw}}^t : \mathbb{I}^{\mathrm{skw}} : \dot{\tilde{\boldsymbol{F}}}\tilde{\boldsymbol{F}}^{-1} + \dot{\boldsymbol{\varphi}}\cdot\rho_0\,\boldsymbol{B}\right]\mathrm{d}V + \int_{\partial_T\mathscr{B}_0}\dot{\boldsymbol{\varphi}}\cdot\bar{\boldsymbol{T}}\,\mathrm{d}A + \int_{\partial_\varphi\mathscr{B}_0}\dot{\boldsymbol{\varphi}}\cdot\boldsymbol{R}\,\mathrm{d}A\right\}\mathrm{d}t\end{aligned}}$$

Potential energy balance and algorithmic stresses

Balance law of potential energy Π^{int}

$$\Pi^{\text{int}}_{n+1} - \Pi^{\text{int}}_{n} = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left[\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \dot{\hat{\Psi}}_K\right] \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left[\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} : \dot{\tilde{\boldsymbol{K}}}\right] \mathrm{d}V\,\mathrm{d}t$$

Setting $\delta_* \dot{\tilde{\boldsymbol{K}}} = \dot{\tilde{\boldsymbol{K}}}$ and $\delta_* \boldsymbol{S}_K = \boldsymbol{S}_K$, leading to

$$\int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \dot{\tilde{\boldsymbol{K}}} : \left[\frac{\partial \hat{\Psi}_K}{\partial \tilde{\boldsymbol{K}}} + \bar{\boldsymbol{S}}_K - \boldsymbol{S}_K\right] \mathrm{d}V\,\mathrm{d}t = 0 \quad \text{and} \quad \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{K}}} - \dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} - \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}}\right] \mathrm{d}V\,\mathrm{d}t = 0$$

we arrive at

$$\Pi^{\text{int}}_{n+1} - \Pi^{\text{int}}_{n} - \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left\{\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \boldsymbol{S}_K : \dot{\tilde{\boldsymbol{K}}}\right\} \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\,\mathrm{d}V\,\mathrm{d}t$$

$$\Pi^{\text{int}}_{n+1} - \Pi^{\text{int}}_{n} - \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \left\{\dot{\hat{\Psi}}_M + \dot{\hat{\Psi}}_F + \boldsymbol{S}_K : \left[\dot{\tilde{\boldsymbol{F}}}^t \tilde{\boldsymbol{G}} + \tilde{\boldsymbol{F}}^t \dot{\tilde{\boldsymbol{G}}}\right]\right\} \mathrm{d}V\,\mathrm{d}t = \int_{t_n}^{t_{n+1}}\int_{\mathscr{B}_0} \bar{\boldsymbol{S}}_K : \dot{\tilde{\boldsymbol{K}}}\,\mathrm{d}V\,\mathrm{d}t$$

Algorithmic curvature-twist stress $\bar{\boldsymbol{S}}_K$ (cp. Simo & Tarnow [1992], Betsch & Steinmann [2002])

Local algorithmic constraint:

$$\hat{\Psi}_{K_{n+1}} - \hat{\Psi}_{K_n} - \int_0^1 \boldsymbol{S}_K : \frac{\partial \tilde{\boldsymbol{K}}}{\partial \alpha}\,\mathrm{d}\alpha = \int_0^1 \bar{\boldsymbol{S}}_K : \frac{\partial \tilde{\boldsymbol{K}}}{\partial \alpha}\,\mathrm{d}\alpha \qquad \alpha(t) := \frac{t - t_n}{t_{n+1} - t_n}$$

