Mixed finite element formulations for finite anisotropic elastodynamics

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Motivation

- Dynamic simulations of fiber-reinforced materials in light-weight structures

- Keywords:
  - Anisotropic material behavior
  - Nearly incompressible material behavior
  - Thermo-viscoelastic material behavior
  - Long-term stimulations

- Solution strategy:
  1. Mixed finite elements for finite anisotropic elastodynamics for higher-order time integrators to reduce locking effect [polyconvex material formulations, continuous Galerkin cG(k)]
  2. Energy-momentum conserving time integrators for stable long-term simulations [Discrete gradient eG(k)]
  3. Extension to an thermo-viscoelastic material behavior

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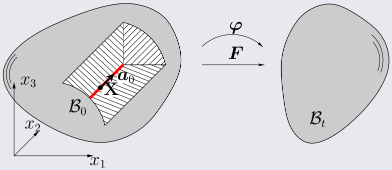
Transversely isotropic material



Continuum mechanics

$$\boldsymbol{C} = \boldsymbol{F}^T\boldsymbol{F} \qquad \boldsymbol{M} = \boldsymbol{a}_0 \otimes \boldsymbol{a}_0$$

$$J = I_3(\boldsymbol{F}) = \det[\boldsymbol{F}]$$

$$I_4(\boldsymbol{C}) = \mathrm{tr}[\boldsymbol{C}\boldsymbol{M}] \quad I_5(\boldsymbol{C}) = \mathrm{tr}[\mathrm{cof}[\boldsymbol{C}\boldsymbol{M}]]$$

$\boldsymbol{F}$ ... Deformation gradient
$\boldsymbol{C}$ ... Right Cauchy-Green tensor
$\boldsymbol{a}_0$ ... Fiber direction

Hyperelastic, transversely isotropic, polyconvex material formulation

$\Psi = \Psi^{iso} + \Psi^{ti}$ [Schröder, Wriggers, Balzani 2011]

$$\Psi^{iso}(\boldsymbol{C}) = \frac{\epsilon_1}{2}(\mathrm{tr}[\boldsymbol{C}])^2 + \frac{\epsilon_2}{2}(\mathrm{tr}[\mathrm{cof}[\boldsymbol{C}]])^2 - \epsilon_3 \ln(\sqrt{\det[\boldsymbol{C}]}) + \epsilon_4(\det[\boldsymbol{C}]^{\epsilon_5} + \det[\boldsymbol{C}]^{-\epsilon_5} - 2)$$

$$\Psi^{ti}(\boldsymbol{C}) = \epsilon_6\left(\frac{1}{\epsilon_7+1}(\mathrm{tr}[\boldsymbol{C}\boldsymbol{M}])^{\epsilon_7+1} + \frac{1}{\epsilon_8+1}(\mathrm{tr}[\mathrm{cof}[\boldsymbol{C}]\boldsymbol{M}])^{\epsilon_8+1} + \frac{1}{\epsilon_9}\det[\boldsymbol{C}]^{-\epsilon_9}\right)$$

Hu-Washizu functionals

$$\Pi^{D}_{HW}(\boldsymbol{q}) = \int_{\mathcal{B}_0} \Psi(\boldsymbol{C}(\boldsymbol{q}))\mathrm{d}V \qquad \Pi^{DP}_{HW}(\boldsymbol{q},\Theta,p) = \Pi^{D} + \int_{\mathcal{B}_0} p(J(\boldsymbol{q})-\Theta)\mathrm{d}V$$ [Simo 85]

$$\Pi^{CoFEM}_{HW}(\boldsymbol{q},\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]},\boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]},\dots) = \Pi^{DP}_{HW} + \int_{\mathcal{B}_0} \boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]} : (\mathrm{cof}[\boldsymbol{C}(\boldsymbol{q})] - \boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]})\mathrm{d}V$$ [Schr 11]

$$\Pi^{CoA}_{HW}(\boldsymbol{q},h_{I_4},b_{I_4},h_{I_5},b_{I_5},\dots) = \Pi^{CoFEM}_{HW} + \int_{\mathcal{B}_0} b_{I_4}(I_4(\boldsymbol{C})-h_{I_4})\mathrm{d}V + \int_{\mathcal{B}_0} b_{I_5}(I_5(\boldsymbol{C})-h_{I_5})\mathrm{d}V$$

Veubeke-Hu-Washizu functional

$$\Pi_{VHW}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{v},\boldsymbol{p},\dots) = \boxed{\int_T\int_{\mathcal{B}_0} \frac{1}{2}\rho_0 \boldsymbol{v}^T\boldsymbol{v}\mathrm{d}V\mathrm{d}t + \int_T\int_{\mathcal{B}_0} \boldsymbol{p}(\dot{\boldsymbol{q}}-\boldsymbol{v})\mathrm{d}V\mathrm{d}t} - \int_T \Pi_{HW}(\boldsymbol{q},\dots)\mathrm{d}t$$

$$+ \int_T \Pi^{ext}(\boldsymbol{q})\mathrm{d}t \quad \text{with} \quad \Pi^{ext}(\boldsymbol{q}) = \int_{\mathcal{B}_0} \rho_0 \boldsymbol{g}\cdot\boldsymbol{q}\mathrm{d}V + \int_{\partial\mathcal{B}_0} \boldsymbol{t}\cdot\boldsymbol{q}\mathrm{d}A$$

