Let $\mathbb{V}$ be a complex affine line, and $\mathcal{M}$ a holonomic $\mathcal{D}_{\mathbb{V}}$-module. The Fourier-Laplace transform $\mathcal{M}^{\wedge}$ of $\mathcal{M}$ is a holonomic $\mathcal{D}$-module on the dual affine line $\mathbb{V}^*$. Even if $\mathcal{M}$ is regular, $\mathcal{M}^{\wedge}$ is irregular in general. It is natural and important to try to describe the Stokes structure of $\mathcal{M}^{\wedge}$ in terms of the Stokes structure of $\mathcal{M}$. In the literature dealing with this problem, let us mention in particular the papers by Malgrange [4], Mochizuki [5],[6], Hien-Sabbah [3], D'Agnolo-Hien-Morando-Sabbah [2].

Malgrange gave a comprehensive treatment in [4]. Mochizuki has given a recipe for a complete description of the Fourier-Laplace transform of a general $\mathcal{M}$ using the rapid decay homology theory introduced by Bloch-Esnault. For a particular kind of $\mathcal{D}_{\mathbb{V}}$-module, so-called elementary, Hien-Sabbah gave a more explit description. Using the Riemann-Hilbert correspondence of Deligne-Malgrange, they introduced a topological local Laplace transformation at the level of Stokes-filtered local systems, and computed it in terms of Cech cohomology.

A different point of view to the study of the Stokes phenomena is given by the Riemann-Hilbert correspondence, as stated by D'Agnolo-Kashiwara [1]. This associates to a holonomic $\mathcal{D}$-module the enhanced ind-sheaf of its enhanced solutions. Moreover, by functoriality, such correspondence interchanges Fourier-Laplace transform for holonomic $\mathcal{D}$-modules with Fourier-Sato transform for enhanced ind-sheaves. Using this point of view, D'Agnolo-Hien-Morando-Sabbah explicitly computed the Stokes structure of $\mathcal{M}^{\wedge}$, for $\mathcal{M}$ regular holonomic.

In this thesis, using this same point of view, our aim is to get a description of the Fourier-Laplace transform of an elementary $\mathcal{D}_{\mathbb{V}}$-module. Unlike Hien-Sabbah, our approach is purely topological. Like D'Agnolo-Hien-Morando-Sabbah, it is based on computations in terms of Borel-Moore homology classes. For that, we choose the most natural classes, namely those attached to steepest descent cycles.

References

[1] Andrea D'Agnolo and Masaki Kashiwara. (2016). *Riemann-Hilbert correspondence for holonomic D-modules*. Publ. Math. Inst. Hautes Études Sci. **123**, 69–197

[2] Andrea D'Agnolo, Marco Hien, Giovanni Morando and Claude Sabbah. (2017). *Topological computation of some Stokes phenomena on the affine line*, `arXiv:1705.07610`

[3] Marco Hien and Claude Sabbah. (2014). *The local Laplace transform of an elementary irregular meromorphic connection.* Rend. Semin. Mat. Univ. Padova **134**, 133–196.

[4] Bernard Malgrange. (1991). *Équations différentielles à coefficients polynomiaux*, Birkhauser, Advances in Mathematics, 232 pp.

[5] Takuro Mochizuki. (2010). *Note on the Stokes structure of Fourier transform.* Acta Math. Vietnam. **35**, no. 1, 107–158

[6] Takuro Mochizuki. (2018). *Stokes shells and Fourier transforms.* `arXiv:1808.01037`