Multiscale time integration of dissipative thermodynamics with energy-momentum consistent time finite element methods

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ABSTRACT

Presently, there is a significant effort to find more robust and reliable time integration methods for thermo-mechanically coupled dynamics. One possible way of designing such methods is the structure-preserving time integration. The goal here is to design a method, which ensures that physical features of the underlying problem are inherited by the discrete solution. Structure-preserving methods have already proven their excellent robustness as well as their reliability of finding meaningful solutions of different thermo-mechanical problems (see e.g. Romero [1] and Krüger et al. [2]).

However, the ability of these methods to take into account the usely present different time scales in the different sub-solutions has hitherto based on a fractional step method (see Kassiotis et al.[3] and Romero [4]). Such partioned solution strategies are initially introduced in order to reduce the numerical effort of monolithic strategies (see Park & Felippa [5]). Using a fractional step method, we obtain a so-called h-adaption of the time axis, because the problem is solved sequentially on so-called micro time steps (compare also Leyendecker & Ober-Blöbaum [6] for constraint systems). An alternative way is to use a Galerkin-based structure-preserving method in order to increase the degree of the ansatz functions in time. This corresponds to a p-adaption method solving the problem simultaneously on all micro time steps (see Figure 1).

In this paper, we present such a Galerkin-based method for problems on different time scales. We consider the finite thermo-viscoelastodynamics based on the strong coupling of the local momentum balance equation, the local entropy balance equation and a nonlinear viscous evolution equation. The sub-solution of this system of differential equations is represented by the deformation mapping, the velocity field, the temperature field and a locally defined tensor-valued internal variable. The time scales of these variables are generally different. The shortest scale is usually present in the internal variables time evolution and the largest time scale is associated with the temperature evolution. The time scale of the deformation mapping lies in between (see Ibrahimbegovic [7]). Therefore, we define the largest order of finite elements in time in the viscous evolution equation, and the smallest in the entropy evolution equation. By means of numerical examples in the context of nonlinear thermodynamics with different boundary conditions, we show the advantage of this strategie in conjunction with the excellent robustness of energy-momentum consistent time integration. We also compare the numerical effort of multiscale time integration to calculations with equal orders of finite elements in time.

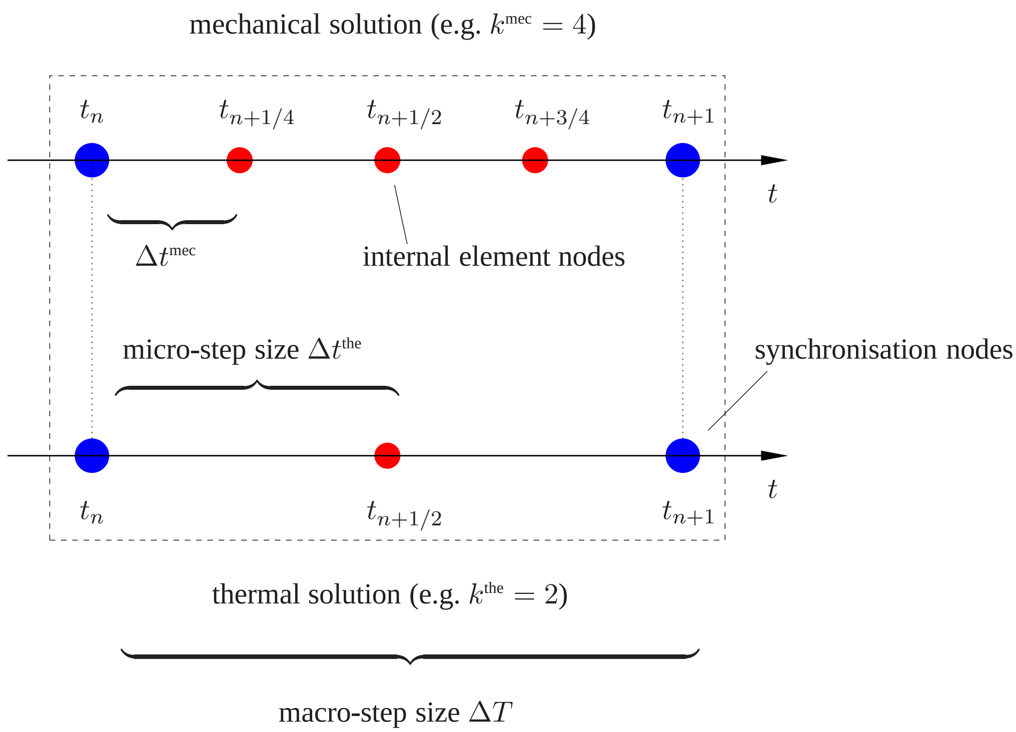

Figure 1: Macro-step size ΔT and micro-step sizes Δt^{mec} and Δt^{the} in the Galerkin-based time integration of thermo-mechanically coupled problems (compare Fig. 1 in Kassiotis et al. [3]).

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