$\bar{\boldsymbol{S}}_K$ vanishes with

$$\tilde{\boldsymbol{K}} := \sum_{I=1}^{k+1} M_I(\alpha)\,\tilde{\boldsymbol{K}}^I \qquad M_I(\alpha) := \prod_{\substack{J=1\\ J\neq I}}^{k+1} \frac{\alpha - \alpha_J}{\alpha_I - \alpha_J} \qquad I_{\text{Gauss}}\{f\} = \sum_{l=1}^{k} f(\xi_l)\,w_l$$

for the Saint Venant-Kirchhoff curvature-twist model

$$\Psi^{\text{cur}}(\tilde{\boldsymbol{K}}; \boldsymbol{A}_0) = \frac{\mu_{\text{twi}}}{2}\left[\boldsymbol{A}_0 \tilde{\boldsymbol{K}} : \boldsymbol{I}\right]^2 + \mu_{\text{ben}}\,\boldsymbol{A}_0 \tilde{\boldsymbol{K}} : \boldsymbol{A}_0 \tilde{\boldsymbol{K}}$$

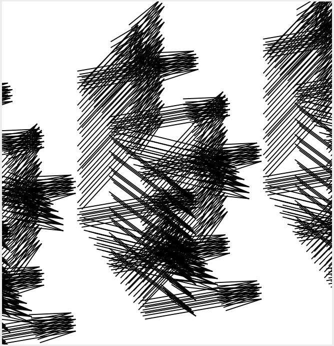
TECHNISCHE UNIVERSITÄT CHEMNITZ

A variational-based energy-momentum time integration of metamaterials in non-isothermal rotordynamical systems

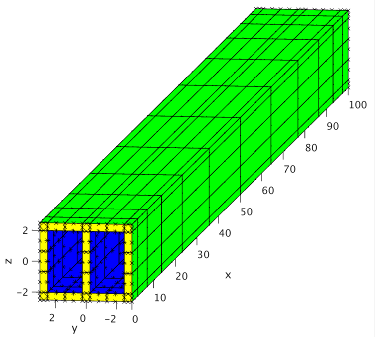
Michael Groß, Julian Dietzsch, Chris Röbiger

Dynamic torsion of a long composite square pipe
(Boundary and initial conditions; 121-em with H20-mixed-Bbar)

Material Status Monitor



Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches: Torque load $W_x^A = \pm \hat{W}_x \, f(t)$

Temperature control $\Theta^A = \hat{\Theta} \, f(t)$

Blue patches: Outward heat flux $\bar{Q}^A := -\hat{Q} \, f(t))$

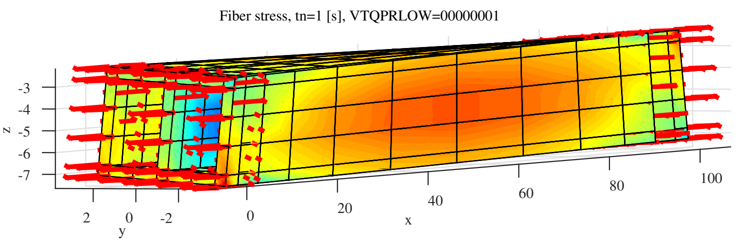
Green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

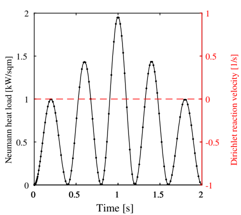
$$\begin{aligned} \boldsymbol{\varphi}_0^A &= \boldsymbol{X}^A & \boldsymbol{\alpha}_0^A &= \mathbf{0} \\ \boldsymbol{v}_0^A &= \mathbf{0} & \boldsymbol{\omega}_0^A &= \mathbf{0} \\ \Theta_0^A &= \Theta_\infty & \eta_0^A &= 0 \\ \boldsymbol{\tau}_{\mathrm{skw}}^t &= \boldsymbol{O} & \Theta_\infty &= 298.15 \end{aligned}$$

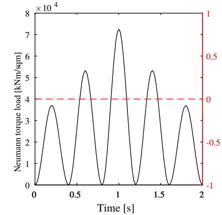
Dynamic torsion of a long composite square pipe
(Motion and loads with no curvature stiffness $\mu_{\mathrm{twi}} = 0 = \mu_{\mathrm{ben}}$)