- Variation with respect to all unknowns

$\Pi^{D}_{VHW}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{v},\boldsymbol{p})$ $\quad \Pi^{DP}_{VHW}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{v},\boldsymbol{p},\Theta,p)$ $\quad \Pi^{CoFEM}_{VHW}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{v},\boldsymbol{p},\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]},\boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]},\Theta,p)$

$\Pi^{CoA}_{VHW}(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{v},\boldsymbol{p},\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]},\boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]},\Theta,p,h_{I_4},b_{I_4},h_{I_5},b_{I_5})$

Weak Form (CoFEM) $[\Psi = \Psi(\boldsymbol{C}, \mathrm{cof}[\boldsymbol{C}], J) = \Psi(\boldsymbol{C}, \boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}, \Theta)]$

$$\int_T \int_{\mathcal{B}_0} (\mathrm{div}[\boldsymbol{F}\underbrace{(2\frac{\partial \Psi}{\partial \boldsymbol{C}} + pJ^{-1}\mathrm{cof}[\boldsymbol{C}] + 2\boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]} : \mathbb{P})}_{\boldsymbol{S}}] - \dot{\boldsymbol{p}})\delta \boldsymbol{q} \mathrm{d}V \mathrm{d}t = 0 \qquad \mathbb{P} = \frac{\partial \mathrm{cof}[\boldsymbol{C}]}{\partial \boldsymbol{C}}$$

$$\boxed{\int_T \int_{\mathcal{B}_0} (\frac{1}{\rho_0}\boldsymbol{p} - \dot{\boldsymbol{q}})\delta \boldsymbol{p} \mathrm{d}V \mathrm{d}t = 0} \qquad \int_T \int_{\mathcal{B}_0} (p - \frac{\partial \Psi}{\partial \Theta})\delta \Theta \mathrm{d}V \mathrm{d}t = 0 \qquad \int_T \int_{\mathcal{B}_0} (\Theta - J)\delta p \mathrm{d}V \mathrm{d}t = 0$$

$$\int_T \int_{\mathcal{B}_0} (\boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]} - \frac{\partial \Psi}{\partial \boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}})\delta \boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]} \mathrm{d}V \mathrm{d}t = 0 \qquad \int_T \int_{\mathcal{B}_0} (\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]} - \mathrm{cof}[\boldsymbol{C}])\delta \boldsymbol{B}_{\mathrm{cof}[\boldsymbol{C}]} \mathrm{d}V \mathrm{d}t = 0$$

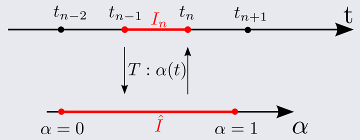
Space Discretization [Wrig 08]

$$\int_{\mathcal{B}_0} \ldots \mathrm{d}V = \sum_{e=1}^{n_{el}} \int_{\Omega_0^e} \ldots \mathrm{d}V \rightarrow \int_{\Omega^{\square}} \ldots \det[\boldsymbol{J}^e]\mathrm{d}\Omega$$

Time Discretization [Bets 01]

$$\int_T \ldots \mathrm{d}t = \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \ldots \mathrm{d}t \rightarrow h_n \int_0^1 \ldots \mathrm{d}\alpha$$

Time transformation



$h_n \ldots$ Time step size

Approximation $k =$ Polynomial degree in time

$$\boldsymbol{q}^{e,h} = \sum_{I=1}^{k+1} \sum_{A=1}^{nno_{\mathsf{u}}} M_I \mathrm{N}_{\mathsf{u}}^A \boldsymbol{q}_I^{eA} \qquad \delta\boldsymbol{q}^{e,h} = \sum_{J=1}^{k} \sum_{A=1}^{nno_{\mathsf{u}}} \tilde{M}_J \mathrm{N}_{\mathsf{u}}^A \delta\boldsymbol{q}_I^{eA} \qquad \dot{\boldsymbol{q}}^{e,h} = \frac{1}{h_n} \sum_{I=1}^{k+1} \sum_{A=1}^{nno_{\mathsf{u}}} M_I' \mathrm{N}_{\mathsf{u}}^A \boldsymbol{q}_I^{eA}$$

- Lagrangian shape functions in time

$$M_i(\alpha) = \prod_{\substack{j=1\\ j\neq i}}^{k+1} \frac{\alpha - \alpha_j}{\alpha_i - \alpha_j},\ 1 \leq i \leq k+1 \qquad \tilde{M}_i(\alpha) = \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\alpha - \alpha_j}{\alpha_i - \alpha_j}, 1 \leq i \leq k$$

- The other quantities are approximated in the same manner, e. g.:

$$\Theta^{e,h} = \sum_{I=1}^{k+1} \sum_{A=1}^{nno_{\Theta}} M_I(\alpha) \mathrm{N}_{\Theta}^A(\boldsymbol{\xi}) \Theta_I^{eA} \qquad \delta\Theta^{e,h} = \sum_{J=1}^{k} \sum_{A=1}^{nno_{\Theta}} \tilde{M}_J(\alpha) \mathrm{N}_{\Theta}^A(\boldsymbol{\xi}) \delta\Theta_I^{eA}$$

