Prof. Dr. Vladimir Shikhman
Professur für Wirtschaftsmathematik
Technische Universität Chemnitz

Übungsleiter: David Müller
david.mueller@mathematik.tu-chemnitz.de

Mathematical Foundation of Big Data Analytics (SS 2019)
Sparse Recovery

Ex. 1 Look at the following function:

$$f(w) = \frac{1}{2}\|\Phi w - y\|^2$$

a) Show that for a matrix Φ and its spectral norm $\|\Phi\|$ the following holds:

$$\|\Phi\| = \sqrt{\lambda_{max}(\Phi^T\Phi)}$$

, where λ_{max} denotes the largest eigenvalue.

b) Show that the gradients of $f(w)$ are bounded and that their Lipschitz constant can be expressed in terms of the spectral norm of Φ.

Ex. 2 The ridge regression problem is given by

$$\min_w \frac{1}{2}\|\Phi w - y\|^2 + \frac{1}{2}\alpha\|w\|^2.$$

a) Derive the problem from the maximum posterior Bayes approach (MAP) by choosing a suitable prior distribution for the weights $w_j, j = 0, \ldots, M$. Compare it to the LASSO-problem.

b) Calculate the optimal solution. Is this solution always unique?

Ex. 3 Determine expected value and variance of the Laplace distribution with density

$$f(x; \tau, \sigma) = \frac{1}{2\sigma}e^{-\frac{|x-\tau|}{\sigma}}.$$