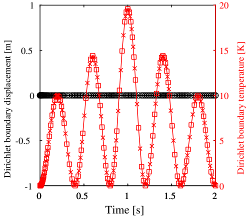
Current fiber stress at $t_n = 1.0\,\mathrm{s}$ $(\lambda = 10^{10},\ \mu = 2.5 \cdot 10^9)$



Time evolution of the loads (wave function f)



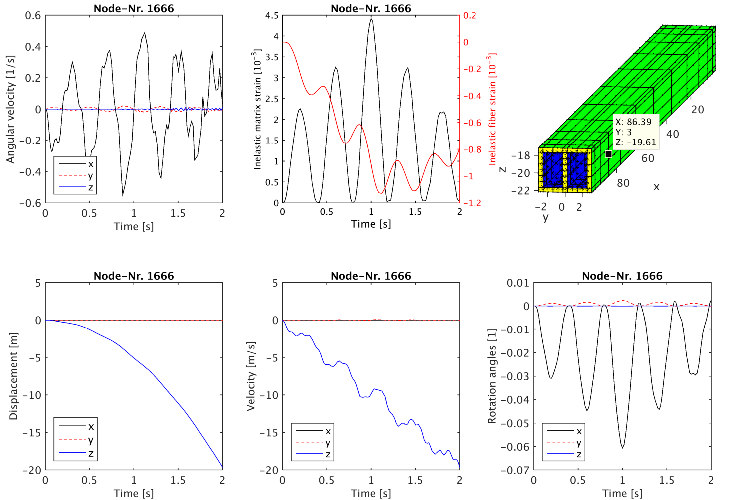




Dynamic torsion of a long composite square pipe

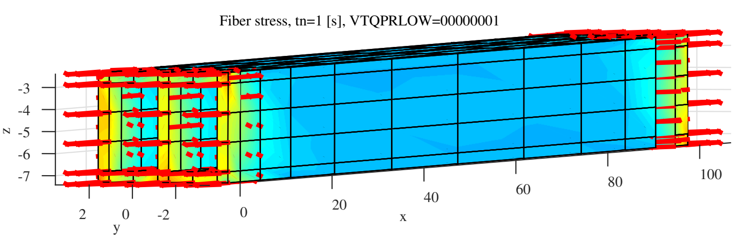
(Solution vs. time with no curvature stiffness $\mu_{\text{twi}} = 0 = \mu_{\text{ben}}$)

Time evolutions of a node on the free boundary

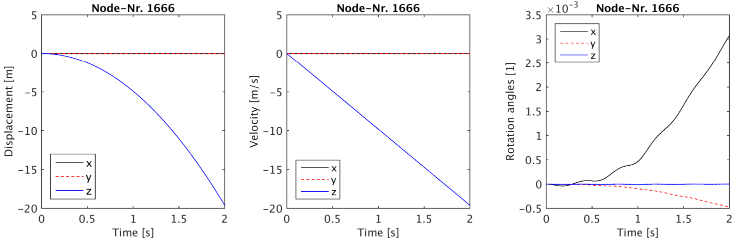


Dynamic torsion of a long composite square pipe
(Motion/solution with curvat. stiff. $\mu_{\mathrm{twi}} = 10^{14}$, $\mu_{\mathrm{ben}} = 5 \cdot 10^{12}$)

Current fiber stress at $t_n = 1.0\,\mathrm{s}$ $(\lambda = 10^{10},\ \mu = 2.5 \cdot 10^{9})$