Time structure matrices:

$$\mathbf{A}' = \int_0^1 \begin{bmatrix} \tilde{M}_1 M_2 & \dots & \tilde{M}_1 M_{k+1} \\ \vdots & & \vdots \\ \tilde{M}_k M_2 & \dots & \tilde{M}_k M_{k+1} \end{bmatrix} \mathrm{d}\alpha \quad \mathbf{A}'' = \int_0^1 \begin{bmatrix} \tilde{M}_1 M_2' & \dots & \tilde{M}_1 M_{k+1}' \\ \vdots & & \vdots \\ \tilde{M}_k M_2' & \dots & \tilde{M}_k M_{k+1}' \end{bmatrix} \mathrm{d}\alpha \quad \mathbf{b} = \begin{bmatrix} \tilde{M}_1 \\ \vdots \\ \tilde{M}_k \end{bmatrix}$$

$$\mathbf{b}' = \int_0^1 \left[\tilde{M}_1 M_1 \quad \dots \quad \tilde{M}_k M_1\right]^T \mathrm{d}\alpha \qquad \mathbf{b}'' = \int_0^1 \left[\tilde{M}_1 M_1' \quad \dots \quad \tilde{M}_k M_1'\right]^T \mathrm{d}\alpha$$

Residuals on element level (CoFEM-Element and cG(k))

$$\boxed{\boldsymbol{r}_q^e = \mathbf{b}'' \boxtimes \mathbf{p}_1^{e,h} + [\mathbf{A}'' \boxtimes \mathbf{I}^{[3\cdot n_{no\mathsf{u}}]}]\mathbf{p}_{\text{new}}^{e,h} + h_n \int_0^1 \int_{\Omega_0^e} \mathbf{b} \boxtimes \left[(\mathbf{B}^e)^T \cdot [\overset{V}{\boldsymbol{S}^e}] \right] \mathrm{d}V \mathrm{d}\alpha + \mathbf{f}^{ext}}$$

$$\boxed{\boldsymbol{r}_p^e = \mathbf{b}'' \boxtimes \mathbf{q}_1^{e,h} + [\mathbf{A}'' \boxtimes \mathbf{I}^{[3\cdot n_{no\mathsf{u}}]}]\mathbf{q}_{\text{new}}^{e,h} - \frac{h_n}{\rho_0^e} \left([\mathbf{b}' \boxtimes \mathbf{p}_1^{e,h}] + [\mathbf{A}' \boxtimes \mathbf{I}^{[3\cdot n_{no\mathsf{u}}]}]\mathbf{p}_{\text{new}}^{e,h} \right)}$$

$$\mathbf{B}^e = \left[[\overset{V}{\boldsymbol{\mathcal{B}}^{e1}}] \, [\overset{V}{\boldsymbol{\mathcal{B}}^{e2}}] \dots [\overset{V}{\boldsymbol{\mathcal{B}}^{eA}}] \right] \qquad \boldsymbol{S}^e = 2 \left(\frac{\partial \Psi(\boldsymbol{C}^e, \boldsymbol{H}^{e,h}_{\mathrm{cof}[\boldsymbol{C}]}, \Theta^{e,h})}{\partial \boldsymbol{C}^e} + \frac{p^{e,h}}{2J^e} \mathrm{cof}[\boldsymbol{C}^e] + \boldsymbol{B}^{e,h}_{\mathrm{cof}[\boldsymbol{C}]} : \mathbb{P}^e \right)$$

$$\mathbf{q}_I^e = [\mathbf{q}_I^{e1} \, \mathbf{q}_I^{e2} \dots \mathbf{q}_I^{eA}]^T \quad \mathbf{q}_I^{e,h} = (\mathbf{H}_{\mathsf{uu}} \boxtimes \mathbf{I}^{[3\times 3]})\mathbf{q}_I^e \quad \mathbf{q}_{\text{new}}^{e,h} = [\mathbf{q}_2^{e,h} \, \mathbf{q}_3^{e,h} \dots \mathbf{q}_{k+1}^{e,h}]^T \quad \text{(same for } \mathbf{p}\text{)}$$

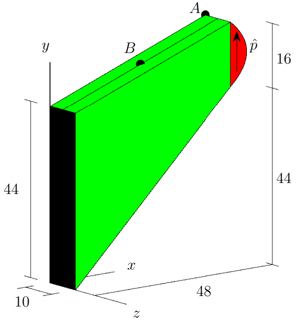
$$\mathbf{H}_{\mathsf{uu}} = \int_{\Omega_0^e} \mathbf{N}_{\mathsf{u}} \boxtimes \mathbf{N}_{\mathsf{u}}^T \mathrm{d}V \qquad \mathbf{N}_{\mathsf{u}} = [\mathrm{N}_{\mathsf{u}}^1 \, \mathrm{N}_{\mathsf{u}}^2 \dots \mathrm{N}_{\mathsf{u}}^A]^T$$

- Solve $\Theta_{\text{new}}^e, p_{\text{new}}^e,$ .... on element, e.g.

