

# Third Central European Set-Valued and Variational Analysis Meeting

## CESVVAM, TU Chemnitz, 25.11.2017

The Central European Set-Valued and Variational Analysis Meeting is an informal meeting to hear the latest developments from the "local" community in set-valued and variational analysis. It has been initiated by Russell Luke in [Göttingen, 2015](#). The second edition took place in [Jena, 2016](#) organized by Andreas Löhne. We are glad to welcome the participants from Austria, Czech Republic, Germany, Netherlands, and Poland for the third CESVVAM in Chemnitz.

**Venue:** TU Chemnitz, [Straße der Nationen 62](#), [Altes Heizhaus](#)

### Schedule

- Friday, November 24, 2017, 7:00 pm, informal dinner at [Michaelis Restaurant](#) (Am Düsseldorfer Platz 1, 09111 Chemnitz)
- Saturday, November 25, 2017, 8:50 am - 6:00 pm, meeting
- Saturday, November 25, 2017, from 7:00 pm, mini-city tour and informal get-together

### Organizing team:

- [Sorin-Mihai Grad](#)
- [Thomas Jahn](#)
- [Vladimir Shikhman](#)
- [Gerd Wachsmuth](#)
- [Oleg Wilfer](#)

**Program** (TU Chemnitz, [Straße der Nationen 62](#), [Altes Heizhaus](#), Saturday, November 25, 2017, 8:50 am - 6:00 pm)

<b>Time</b>	<b>Activity</b>
<b>8:50 – 09:00</b>	<b>Opening</b>
9:00 – 9:30	Andrzej Cegielski, Zielona Góra Regular sequences of quasi-nonexpansive operators and their applications
9:30 – 10:00	Oleg Wilfer, Chemnitz Proximal methods for solving nonlinear minimax location problems with perturbed minimal time functions via conjugate duality
10:00 – 10:30	Sandy Bitterlich, Chemnitz Proximal-AMA and application in image deblurring and denoising
<b>10:30 – 11:00</b>	<b>Coffee Break</b>
11:00 – 11:30	Patrick Mehrlitz, Freiberg On optimal control problems with control complementarity constraints
11:30 – 12:00	Matus Benko, Linz Calculus for directional limiting normal cones and subgradients
12:00 – 12:30	Thomas Surowiec, Marburg Aspects of Variational Analysis in Risk-Averse PDE-Constrained Optimization
<b>12:30 – 14:30</b>	<b>Lunch</b>
14:30 – 15:00	Christian Günther, Halle-Wittenberg Jahn-Graef-Younes type algorithms for discrete vector optimization based on cone-monotone sorting functions
15:00 – 15:30	Yurii Malitsky, Göttingen Golden Ratio Algorithms for Variational Inequalities
15:30 – 16:00	Anna-Lena Martins, Göttingen Relaxed Averaged Alternating Reflections (RAAR): an approach to inconsistent feasibility problems
<b>16:00 – 16:30</b>	<b>Coffee Break</b>
16:30 – 17:00	Hieu Thao Nguyen, Delft On linear convergence of projection algorithms for phase retrieval problem
17:00 – 17:30	Matthew Tam, Göttingen Symbolic convex analysis
17:30 – 18:00	Helmut Gfrerer, Linz Stability Analysis for Parameterized Equilibria with Conic Constraints
18:00	<b>Closing</b>

**9:00 – 9:30 Andrzej Cegielski, Zielona Góra,  
Regular sequences of quasi-nonexpansive operators and their applications**

We present a systematic study of regular sequences of quasi-nonexpansive operators in Hilbert space. We are interested, in particular, in weakly, boundedly and linearly regular sequences of operators. We show that these types of the regularity are preserved under relaxations, convex combinations and products of operators. Moreover, in this connection, we show that the weak, bounded and linear regularity lead to weak, strong and linear convergence, respectively, of various iterative methods. This applies, in particular, to block iterative and string averaging projection methods, which, in principle, are based on the above-mentioned algebraic operations applied to projections. Finally, we show an application of regular sequences of operators to variational inequality problems. In collaboration with Simeon Reich and Rafał Zalas.