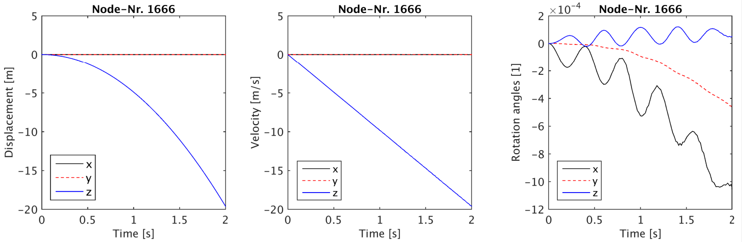


Time evolutions of a node on the free boundary

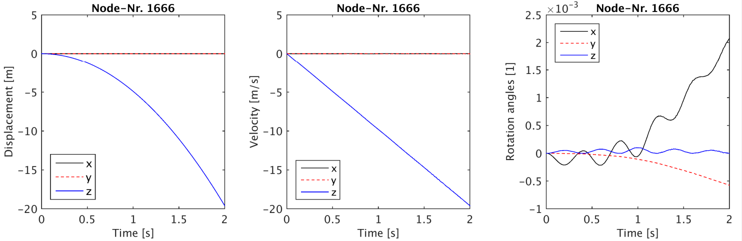


Dynamic torsion of a long composite square pipe
(Solution vs. time with curvature stiffness; H20-mixed-Bbar)

Time evolutions on free boundary with $\mu_{\mathrm{twi}} = 10^{14}$ and $\mu_{\mathrm{ben}} = 0$)

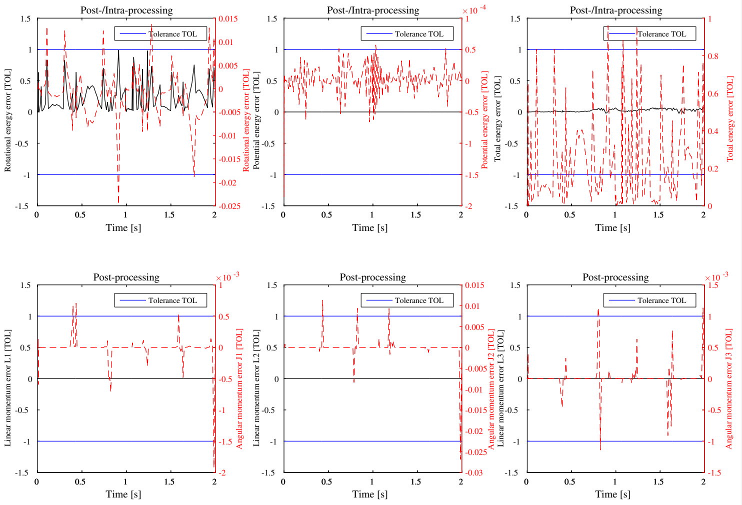


Time evolutions on free boundary with $\mu_{\mathrm{twi}} = 0 \wedge \mu_{\mathrm{ben}} = 5 \cdot 10^{12}$)



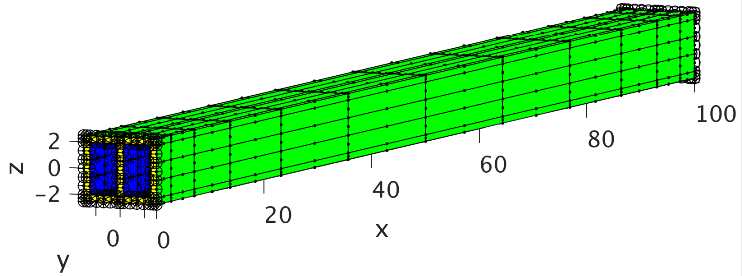
Dynamic torsion of a long composite square pipe
(Balance laws with curvature stiff. $\mu_{\mathrm{twi}} = 10^{14}$, $\mu_{\mathrm{ben}} = 5 \cdot 10^{12}$)

Error of energy and momentum balance laws versus time



Displacement-controlled bending of the long pipe
(Boundary and initial conditions; 121-em with H20-mixed-Bbar)