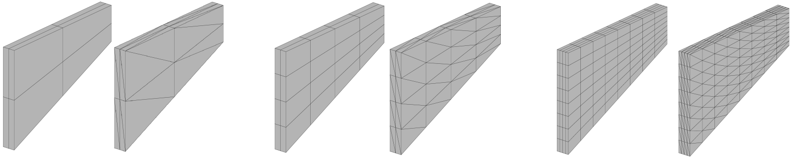
$$\Theta_{\text{new}}^e = [\mathbf{A}' \boxtimes \mathbf{H}_{\Theta\mathsf{p}}]^{-1} \left(\int_0^1 \int_{\Omega_0^e} \mathbf{b} \boxtimes \mathbf{N}_\Theta J^e \mathrm{d}V \mathrm{d}\alpha - \mathbf{b}' \mathbf{H}_{\Theta\mathsf{p}} \Theta_1^e \right) \qquad \mathbf{H}_{\Theta\mathsf{p}} = \int_{\Omega_0^e} \mathbf{N}_\Theta \boxtimes \mathbf{N}_{\mathsf{p}}^T \mathrm{d}V$$

- Eliminate $\mathbf{p}_{\text{new}}^{e,h}$ in first equation and condensate at element level to pure displacement form: $\tilde{\mathbf{K}}_{\mathsf{uu}}^e = \mathbf{K}_{\mathsf{uu}}^e + \mathbf{K}_{\mathsf{up}}^e \mathbf{K}_{\mathsf{p}\Theta}^e \mathbf{K}_{\Theta\Theta}^e \mathbf{K}_{\Theta\mathsf{p}}^e \mathbf{K}_{\mathsf{pu}}^e + \mathbf{K}_{\mathsf{ub}}^e \mathbf{K}_{\mathsf{bh}}^e \mathbf{K}_{\mathsf{hh}}^e \mathbf{K}_{\mathsf{hb}}^e \mathbf{K}_{\mathsf{hu}}^e$

- Quadratic distribution of an in-plane load with the pressure amplitude $\hat{p} = 1500$
- Compare the non-standard mixed elements with the standard displacement element for tetrahedral and hexahedral elements (serendipity formulation)
- $h_n = 0.01, T_{End} = 1, \boldsymbol{a}_0 = [1\ 1\ 1]$



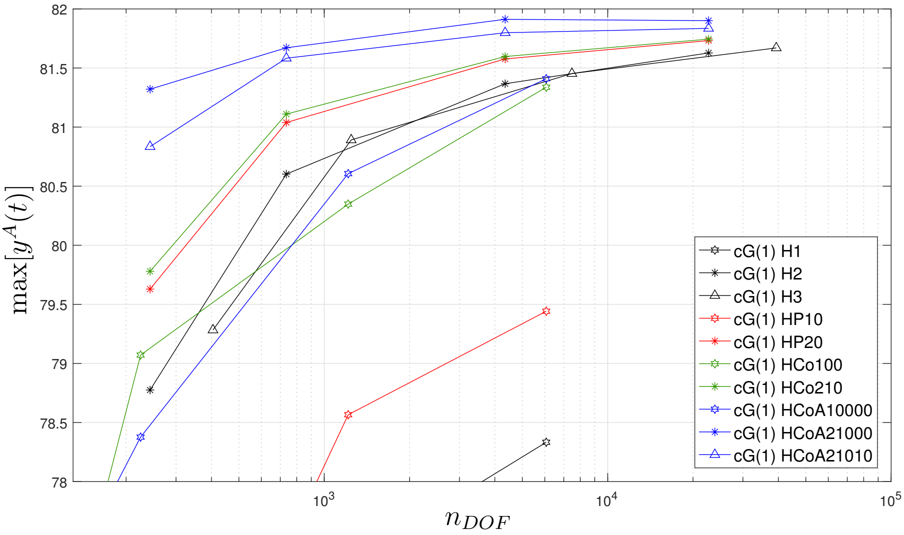
- Refinement levels:



- Material parameters ([Schr 11]):

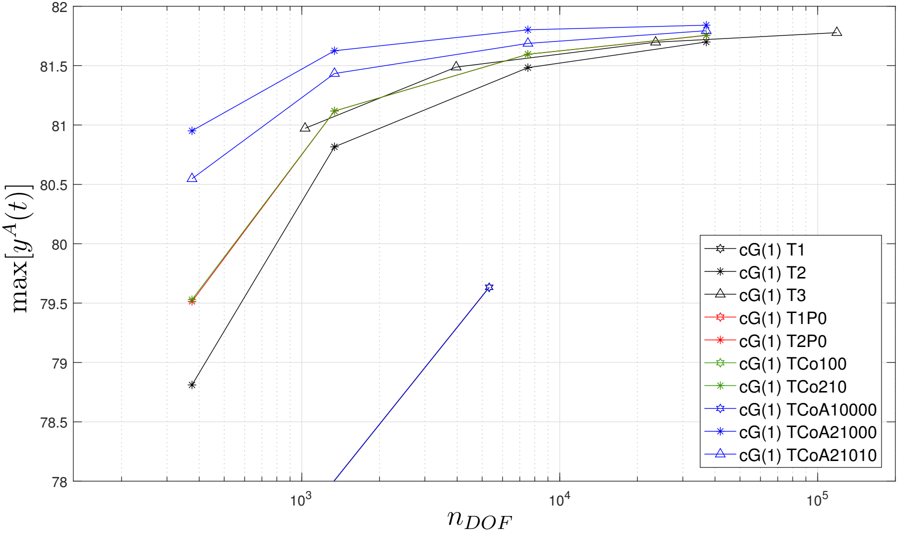
ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ρ_0
42	84	1260	100	10	3000	4	8	1	0.01

