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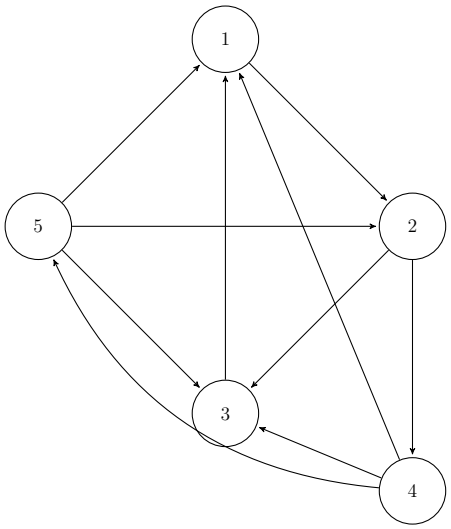
Mathematical Foundations of Big Data Analytics (SS 2019)
Lectorial 1: Ranking I

Exercise 1) Compute the ranking for the network shown in lecture 1:



Exercise 2) Given the dual problem

$$\max_{y,y_{n+1}} y_{n+1} \quad s.t. \quad y \leq P^T \cdot y - y_{n+1} \cdot e, \quad y, y_{n+1} \geq 0.$$

a) Show feasibility of $\bar{y} = e,\ \bar{y}_{n+1} = 0$.

b) Prove optimality of the solution from a).

Exercise 3) Within an exchange economy the Producer P_i manufactures exactly one entity of the good $G_i,\ i = 1, \ldots, n$. For that, the producer P_i employs $a_{ij} \geq 0$ of the goods $G_j,\ j = 1, \ldots, n$. The equilibrium prices of goods are such, that the corresponding cost of the agents' production does not exceed their revenues.

a) Compute and interpret a matrix similar to a transition matrix.

b) Show that the vector of equilibrium prices can be interpreted as a ranking.

c) How is it possible to adjust prices of goods in order to reach an equilibrium?

Exercise 4) Let a stochastic matrix P be given. The sequence of Césaro means is defined by

$$\bar{x}(s) = \frac{1}{s+1} \sum_{t=0}^{s} x(t),$$

where $x(0)$ is a starting distribution and $x(t)$ is formed by

$$x(t) = P \cdot x(t-1) \quad \text{for} \quad t = 1, 2, \ldots.$$

a) Show that $\bar{x}(s)$ is a distribution.

b) Establish the inequality

$$||\bar{x}(s) - P \cdot \bar{x}(s)||_1 \leq \frac{2}{s+1}.$$