Boundary Status Monitor



Dirichlet and Neumann boundary conditions

Yellow patches $x = 0$: Temperature control $\Theta^A = \hat{\Theta}\, f(t)$

Clamping with $\boldsymbol{\varphi}^A = \boldsymbol{X}^A$ and $\boldsymbol{\alpha}^A = \boldsymbol{0}$

Yellow patches $x = 100$: Temperature control $\Theta^A = \hat{\Theta}\, f(t)$

Displacement control $u_z^A = -\hat{u}\, f(t)$

Blue patches: Outward heat flux (cooling) with $\bar{Q}^A(t) = \hat{Q}\, f(t)$

Green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

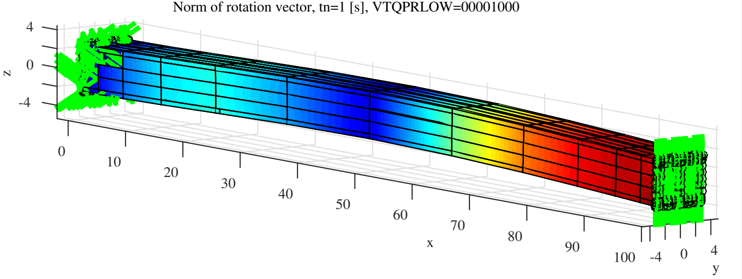
$$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \qquad \boldsymbol{\alpha}_0^A = \boldsymbol{0}$$
$$\boldsymbol{v}_0^A = \boldsymbol{0} \qquad \boldsymbol{\omega}_0^A = \boldsymbol{0}$$
$$\Theta_0^A = \Theta_\infty \qquad \eta_0^A = 0$$
$$\boldsymbol{\tau}_{\mathrm{skw}}^t = \boldsymbol{O} \qquad \Theta_\infty = 298.15$$

Movies

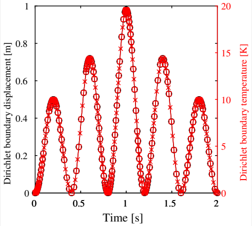
$S_F(\boldsymbol{X})$ $\alpha(\boldsymbol{X})$

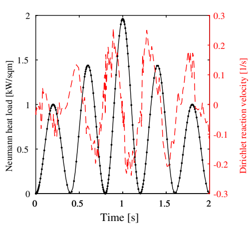
Displacement-controlled bending of the long pipe
(Motion and loads with no curvature stiffness $\mu_{\mathrm{twi}} = 0 = \mu_{\mathrm{ben}}$)

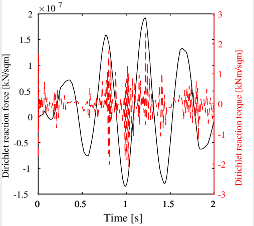
Current rotation angle at $t_n = 1.0\,\mathrm{s}$ $(\lambda = 10^{10},\ \mu = 2.5 \cdot 10^{9})$