Numerical example - Cook cantilever beam
Convergence of the y-coordinate at Point A



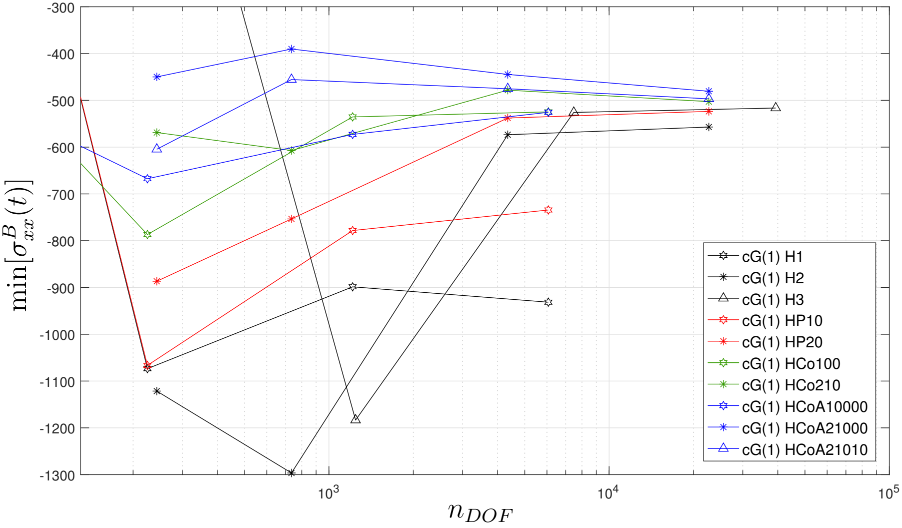
H[$\boldsymbol{q}$] HP[$\boldsymbol{q}$,Θ] HCo[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] HCoA[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]

Numerical example - Cook cantilever beam
Convergence of the y-coordinate at Point A



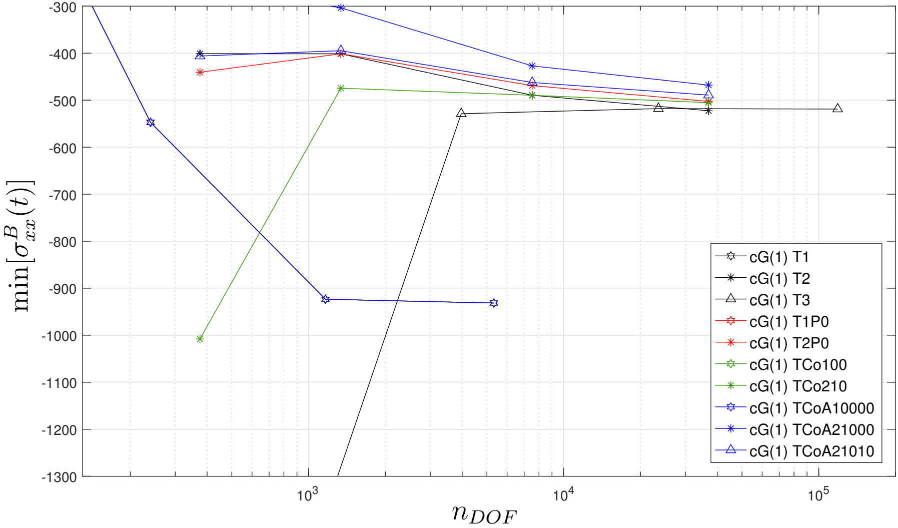
T[q] TP[q,Θ] TCo[q,$H_{\mathrm{cof}[C]}$,Θ] TCoA[q,$H_{\mathrm{cof}[C]}$,Θ,h_{I_4},h_{I_5}]