**9:30 – 10:00 Oleg Wilfer, Chemnitz  
Proximal methods for solving nonlinear minimax location problems with perturbed minimal time functions via conjugate duality**

We investigate minimax location problems formulated by means of an extended perturbed minimal time function via a conjugate duality approach, necessary and sufficient optimality conditions being delivered together with characterizations of the optimal solutions in some particular instances. Afterwards we present a proximal method in order to numerically solve such problems, concrete location problems being solved with MATLAB. In order to introduce the general minimax location problems we propose a new perturbed minimal time function that generalizes the original minimal time function and several of its recent extensions. The motivation to investigate such problems comes from both theoretical and practical reasons, as location type problems arise in various areas of research and real life, such as geometry, physics, economics or medical management, problems from these fields being mentioned as possible interpretations of our results.

**10:00 – 10:30 Sandy Bitterlich, Chemnitz  
Proximal-AMA and application in image deblurring and denoising**

The Alternating Minimization Algorithm (AMA) introduced by Tseng (1991) is an algorithm for separable convex programming problems that minimizes the sum of a strongly convex and a convex function under linear constraints. We propose a Proximal Alternating Minimization Method (Proximal AMA), which is based on the AMA algorithm. Compared to the objective function considered for the AMA algorithm, our objective contains additional smooth functions. Our algorithm performs a proximal step for the convex function which makes it attractive to several applications such as image processing problems.

**11:00 – 11:30 Patrick Mehrlitz, Freiberg  
On optimal control problems with control complementarity constraints**

Complementarity constraints on control functions can be used to model switching requirements or result from transferring a bilevel optimal control problem with lower level control constraints into a single-level optimization problem using lower level optimality conditions. In our talk, we address several different features of optimal control problems with complementarity constraints. First, we point out that the existence of optimal solutions is closely related to the underlying function space setting. Due to the complementarity structure, constraint qualifications of reasonable strength fail to hold at all feasible points of the optimal control problem. However, we show that necessary optimality conditions of strong stationary type are valid at local optimal solutions. Finally, we present a penalization method based on smoothed NCP-functions which can be used to compute global optimal solutions of optimal control problems with control complementarity constraints. This talk is based on a joint ongoing work with Christian Clason.

**11:30 – 12:00 Matus Benko, Linz**

**Calculus for directional limiting normal cones and subgradients**

In the recent years, directional versions of the limiting (Mordukhovich) normal cone, the coderivative of a multifunction, the metric subregularity, etc. have been intensively studied by Gfrerer, yielding very interesting results. In this talk we present some basic calculus rules for these limiting objects valid under mild assumptions. E.g. we provide (upper) estimates for the directional limiting normal cone of a constraint set, the directional limiting subdifferential of a composition of functions, the directional coderivative of a composition of multifunctions, etc. We conclude the talk by mentioning some applications of the proposed calculus.

**12:00 – 12:30 Thomas Surowiec, Marburg**

**Aspects of Variational Analysis in Risk-Averse PDE-Constrained Optimization**

A wide array of problems in the engineering and physical sciences are laden with uncertainty. As a result, the associated optimal decision-making problems are distributed parameter stochastic programs. Taking a cue from risk management studies, we employ convex and coherent risk measures to robustly handle the implicit uncertainty in the objective functionals. In order to take advantage of efficient numerical approximation techniques and optimization algorithms from PDE-constrained optimization, it is often necessary to find a suitable regularization scheme for (generally non-smooth) coherent risk measures. To this aim, we propose a variational smoothing technique, which we call “epi-regularization”. This provides a comprehensive framework for studying the effects on the axioms of coherency and regularity, differentiability, variational convergence, and consistency of minimizers and first-order stationary points. In cases of local convexity of the composite objective functional, rigorous error estimates can be derived. The theoretical results are confirmed by several numerical experiments.