Time evolution of the loads and reactions

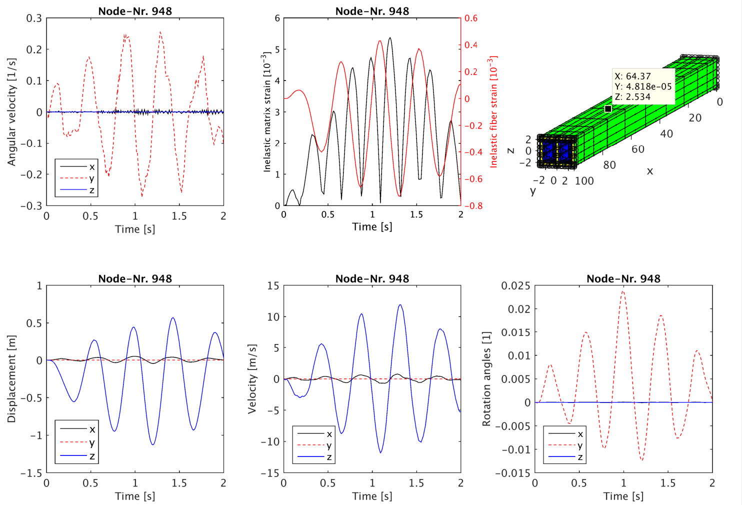






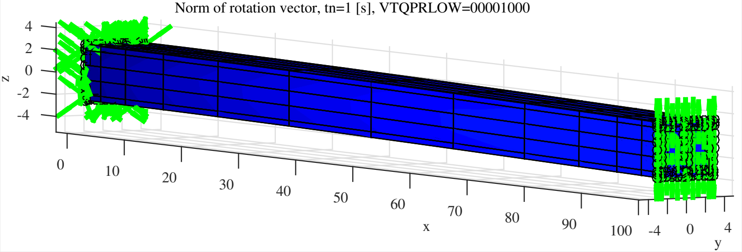
Displacement-controlled bending of the long pipe
(Solution vs. time with no curvature stiffness $\mu_{\text{twi}} = 0 = \mu_{\text{ben}}$)

Time evolutions of a node on the free boundary

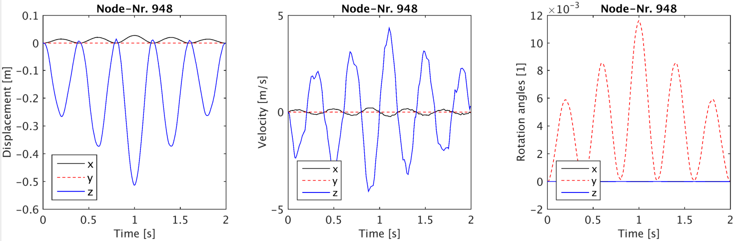


Displacement-controlled bending of the long pipe
(Motion/solution with curvat. stiff. $\mu_{\text{twi}} = 10^{14}$, $\mu_{\text{ben}} = 5 \cdot 10^{12}$)

Current rotation angle at $t_n = 1.0\,\text{s}$ ($\lambda = 10^{10}$, $\mu = 2.5 \cdot 10^9$)



Time evolutions of a node on the free boundary

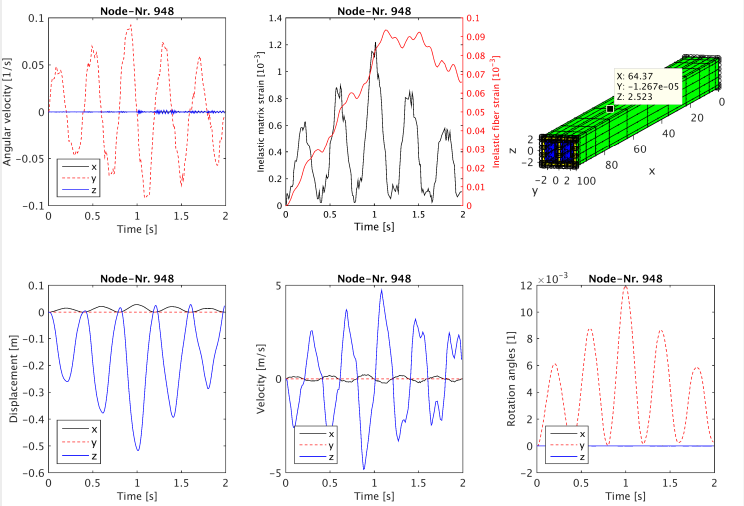


Displacement-controlled bending of the long pipe

(Solution vs. time with curvat. stiff. $\mu_{\mathrm{twi}} = 0$, $\mu_{\mathrm{ben}} = 5 \cdot 10^{12}$)