Numerical example - Cook cantilever beam
Convergence of σ_{xx} at Point B



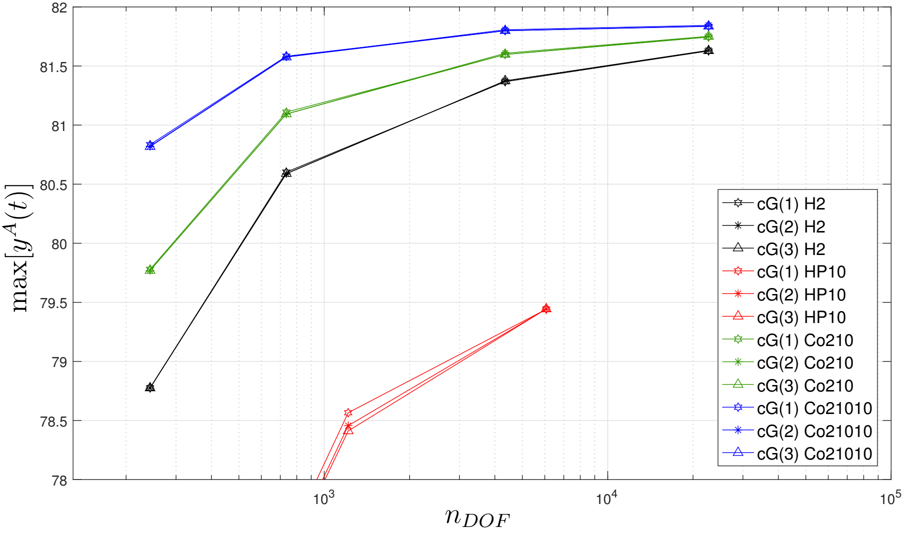
H[q] HP[q,Θ] HCo[q,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] HCoA[$q$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]

Numerical example - Cook cantilever beam
Convergence of σ_{xx} at Point B



T[$\boldsymbol{q}$] TP[$\boldsymbol{q}$,Θ] TCo[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] TCoA[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]

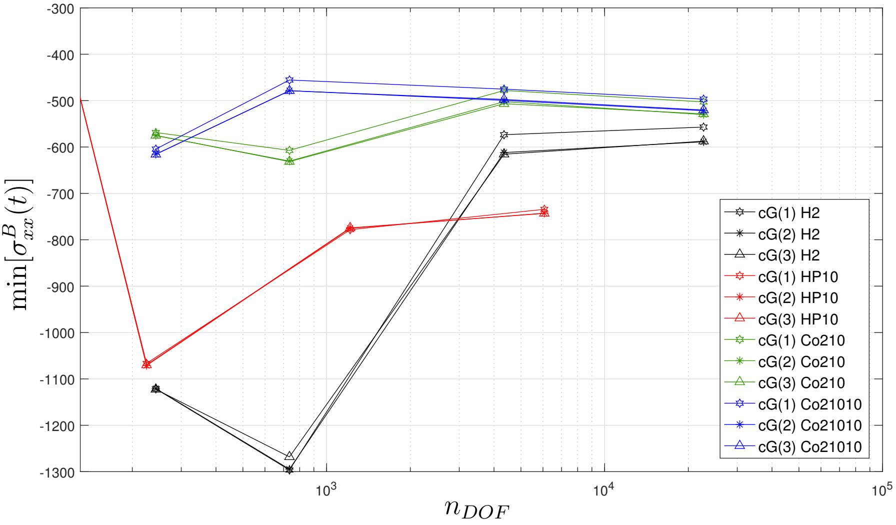
Numerical example - Cook cantilever beam
Influence of time polynomial degree - Convergence of the y-coordinate at Point A



H$[\boldsymbol{q}]$ HP$[\boldsymbol{q},\Theta]$ HCo$[\boldsymbol{q},\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]},\Theta]$ HCoA$[\boldsymbol{q},\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]},\Theta,h_{I_4},h_{I_5}]$

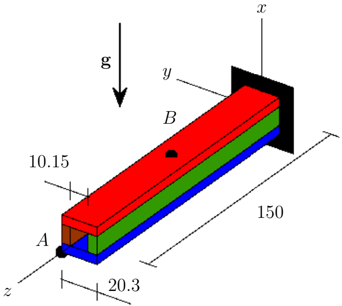
Numerical example - Cook cantilever beam

Influence of time polynomial degree - Convergence of σ_{xx} at Point B

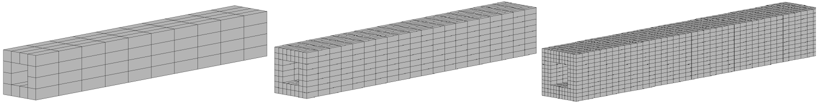


H[q] HP[q,Θ] HCo[q,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] HCoA[$q$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]

- Oscillating in a gravity field
  $$\boldsymbol{g} = \begin{bmatrix} -9.81 & 0 & 0 \end{bmatrix}^T$$
- Compare the non-standard mixed elements with the standard displacement element for hexahedral elements (serendipity formulation)
- $h_n = 0.1, T_{End} = 50$
- Different fiber directions:
  $\boldsymbol{a}_0 = [0 \; -0.3095 \; -0.9509]$
  $\boldsymbol{a}_0 = [0.3095 \; 0 \; 0.9509]$
  $\boldsymbol{a}_0 = [0 \; 0.3095 \; -0.9509]$
  $\boldsymbol{a}_0 = [0.3095 \; 0 \; -0.9509]$



