Adaptivity methods in Pro/Mechanica Structure

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Introduction

This note describes the methods used for polynomial order adaptivity in Pro/Mechanica Structure. The emphasis is on an overall description and explanation of the relative advantages of each approach, rather than on the details of how these methods are implemented and how they can be controlled by the user. Please refer to Pro/Mechanica documentation for more specific usage information.

Pro/Mechanica uses p-type finite elements to compute its solution. One of the key advantages of p-type finite elements is that they allow solution adaptivity without requiring mesh refinement. With standard "h-type" finite elements, once a solution is obtained, the only way to improve its quality consists of repeating the calculation using a finer mesh - this process is well known to be time consuming, complex and problematic in several ways. In contrast, with p-type elements, the maximum polynomial orders of basis functions used to approximate the solution can be increased locally as needed. The solution process can then be repeated on the same mesh, with the new increased polynomial orders. Such an adaptivity step (often called a *pass* in Pro/Mechanica) can be repeated, if desired, to achieve even greater accuracy.

In the p-type finite element method, basis functions are constructed in a way that the maximum polynomial order of approximation can be selected independently for each edge, face, and solid in the mesh. Pro/Mechanica selects polynomial orders independently for each edge, and the polynomial order of approximation on each face and solid is then selected based on the choices made for the underlying edges. Therefore, the goal of a p-order adaptivity method consists of selecting appropriate polynomial orders for each edge which will lead to an overall solution providing good quality results using acceptable elapsed time and computing resource.

These polynomial orders should be high enough to provide the desired degree of accuracy, but as low as possible in order to prevent the introduction of unnecessary degrees of freedom in the solution process, which in turn lead to degraded performance and increased system resource requirements.

Uniform polynomial escalation

The simplest adaptivity method consists of uniform polynomial escalation. The solution is started with a low polynomial order p (typically $p=1$ or $p=2$) and then repeated, increasing p by one at each pass on all edges in the mesh. The process can be stopped when the desired accuracy is achieved. This method is not used in Pro/Mechanica, but it is described here only for the purpose of explanation. In fact, there are several problems with using uniform polynomial escalation.

First of all, since the polynomial order is increased on all edges in the same way, it results in much larger numbers of degrees of freedom than actually necessary. This is because typically a small number of critical edges in regions where results are changing rapidly will drive all the remaining edges to a high polynomial order.

Second, one still has to make a decision on when to stop. This requires estimating the accuracy of the solution obtained at the last pass and stopping if this is better than the desired accuracy. One approach to this consists of comparing the solution obtained at the last pass with the previous one, and using the difference between these two solutions (loosely speaking) as an estimate of the accuracy achieved. If this approach is applied simplistically, situations may occur during a solution which may affect its robustness and reliability.

In particular, it may be possible that the convergence of a model is driven by large differences in one region of the mesh (say region A) in a particular pass of the solution (say pass N). In this case the *p*-levels will be increased in that region for pass N+1. However, there may be other areas of significant differences (although less than in region A) and these regions may not benefit from increased *p*-levels in pass N+1 and therefore the differences in pass N+1 in these regions may be reported as artificially low.

Multi-Pass Adaptivity (MPA)

Multi-Pass Adaptivity (MPA) was the first adaptivity method provided in Pro/Mechanica. Instead of increasing polynomial orders by one on every edge at each pass, the MPA algorithm tries to identify which areas of the model require additional accuracy, and increases by one or two at each pass only the polynomial order of edges in those areas.

To identify the edges which need a polynomial order increase, the MPA algorithm compares displacements and element strain energies at the last pass with the corresponding values at the previous pass. Where the difference is larger than the user-specified accuracy, the polynomial order is increased. Otherwise, it is left unchanged. This process is repeated until the required overall convergence criteria for the solution as specified by the user are met. These criteria may include percentage differences of default local and global quantities appropriate to the problem type such as displacement, strain energy and r.m.s. stress, but can also involve user defined measures.

The MPA algorithm uses several techniques developed over many releases to minimize the chance of missing regions of the model which require increased *p*-levels, although these techniques are primarily based on user experience and by automating the accepted manual method of achieving a converged solution by adding solution refinement to regions where errors are high.

However, although there is no absolute theoretical basis for an approach based on comparing one pass with the next which can be guaranteed to avoid such problems, experience has shown that for most real engineering situations, MPA provides a good, robust solution, especially when used in conjunction with user-defined measures and for generating extremely accurate results in regions of special interest such as at stress concentrations or crack tips.

Single-Pass Adaptivity (SPA)

Within recent years Pro/Mechanica has introduced and refined an alternative and theoretically superior algorithm called Single Pass Adaptivity (SPA) which uses a more rigorous method to estimate and improve solution accuracy. As most finite element analysts know, looking at raw stresses (stresses computed directly from displacement derivatives) can be very selective in identifying model areas of low accuracy. Those stresses are discontinuous at element boundaries, and the amplitude of their jump at the discontinuity is a good estimator of stress accuracy. This fact has a solid mathematical basis – if not in an element-by-element sense, at least in a global way.

By measuring these stress jumps, one can obtain an assessment of the local accuracy of a solution, without reference to a previous pass. In the SPA algorithm, element error indicators are computed which essentially measure the average stress jump around each element. The polynomial orders of edges belonging to elements with large errors are increased. Edges of elements with larger error receive a higher polynomial order increase than edges of elements with lower errors. Polynomial orders on the edges of elements with low error at left unchanged.

Experimentation has shown that most of the accuracy gain occurs after the first pass of adaptivity. Therefore, the most efficient mode of operation has been found to consist of starting with a low uniform polynomial order ($p=3$ was found to be the best compromise between accuracy and performance), computing element error indicators based on this solution, updating polynomial orders, and repeating the solution. The result obtained at this point is taken as the final answer, and element error indicators are recomputed to give the user a global indication of average stress accuracy achieved.

The parameters that control how the polynomial order increase on each edge is related to the element error indicators for that edge have been tuned to make sure that the SPA solution will provide – on the average – good engineering accuracy at a computational cost which is a small fraction of the computational cost incurred with MPA. The cost reduction is due to the fact that the polynomial order will typically stay at $p=3$ for the majority of edges in the model, with a small percent of the edges in critical areas at higher polynomial orders.

It should be noted, however, that in rare cases where users are primarily interested in results which are significantly lower than the maximum values in the model, then the convergence errors in the results at those locations may be greater than expected. This is because the errors relate to a percentage of a much higher result (the maximum value in the model). In these cases, users may benefit from using MPA to drive up the p-orders in areas away from the model maximum by specifying extremely stringent global convergence criteria. In future versions of Pro/Mechanica, automatic methods for identifying and reducing convergence errors using SPA in user defined regions of the model will be introduced.

Experience gained over the past few releases by commercial users and quality assurance engineers at PTC has demonstrated that for the majority of general engineering situations, the SPA algorithm usually provides better accuracy than MPA at a substantially lower computational cost. This experience, in conjunction with the sound theoretical basis underlying the SPA algorithm, means that SPA should be used as the preferred approach wherever appropriate.