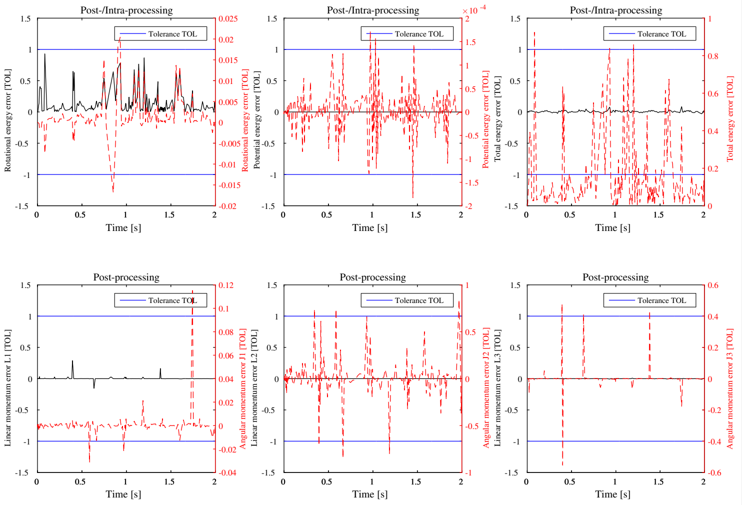
Michael Groß, Julian Dietzsch, Chris Röbiger

Time evolutions of a node on the free boundary



Displacement-controlled bending of the long pipe
(Balance laws with curvature stiff. $\mu_{\text{twi}} = 10^{14}$, $\mu_{\text{ben}} = 5 \cdot 10^{12}$)

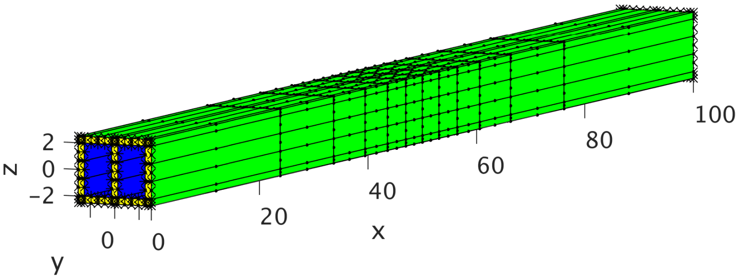
Error of energy and momentum balance laws versus time



Driven rotation of the long pipe under pressure
(Boundary and initial conditions; 121-em with H20-mixed-Bbar)

Boundary Status Monitor

(cf. Nguyen et al. [2015])



Dirichlet and Neumann boundary conditions

Yellow patches $x = 0$: Temperature $\Theta^A = \hat{\Theta}\, f(t)$

Torque load $\boldsymbol{W}^A = +\hat{\boldsymbol{W}}_{\text{drive}}\, f(t)$

Yellow patches $x = 100$: Temperature $\Theta^A = \hat{\Theta}\, f(t)$

Torque load $\boldsymbol{W}^A = -\hat{\boldsymbol{W}}_{\text{friction}}\, f(t)$

Blue patches: 1) Pressure follower load $p^A(t) = \hat{p}\,|\sin(\omega_{\text{load,p}}\, t)|$

2) Outward heat flux (cooling) with $\bar{Q}^A(t) = \hat{Q}\, f(t)$

Green patches: no b.c. (thermal insulation $\bar{Q}^A := 0$)