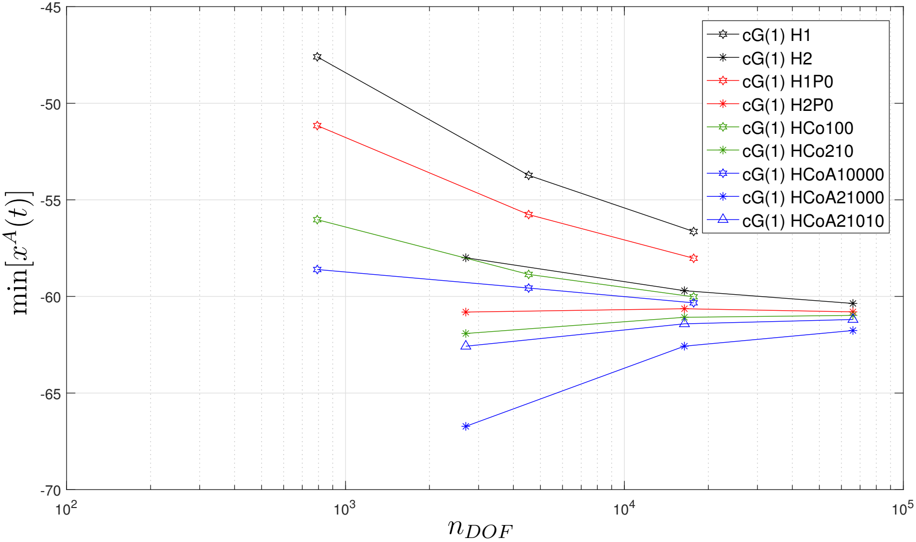
- Refinement levels:



- Material parameters([Schr 11]):

ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_5	ϵ_6	ϵ_7	ϵ_8	ϵ_9	ρ_0
42	84	1260	100	10	3000	4	8	1	0.08

Numerical example - Oscillating cantilever beam
Convergence of the x-coordinate at Point A



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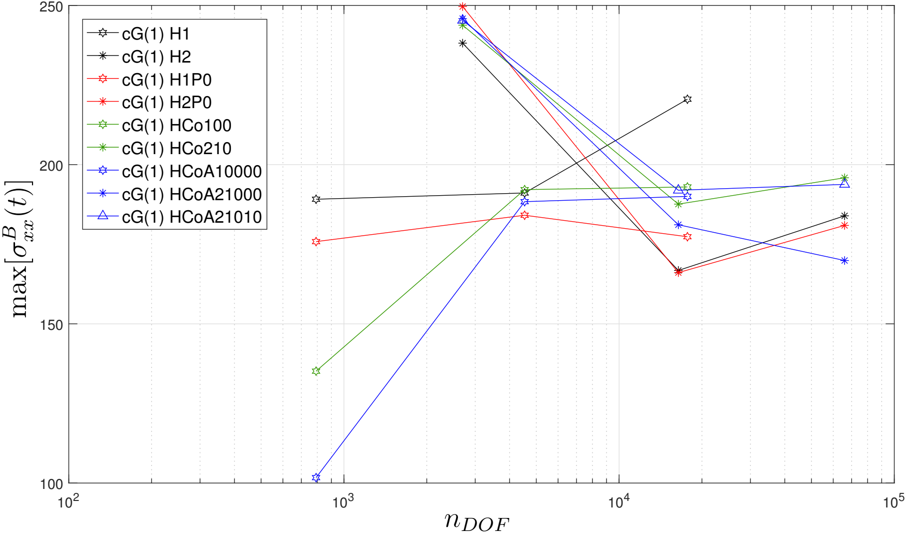
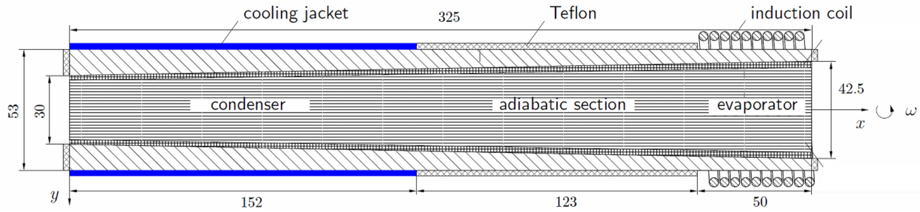
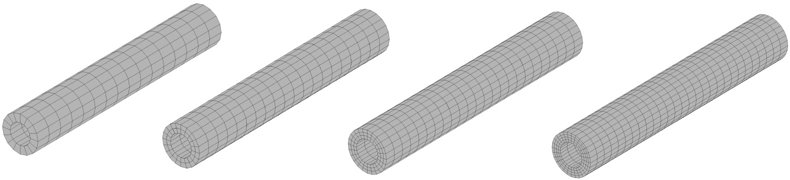
Numerical example - Oscillating cantilever beam
Convergence of σ_{xx} at Point B



H[$\boldsymbol{q}$] HP[$\boldsymbol{q}$,Θ] HCo[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] HCoA[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]



- $\Omega_{[0\ 0\ 0]} = [10\ 0\ 0], h_n = 0.00025, T_{end} = 1, \boldsymbol{a}_0 = [1\ 0\ 0], \hat{p} = 400$
- Refinement levels:



