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**Mathematical Foundation of Big Data Analytics (SS 2019)**  
**Recommendation System I**

**Ex. 1** Recall the sparse rating matrix given in the lecture

$$\begin{bmatrix} 5 & 3 & - & 1 \\ 4 & - & - & 1 \\ 1 & 1 & - & 5 \\ 1 & - & - & 4 \\ - & 1 & 5 & 4 \end{bmatrix}$$

a) Calculate the similarity of the users based on the cosine measure

$$S(i, \ell) = \frac{\sum_{j \in U(i) \cap U(\ell)} (a_{ij} - \bar{a}_i) \cdot (a_{\ell j} - \bar{a}_\ell)}{\sqrt{\sum_{j \in U(i) \cap U(\ell)} (a_{ij} - \bar{a}_i)^2} \cdot \sqrt{\sum_{j \in U(i) \cap U(\ell)} (a_{\ell j} - \bar{a}_\ell)^2}}$$

b) Based on a), estimate the missing ratings by KNN-Algorithm.  
 c) Evaluate the method and compare the results to matrix factorization approach.

**Ex. 2** Calculate the singular value decomposition (SVD) of the following matrix:

$$A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

Suppose  $A$  is a rating matrix. How many relevant features are there?

**Ex. 3** Let  $L \in \mathbb{R}^{n,k}$  and  $R \in \mathbb{R}^{k,m}$  are two matrices, with  $\text{rank}(L) = \text{rank}(R) = k$ ., where  $k < m, n$ . Show that the matrix  $L \cdot R$  has rank k.

**Ex. 4** Use the LR factorization in order to solve

$$\min_{\text{rank}(A_k)=k} \|A - A_k\|_F^2$$

by gradient methods and give the formula for the gradient steps.