TECHNISCHE UNIVERSITÄT
CHEMNITZ

Fakultät für Mathematik
Chemnitzer Mathematisches Colloquium

E i n l a d u n g

In der Reihe „Chemnitzer Mathematisches Colloquium“ der Fakultät für Mathematik der TU Chemnitz spricht

Herr Prof. Dr. Michael Eiermann (TU/BA Freiberg)

über das Thema

The State of the Forsythe-Motzkin Conjecture.

Der Vortrag findet am

Donnerstag, dem 24. Oktober 2019, um 16.00 Uhr im Raum B202, Reichenhainer Straße 70

statt.

Ich möchte Sie hiermit recht herzlich zu dieser Veranstaltung einladen. Das Kolloquium wird von Herrn Prof. Dr. Oliver Ernst geleitet.

Abstract:
In 1951, George E. Forsythe and Theodore S. Motzkin investigated the so-called s-*step optimum gradient method* for solving symmetric systems of linear algebraic equations. They were particularly interested in the special case of $s = 1$ which represents the well known method of steepest descent. In numerical experiments, they observed that the sequence of the associated error vectors is äsymptotically of period 2", showed this statement for systems of dimension $n = 3$ and conjectured that it holds for all n. It was Hirotugu Akaike who first proved this conjecture in 1959. Later, in 1968, Forsythe reconsidered the problem and generalized some (but by far not all) of Akaike's results from the case of $s = 1$ to the case of $s > 1$.

It turns out that the periodicity phenomenon observed by Forsythe and Motzkin is not really related to linear systems but arises when bases of Krylov subspaces are generated by the restarted Lanczos process. For any restart length $s \geq 1$, numerical experiments indicate that the sequence of the thereby generated s-dimensional Krylov subspaces is äsymptotically of period 2", i.e., alternates asymptotically between two s-dimensional spaces. Despite many attempts, this observation has not been proven so far.

Here, we consider the Forsythe-Motzkin conjecture, which, in its original version is a linear algebra problem, in the context of orthogonal polynomials for modified weight functions. We describe the current state of the conjecture und explain why we believe that it is a difficult problem.

Prof. Dr. Oliver Ernst
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