**14:30 – 15:00 Christian Günther, Halle-Wittenberg**

**Jahn-Graef-Younes type algorithms for discrete vector optimization based on cone-monotone sorting functions**

In this talk we present new Jahn-Graef-Younes type algorithms for solving discrete vector optimization problems. In order to determine the minimal elements of a finite set with respect to an ordering cone, the original algorithmic approach by Jahn (2006) consists of a forward iteration (Graef-Younes method) in a first phase, followed by a backward iteration in a second one. Our approach is based on a pre-sorting scheme via certain cone-monotonic functions. We derive new implementable algorithms for solving discrete vector optimization problems. In particular, we analyze the case where the ordering cone is polyhedral. In our computational studies we use the "Multiobjective Search Algorithm with Subdivision Technique (MOSAST)" (see Jahn, 2006) that is based on the subdivision technique introduced by Dellnitz et al. (2005). Using MOSAST it is possible to approximate the set of Pareto efficient solutions of continuous multi-objective optimization problems by applying algorithms for discrete vector optimization problems. We present computational results obtained by our algorithms in MATLAB and make comparisons with the original Jahn-Graef-Younes method.

**15:00 – 15:30 Yurii Malitsky, Göttingen**

**Golden Ratio Algorithms for Variational Inequalities**

We present several novel methods for solving general (pseudo-) monotone variational inequalities. The first method uses fixed stepsize and is similar to the proximal reflected gradient method: it also requires only one value of operator and one prox-operator per iteration. However, its extension — the dynamic version — has a notable distinction. In every iteration it defines a stepsize, based on a local information about operator, without running any linesearch procedure. Thus, the iteration costs of this method are almost the same as in the first one with a fixed stepsize, but it converges without the Lipschitz assumption on the operator. We further discuss possible generalizations of the methods, in particular for solving large-scale nonlinear saddle point problems. Some numerical experiments are reported.

**15:30 – 16:00 Anna-Lena Martins, Göttingen**

**Relaxed Averaged Alternating Reflections (RAAR): an approach to inconsistent feasibility problems**

Due to its good numerical performance, the RAAR algorithm is of great interest for the phase retrieval problem. Despite its reputation, relatively little work has been done on understanding the algorithm in the inconsistent setting; the setting that is, in fact, the natural one for many real-world problems. Utilizing a recent result of Luke, Nguyen and Tam regarding convergence of Picard iterations, we investigate this case. By exploiting the location of fixed points of the RAAR algorithm, we are able to prove local linear convergence in the inconsistent case.

**16:30 – 17:00 Hieu Thao Nguyen, Delft**

**On linear convergence of projection algorithms for phase retrieval problem**

We report on an ongoing project about efficient projection algorithms for phase retrieval problem. A linear convergence result is established under natural assumptions on the problem data. The question of how likely these assumptions are satisfied in the context of phase retrieval problem will be discussed. In collaboration with Michel Verhaegen.

**17:00 – 17:30 Matthew Tam, Göttingen**

**Symbolic convex analysis**

In this talk we consider a class of monotone operators which are appropriate for symbolic representation and manipulation within a computer algebra system. Various structural properties of the class (e.g., closure under taking inverses, resolvents) are investigated as well as the role played by maximal monotonicity within the class. In particular, we show that there is a natural correspondence between our class of monotone operators and the subdifferentials of convex functions belonging to a class of convex functions deemed suitable for symbolic computation of Fenchel conjugates which were previously studied by Bauschke & von Mohrenschildt and by Borwein & Hamilton. A number of illustrative computational examples utilising the introduced class of operators will be provided including computation of proximity operators, recovery of a convex penalty function associated with the hard thresholding operator, and computation of superexpectations, superdistributions and superquantiles with specialization to risk measures.

**17:30 – 18:00 Helmut Gfrerer, Linz**

**Stability Analysis for Parameterized Equilibria with Conic Constraints**

We consider parameterized equilibria governed by generalized equations whose multivalued parts are modeled via regular normals to nonconvex conic constraints. The main goal is to derive sufficient conditions for isolated calmness of the solution map. In doing so we compute that part of the graphical derivative of the normal cone mapping which is important for our analysis. This talk is based on joint work with B. Mordukhovich.