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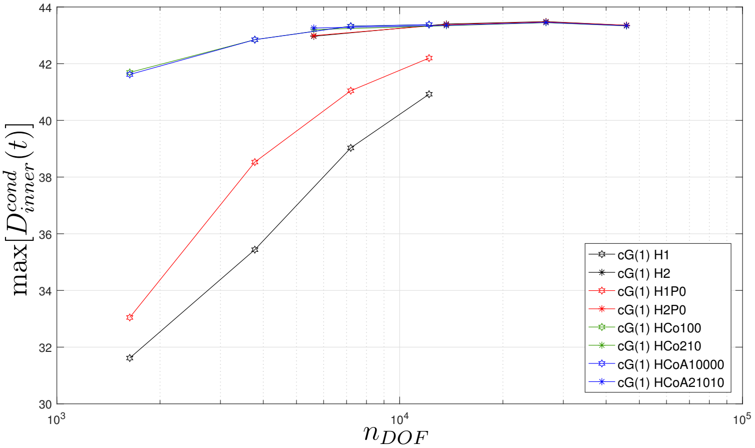
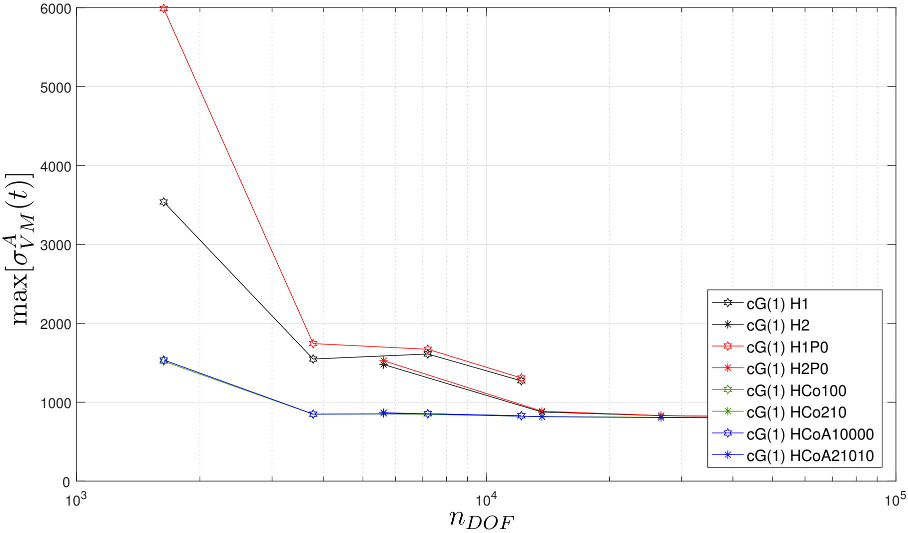
42	84	1260	100	10	3000	4	8	1	0.01

Numerical example - Rotating heatpipe
Convergence of the inner diameter at condenser site



H[q] HP[q,Θ] HCo[q,$H_{\mathrm{cof}[C]}$,Θ] HCoA[q,$H_{\mathrm{cof}[C]}$,Θ,h_{I_4},h_{I_5}]

Numerical example - Rotating heatpipe
Convergence of σ_{VM} on outer surface



H[$\boldsymbol{q}$] HP[$\boldsymbol{q}$,Θ] HCo[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$] HCoA[$\boldsymbol{q}$,$\boldsymbol{H}_{\mathrm{cof}[\boldsymbol{C}]}$,$\Theta$,$h_{I_4}$,$h_{I_5}$]

Conclusion

- Motivation:
  - Dynamic simulations of fiber-reinforced materials in light-weight structures
- Strategy:
  - Mixed finite elements for finite anisotropic elastodynamics
  - Higher-order time integrators [continuous Galerkin cG(k)]
- Important results:
  - Mixed element reduce locking effect
  - Higher-order time integrators increase accuracy
- Outlook:
  - Investigate other mixed element formulations (e.g. SKA-Element [Schr 16])
  - Apply discrete gradient to mixed formulations for energy-momentum conserving

Literatur I

P. Betsch and P. Steinmann.
"Conservation properties of a time FE method—part II: Time-stepping schemes for non-linear elastodynamics".
International Journal for Numerical Methods in Engineering, Vol. 50, No. 8, pp. 1931–1955, 2001.

J. Schröder, P. Wriggers, and D. Balzani.
"A new mixed finite element based on different approximations of the minors of deformation tensors".
Computer Methods in Applied Mechanics and Engineering, Vol. 200, No. 49, pp. 3583–3600, 2011.

Literatur II

J. Schröder, N. Viebahn, D. Balzani, and P. Wriggers.
"A novel mixed finite element for finite anisotropic elasticity; the SKA-element Simplified Kinematics for Anisotropy".
Computer Methods in Applied Mechanics and Engineering, Vol. 310, No. , pp. 475 – 494, 2016.

J. Simo, R. Taylor, and K. Pister.
"Variational and projection methods for the volume constraint in finite deformation elasto-plasticity".
Computer Methods in Applied Mechanics and Engineering, Vol. 51, No. 1, pp. 177–208, 1985.

P. Wriggers.
Nonlinear finite element methods .
Vol. , Springer, Berlin, 2008.
Hier auch später erschienene, unveränderte Nachdrucke.