Initial conditions

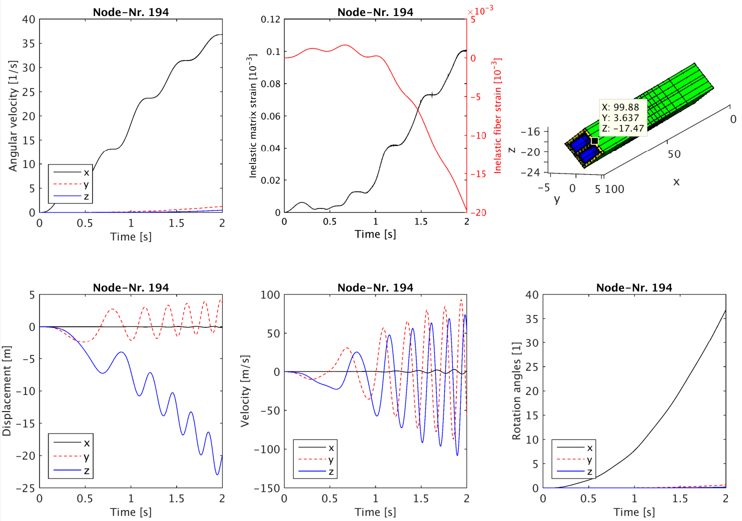
$$\boldsymbol{\varphi}_0^A = \boldsymbol{X}^A \qquad \boldsymbol{\alpha}_0^A = \boldsymbol{0}$$
$$\boldsymbol{v}_0^A = \boldsymbol{0} \qquad \boldsymbol{\omega}_0^A = \boldsymbol{0}$$
$$\Theta_0^A = \Theta_\infty \qquad \eta_0^A = 0$$
$$\boldsymbol{\tau}^t_{\text{skw}} = \boldsymbol{O} \qquad \Theta_\infty = 298.15$$

Movies

$S_F(\boldsymbol{X})$ $\alpha(\boldsymbol{X})$

Driven rotation of the long pipe under pressure
(Solution vs. time with no curvature stiffness $\mu_{\text{twi}} = 0 = \mu_{\text{ben}}$)

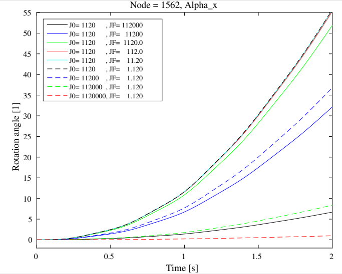
Time evolutions of a node on the thermal Dirichlet boundary $\partial_{\dot{\Theta}}\mathscr{B}_0$

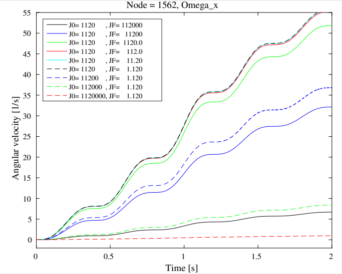


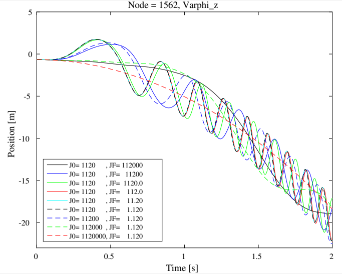
(cf. Weidenhammer [1958]; Gasch, Markert & Pfützner [1979]; Wauer [1981])

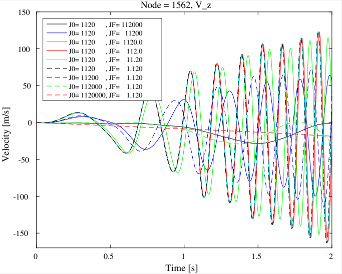
Driven rotation of the long pipe under pressure
(Inertia comparison with no curvature stiffness $\mu_{\text{twi}} = 0 = \mu_{\text{ben}}$)

Time evolutions of a node on the free boundary









Summary

1. Motivation:
   - Precise simulations of fiber roving composites with
   - fiber twisting and bending stiffness on the micro scale
2. Goals:
   - Energy-momentum schemes for constrained micropolar continua
   - derived by variational principles for generalized Cauchy continua
3. Strategy:
   - Independent fields for rotation and rotation gradient
   - Discretization of a mixed principle of virtual power
4. Important results:
   - Curvature stiffness of fibers increases the total stiffness
   - Micro inertia influences rotational/translational motion
5. Next steps:
   - Investigation of nonlinear curvature strain energy functions
   - Introduction of an algorithmic curvature-twist stress tensor